Corporate Taxation and the Distribution of Income

James R. Hines Jr.
University of Michigan and NBER

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ABSTRACT

Corporate taxation affects the distribution of income both by changing relative returns to capital and labor and by reducing the share of corporate activity in the economy. Corporate investments are safer and have more diversified ownership than noncorporate alternatives, so a tax-induced reduction in corporate activity contributes to income dispersion and thereby increases income inequality. The dispersion effect is so large that higher corporate taxes can be associated with greater income inequality even when the corporate tax burden falls entirely on capital owned disproportionately by the rich. Income dispersion created by a ten percent higher U.S. corporate tax rate increases by 0.8-2.2 percent the fraction of U.S. income received by the top one percent, which may more than offset the distributional effects of reducing average returns to capital.

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James R. Hines Jr.
Department of Economics
University of Michigan
343 Lorch Hall
611 Tappan Street
Ann Arbor, MI 48109-1220

jrhines@umich.edu

1. Introduction

The incidence of the corporate income tax is an important and complex subject, and a timely one too, with recent reforms such as the 2017 U.S. Tax Cuts and Jobs Act making significant changes to corporate taxation. Interest in the incidence of the corporate income tax dovetails with ongoing concerns over income distributions in high-income economies, since the taxation of corporate income may serve as a backdoor method of achieving tax progressivity. High-income individuals tend to own corporate shares, along with other forms of capital, so imposing greater tax burdens on corporations might implicitly tax the wealthy owners of corporations. As is now well understood, however, this possibility depends critically on certain general equilibrium aspects of the incidence of the corporate tax. While it is perhaps intuitive that the burden of corporate taxation would fall on capital owners, there are realistic settings in which greater corporate taxation depresses business demand for labor and thereby reduces market wages; and these effects can be so strong that labor bears all, or potentially even more than all, of the corporate tax burden.

This paper considers the effect of corporate taxation on the distribution of after-tax income, which requires a somewhat different perspective than the usual tax incidence calculation. Tax incidence evaluates the extent to which differently situated groups, typically defined on an ex ante basis, bear the burdens of subsequent tax changes. To the extent that taxation also affects the riskiness of economic activity, it will change the pattern of returns, and thereby alter the distribution of income. In particular, corporate taxation changes the nature of business organization and business ownership, which changes the ex post distribution of business returns. The purpose of this paper is to analyze the implications of endogenous business forms for the effects of the corporate tax on income distribution.

Since the publication of Harberger (1962) it has been clear that one of the important forces determining the incidence of the corporate tax is the effect of the tax in encouraging noncorporate business activity.[1] Higher corporate taxes discourage corporate activity, and in reducing corporate factor demand create greater opportunities for unincorporated businesses.

Harberger and subsequent analysts consider the effect of this reallocation of economic resources on market returns to labor and capital; and this reallocation has the potential (if the noncorporate sector is particularly capital-intensive) to impose considerable burdens on labor. But there is a second sense in which this reallocation affects the distribution of income: rising levels of noncorporate business activity have the potential to increase levels of idiosyncratic risk in the economy, thereby leading to greater disparities in economic outcomes.

There is growing evidence that, from the standpoint of individual investors, noncorporate business investments are significantly riskier than corporate investments. In part, this reflects the characteristics of the business activities that tend to be undertaken by unincorporated firms, and in part it reflects the nature of their ownership. U.S. investors in unincorporated business ventures generally incur much greater idiosyncratic risk than do investors in corporate businesses. Partnerships, including LLCs, must specify their owners in partnership agreements, making it difficult to have diversified ownership and costly and cumbersome to change ownership shares at all. S corporations must have 100 or fewer shareholders, all must be U.S. citizens or permanent residents, and all must hold stock with equal rights. C corporations, whose income is subject to the corporate tax, suffer from none of these restrictions (though they are subject to others), and as a result, can much more easily have diversified ownership. Due to their undiversified ownership, the returns received by owners of unincorporated businesses can be very risky, quite apart from the undoubtedly larger business risks that these smaller businesses tend to face.

The consequences of the risky profile of noncorporate investment are predictable: some noncorporate business owners are very successful, whereas others lose significant portions of their investments. Many who engage in uninicorporated business ventures experience returns similar to those available by playing the lottery. Of course what they do is very different than a lottery, in that noncorporate business investors are also typically active managers, and full time (or more) employees of their firms. Nonetheless, their returns can be highly uncertain.

[1] For ease of exposition firms not subject to the corporate tax are denoted "noncorporate," even though in the United States this category includes S corporations, which while corporations are not generally subject to a separate corporate-level layer of tax.

Higher levels of corporate taxation directly and indirectly encourage greater levels of investment in risky noncorporate business activity. This reallocation of economic activity represents substitution away from relatively safer economic forms and styles of business organization into those that offer high returns to some and low returns to others. When the corporate tax increases, McDonald's, Walmart, Ford, and Apple do not expand as much as they would otherwise, so there is greater opportunity for mom and pop restaurants, small stores, and any other start-up that can use productive factors not employed by corporations. As a consequence, there will be greater numbers of entrepreneurs who are highly successful and join the ranks of the rich, just as there will be greater numbers of unsuccessful business people. The result is to make the distribution of income less equal – and this effect can be so large that it exceeds the distributional effect of the corporate tax from reducing economic returns earned by high-income capital owners.

Section 2 of the paper reviews the incidence of the corporate tax and its implications for income distribution. Section 3 considers an example in which the burden of the corporate tax is borne entirely by capital owners, who have disproportionately high incomes, yet a higher corporate tax is associated with a less equal income distribution due to the dispersion of outcomes attributable to greater noncorporate investment. Section 4 generalizes the analysis of section 3 by considering the effect of corporate taxes on income distribution in a stylized model of the U.S. economy, identifying the extent to which encouragement of relatively risky business activity dampens or possibly even reverses the effect of corporate taxes on the concentration of higher incomes. Section 5 reviews empirical evidence on the nature of business activity and business risks in the United States. Section 6 is the conclusion.

2. Corporate Tax Burdens

The standard approach in evaluating the effect of corporate taxes on income distribution is to consider the extent to which the burden of taxation falls on different income groups in the population. It is perhaps natural to expect the burden of the corporate tax to be borne largely by high-income owners of corporate shares, but one of the contributions of Harberger (1962) was to point out that such an outcome would be generally inconsistent with capital market equilibrium,[2] which requires corporate and noncorporate investments to offer investors equivalent expected after-tax returns. In the Harberger (1962) model, higher corporate taxes increase the cost of corporate capital and therefore encourage corporations to substitute labor for capital, thereby depressing demand for capital generally – not merely corporate capital – and reducing its after-tax return in a closed economy. This implication is the basis of the current U.S. Treasury approach to distributing the burden of corporate taxes (Cronin et al., 2013), which is to attribute 82 percent of the burden to all capital income in the economy, and 18 percent to labor income.

Even in the closed economy framework of the Harberger (1962) model, however, induced intersectoral reallocations of resources can reduce or possibly eliminate any burden of the tax on capital owners. Corporate taxation increases the cost of producing corporate output, thereby raising output prices, depressing demand, and shifting output from the corporate sector of the economy to the noncorporate sector. This reallocation affects factor demands to the extent that factor input ratios differ between the corporate and noncorporate sectors of the economy. If the corporate sector of the economy has a lower capital/labor ratio than the noncorporate sector, then the introduction of a corporate tax shifts resources into the noncorporate sector and thereby increases the demand for capital. If this effect is large enough, then it has the potential to exceed in magnitude the countervailing impact of factor substitution, thereby implying that higher rates of corporate tax are associated with greater after-tax returns to capital – including capital invested in corporations. It would then follow that labor bears all, or even more than all, of the burden of the corporate tax in the form of lower real wages.[3]

Open economy considerations further complicate the simple intuition that capital owners bear the full burden of the corporate tax. As noted by Diamond and Mirrlees (1971), and applied to open economies by Gordon (1986), Kotlikoff and Summers (1987) and Gordon and Hines (2002), any source-based capital income tax falls entirely on fixed local factors – typically labor – in a setting with perfect capital mobility and product substitution. It follows from the assumption of perfect capital mobility that after-tax rates of return to capital cannot differ

[2] Though Auerbach (2006) notes that, in the presence of significant adjustment costs, capital markets will not equilibrate immediately, so values of corporate shares should decline in response to surprise announcements of higher corporate taxes.
[3] McLure (1975) and Kotlikoff and Summers (1987) offer reviews and further elaborations of the Harberger model.

between countries, so higher corporate tax rates must discourage investment and thereby drive up pretax rates of return to the point that after-tax returns remain equal. Since after-tax rates of return to local capital do not change with corporate tax changes, it must be the case that local labor and any other local factor whose location is fixed, the archetypal example being land, bear the full burden of corporate taxes. Additionally, corporate taxes in large open economies may have spillover effects. With fixed world supplies of capital, higher tax rates in one country discourage local investment and drive investment to other countries, where it can reduce rates of return to capital and increase wages.

Harberger (1995, 2006, 2008) and Randolph (2006) explore the sensitivity of conclusions drawn from models of perfect capital mobility and fixed world capital supplies. They calibrate models that incorporate what they argue are realistic estimates of relative capital intensities, capital mobility, and product substitutability, finding that labor is apt to bear a significant fraction of the burden of the corporate tax. In the classes of simple models used in these papers, any imperfect substitutability between foreign and domestic traded goods effectively operates as a form of imperfect capital mobility. As the trade and capital accounts must balance, imperfect substitutability between traded goods implies that extensive net borrowing is expensive as it entails importing large volumes of foreign goods for which there is diminishing marginal substitution. Gravelle and Smetters (2006) use a computable general equilibrium model to allow for subtler variants of imperfect product competition, concluding that corporate capital owners may bear the lion's share of the corporate tax burden despite the availability of capital inflows and outflows.[4]

Empirical studies of corporate tax incidence, including Felix and Hines (2009), Arulampalam et al. (2012), Altshuler and Liu (2013), Hasset and Mathur (2015), Suarez Serrato and Zidar (2016), and Fuest, Peichl, and Siegloch (2018), commonly consider the extent to which higher corporate taxes influence wages, thereby indirectly assessing the incidence of the corporate tax. Felix and Hines (2009) analyze the effects of higher state corporate taxes on wage premiums earned by unionized workers, concluding that workers in fully unionized firms capture 54 percent of the benefits of low tax rates. Arulampalam et al. (2012) compare wages paid by

[4] For more on the importance of product substitutability, see Davidson and Martin (1985) and Gravelle and Kotlikoff (1989, 1993).

firms with differing tax obligations, concluding that about half of the corporate tax is passed on to labor. Liu and Altshuler (2013) compare wages paid by firms in U.S. industries subject to differing levels of taxation, and after adjusting for differing industry concentrations, report that wages absorb 60-80 percent of the corporate tax burden. Hassett and Mathur (2015) find that economies with higher corporate tax rates tend to have lower wages, though Clausing (2013) and Gravelle (2013) call attention to contrary evidence and note that the implied effects of corporate taxes may be implausibly large. Suarez Serrato and Zidar (2016) estimate a model of the effect of U.S. state corporate taxes with imperfect labor and firm mobility, reporting coefficients that imply that firm owners bear 40 percent of the corporate tax burden, workers bear one-third, and landowners the rest. Fuest, Peichl and Siegloch (2018) estimate the effect of subnational German corporate taxes, finding that workers bear roughly half of the tax burden. And Nallareddy, Rouen and Suarez Seratto (2019) find that state corporate tax cuts are associated with greater state after-tax income dispersion.

It appears from this evidence that workers may bear substantial portions of corporate tax burdens in the form of lower real wages, with capital owners also bearing significant burdens. There remain open questions about the distribution of these tax burdens among richer and poorer workers and capital owners, particularly when economic activities have uncertain returns. A small literature extends standard corporate tax incidence models to incorporate economic uncertainty,[5] but its focus remains on the effect of corporate taxes on expected returns to labor and capital. In settings with economic uncertainty, expected returns are ex ante concepts, whereas the income distribution is an ex post realization. Consequently, in order to understand the effect of corporate taxes on the income distribution it is necessary to consider its effects on the distribution of realized outcomes.

3. *Possibility of Second-Order Stochastic Dominance*

This section explores an avenue by which investment return riskiness can interact with the tax system to influence the distribution of income. The model considers a simple setting in which corporate taxation does not change expected pre-tax investment returns, so the burden of corporate income taxation falls entirely on capital owners. There are just two types of people, rich and poor, with the factor endowment of the rich relatively capital-intensive. In the absence of uncertainty, higher corporate income taxes that generate revenue used to reduce labor income taxes would reduce income inequality by imposing greater tax burdens on high-income individuals whose income is largely the return to capital, and reduced tax burdens on low-income individuals whose incomes derive mostly from labor. But if higher corporate taxes also encourage the expansion of risky unincorporated businesses, then higher corporate taxes may increase income inequality.

The two types are denoted *A* and *B*, with *A* poor and *B* rich, and equal numbers of each. Both types are endowed with fixed supplies of capital and labor, with individuals of type *A* relatively more heavily endowed with labor, and individuals of type *B* (therefore necessarily) relatively more heavily endowed with capital. Capital and labor can move freely between the corporate and noncorporate sectors, but (counterfactually) cannot cross national borders, so the economy's total capital stock and total labor supply are fixed. Capital and labor markets are perfectly competitive, and individuals and firms are risk-neutral, in that they evaluate after-tax investment returns based on their expected values. Corporate and noncorporate firms produce identical outputs.

Appendix A analyzes the effects of corporate taxation on expected after-tax returns in this simple model. Holding the government's budget constraint fixed, higher corporate tax rates finance a reduction in labor income taxes, the net effect of which is to increase the expected after-tax incomes of type *A* individuals from $\hat{y}_A$ to $\hat{y}'_A$, and to reduce the expected after-tax incomes of type *B*s from $\hat{y}_B$ to $\hat{y}'_B$. Figure 1 plots the distribution of income in the economy and accompanying Lorenz curve prior to an increase in the corporate tax, and Figure 2 plots the same schedules for expected incomes (denoted $\hat{y}'_A$ and $\hat{y}'_B$) after introduction of the corporate tax.

In addition to affecting expected incomes, corporate taxes affect forms of business ownership by discouraging corporate investments and thereby implicitly encouraging noncpporate investments. Suppose that noncorporate investments earn the same expected pretax rate of return, *r*, earned by corporate investments, but are taxed at the ordinary income tax

[5] See, for example, Batra (1975), Ratti and Shome (1977), and Baron and Forsythe (1981).

rate t rather than the corporate tax rate τ. Workers in the noncorporate sector receive the same wages as do workers in the corporate sector. An individual i choosing to invest in an unincorporated business puts all of his or her potential income at risk, taking a fair gamble in which with probability 0.5 the investment is successful, increasing after-tax income by a fraction c, and with probability 0.5 the investment is unsuccessful, reducing after-tax income by the same fraction c. Investors evaluate their corporate and noncorporate options based on expected returns plus a psychic return of $K_i\beta_i$ if the investor chooses a noncorporate investment, in which K_i is investor i's capital endowment, and β_i is a preference parameter. β_i can be positive or negative, reflecting the joys and frustrations of participating in a risky small business. As a result, the net expected return difference between noncorporate and corporate investment is:

(1) $$K_i\left[r\left(\tau - t\right) + \beta_i\right].$$

Investors for whom $r\left(\tau - t\right) + \beta_i > 0$ will choose noncorporate investments, and those for whom $r\left(\tau - t\right) + \beta_i < 0$ will choose corporate investments.

Assuming that types A and B share the same distribution of β_is, it follows that there will be equal numbers of type A noncorporate investors and type B noncorporate investors. If prior to a tax change $r\left(\tau - t\right) + \beta_i \leq 0$ for every β_i value in the population, then there will be no noncorporate investment. A higher corporate tax depresses after-tax corporate returns and encourages capital to flow into noncorporate investments, with these investments coming from individuals with the highest β_i parameters. If after the tax increase $r\left(\tau - t\right) + \beta_i > 0$ for some individuals, then they will switch from corporate to noncorporate investments as a result.
In order to simplify the resulting analysis, it is useful to consider the case in which $\tau < t$ prior to the tax change, and the change is one that makes these two tax rates equal.[6]

Figure 3 plots the effect of a higher corporate tax rate on the distribution of income. The higher corporate tax rate reduces the expected incomes of type B individuals from $\hat{y}_B$ to $\hat{y}'_B$, but a fraction p of these individuals will be induced to invest in noncorporate businesses. Of these type B noncorporate investors, half will be successful, with resulting after-tax incomes $\hat{y}'_B(1+c)$, and half will be unsuccessful, with resulting after-tax incomes $\hat{y}'_B(1-c)$. Consequently, the tax increase changes the income distribution so that a fraction $p/2$ of the population has incomes of $\hat{y}'_B(1+c)$. This increases the concentration of income at the very top if $\hat{y}'_B(1+c) > \hat{y}_B$, which requires that

(2) $$c > \frac{\hat{y}_B - \hat{y}'_B}{\hat{y}'_B}.$$

One of the consequences of a higher corporate tax rate is to increase income dispersion among top income earners, which has the effect of increasing the concentration of realized income at the very top. Condition (2) identifies circumstances in which tax-induced greater dispersion is of sufficient magnitude that, from the standpoint of the top $p/2$ fraction of income earners, it more than offsets the effect of the tax change in reducing expected incomes. If condition (2) holds, then the corporate tax change increases the incomes of the top $p/2$ fraction of income earners, even though the burden of the corporate tax is fully borne by owners of capital. It is noteworthy that (2) does not depend on the value of p, so this effect appears for any degree of tax-induced shifting between corporate and noncorporate investment.

Higher corporate tax rates for which (2) holds, and which therefore increase the concentration of top incomes, may also make the entire income distribution less equal in the sense of second-order stochastic dominance. The greater income dispersion induced by a higher corporate tax rate raises this possibility; but another necessary (and as it happens, sufficient) condition for second-order stochastic dominance is that the tax change reduces the aggregate incomes of those whose realized after-tax incomes fall below the median. Since $\hat{y}'_A > \hat{y}_A$ and investments are fair gambles, it cannot be the case that a higher corporate tax rate reduces low incomes if all of those with below-median realized incomes are of type A. Hence a necessary condition for a tax-induced reduction in below-median incomes is that unsuccessful type B

[6] The purpose of assuming that $\tau = t$ after the tax change is that the tax-induced changes in the share of corporate investment then do not affect total tax collections. The example readily generalizes to cases in which $\tau \neq t$.

noncorporate investors have lower realized incomes than successful type *A* noncorporate investors. This is the scenario depicted in Figure 3.

The tax change reduces average incomes below the median if:

(3) $$\frac{p}{2}\left[\hat{y}_A'(1-c)+\hat{y}_B'(1-c)\right]+(1-p)\hat{y}_A' < \hat{y}_A .$$

The left side of (3) reflects that the lower half of the after-tax income distribution consists of type *A* individuals who make corporate investments, and type *A* and *B* individuals who make unsuccessful noncorporate investments. The right side of (3) is simply the average below-median income before the tax change. Condition (3) implies:

(4) $$\hat{y}_A' - \hat{y}_A < \frac{p}{2}\left[c\left(\hat{y}_A' + \hat{y}_B'\right) - \left(\hat{y}_B' - \hat{y}_A'\right)\right].$$

Since by construction $\hat{y}_A' > \hat{y}_A$, the right side of (4) must be positive in order for (4) to be satisfied. This requires that

(5) $$\hat{y}_A'(1+c) > \hat{y}_B'(1-c),$$

which is simply the condition that unsuccessful type *B* noncorporate investors have lower realized incomes than successful type *A* noncorporate investors. And (4) also requires that

(6) $$\hat{y}_A > \hat{y}_A'(1-c) + \left[\hat{y}_A c(1-2p) + p(1-c)\left(\hat{y}_B' - \hat{y}_A'\right)\right].$$

Since the term in brackets on the right side of (6) is positive, it follows that (4) implies that $\hat{y}_A'(1-c) < \hat{y}_A$, so a higher corporate tax widens the left tail of the income distribution. A condition analogous to (6) shows that $\hat{y}_B'(1+c) > \hat{y}_B$ is also an implication of (4).

If condition (4) is satisfied then higher corporate taxes reduce below-median incomes. Since higher corporate taxes also induce greater income dispersion, the resulting income distribution is then dominated in a second-order stochastic sense by the original income distribution. Condition (4) indicates that this rather strong sense in which higher corporate taxes makes the income distribution less equal will materialize only for sufficiently high values of p and c. If the corporate tax has little effect on levels of noncorporate activity, then the effect of the corporate tax on expected returns will dominate its impact on the distribution of income, and higher corporate tax rates are associated with greater equality. But if the corporate tax has a large effect on noncorporate activity, and if the returns to noncorporate businesses are sufficiently risky, then it is possible that a higher corporate tax makes after-tax incomes less equal.

Figure 4 plots the potential effects of higher corporate taxes on the economy's Lorenz curve. The diagram in the top panel of Figure 4 depicts the Lorenz curve for the case in which noncorporate activity is sufficiently risky, and there is a sufficient effect of corporate taxation on noncorporate activity, that condition (4) holds. In this instance the income distribution after introduction of the corporate tax is less equal than the initial income distribution in the sense of second-order stochastic dominance – even though the burden of the corporate tax is 100 percent on capital, which tends to be owned by the rich. This surprising possibility reflects that after introduction of the corporate tax a portion of the low-income part of the population consists of unsuccessful business owners. An alternative possibility appears in the diagram presented in the bottom panel of Figure 4. In the scenario depicted in this diagram condition (4) does not hold, so the Lorenz curves for the economy before and after the higher corporate tax intersect at multiple points. In this example, the corporate tax stimulates greater income dispersion at very low and very high incomes, but also narrows income disparities for the large part of the population that does not make risky investments. How one evaluates this change in income distribution then depends on weights attached to outcomes of individuals with differing incomes.

4. Corporate Taxation and the Concentration of High Incomes

It is possible to generalize the implications of the example in section 3, though any more general case immediately requires a method of assessing income distributions. For this purpose, it is instructive to consider the concentration of national income in the hands of the most wealthy individuals.

It simplifies matters to assume that individuals in the population differ in a scalar characteristic θ that can be interpreted as individual factor endowments and other unchanging features relevant to income production. The values of θ are continuously distributed with a cumulative density given by $F(\theta)$, and accompanying marginal density $dF(\theta)$. The total population is normalized to one. The returns to corporate investments are certain, and the after-tax income of someone who chooses to invest in corporations, and not in noncorporate business, is a function simply of θ and of the corporate tax rate. For notational ease this after-tax income is denoted $y(\theta)$, interpreted as being evaluated at the current corporate tax rate, and consists of both capital and labor returns. Furthermore, units are chosen so that $y(\theta) = \theta$ at initial values.

Noncorporate investment is a probabilistic gamble, and entails significant commitment: an individual of type θ whose capital and labor resources would have provided an income $y(\theta)$ if the capital were invested in corporations, and who instead invests in an unincorporated business that succeeds, receives an income of $(1+k)\,y(\theta)$; the same individual, if the noncorporate investment is unsuccessful, receives an income of $(1-m)\,y(\theta)$. Assume that all noncorporate investors are successful with probability ϕ and unsuccessful with probability $(1-\phi)$; furthermore, the stochastic aspect of these investments for simplicity is taken to be a fair gamble, so

(7) $$k\phi = m(1-\phi).$$

The restriction that an investor cannot lose more than he or she has implies that $m \leq 1$, which together with (7) implies

(8) $$k \leq \frac{(1-\phi)}{\phi}.$$

Individuals choose between corporate and noncorporate investments on the basis of idiosyncratic preference parameters, as in the model of section 3. These preference parameters have identical distributions at each value of θ, so the fraction of the population investing in noncorporate businesses, still denoted *p*, is the same for all income groups.

4.1. Aggregate high income.

Focusing attention just on the aggregate amount of income (ψ) earned by those with incomes exceeding the high level $\bar{y}$, it follows that

$$\text{(9)} \qquad \psi = (1-p)\int_{\theta_2}^{\infty} y(\theta)\,dF(\theta) + p\int_{\theta_3}^{\infty} y(\theta)\,dF(\theta) + p\phi\int_{\theta_1}^{\theta_3} y(\theta)(1+k)\,dF(\theta),$$

with $y(\theta_1) = \bar{y}/(1+k)$, $y(\theta_2) = \bar{y}$, and $y(\theta_3) = \bar{y}/(1-m)$. The first term on the right side of (9) reflects that a portion $(1-p)$ of the population has incomes given by $y(\theta)$, so the first term is just the sum of the incomes of this portion whose incomes exceed $\bar{y}$. The second term reflects that the aggregate income earned by those with $y(\theta)$ values exceeding $\bar{y}/(1-m)$ is unaffected by the zero-sum randomizations introduced by noncorporate investment, since even those individuals whose noncorporate investments are unsuccessful have incomes no less than $\bar{y}$. The third term reflects that among those with incomes between $\bar{y}/(1+k)$ and $\bar{y}/(1-m)$, and who make noncorporate investments, only a fraction ϕ wind up with after-tax incomes exceeding $\bar{y}$. Rewriting (9) produces

$$\text{(10)} \quad \psi = \int_{\theta_2}^{\infty} y(\theta)\,dF(\theta) + p\left[\phi\int_{\theta_1}^{\theta_2} y(\theta)\,dF(\theta) + k\phi\int_{\theta_1}^{\theta_3} y(\theta)\,dF(\theta) - (1-\phi)\int_{\theta_2}^{\theta_3} y(\theta)\,dF(\theta)\right].$$

In order to evaluate (10) it is necessary to use a cumulative density function. There is extensive evidence that the distribution of high incomes in the United States closely resembles a Pareto distribution,[7] which, if applied to incomes earned by those without noncorporate investments, would imply that:

$$\text{(11)} \qquad dF(\theta) = \frac{\gamma}{\theta^{1+\alpha}}\,d\theta,$$

with γ a parameter that is generally a function of the income distribution, and $\alpha > 1$ the Pareto parameter. Since observed income distributions include earnings from noncorporate

[7] See, for example, Atkinson, Piketty, and Saez (2011), Jones (2015), Aoki and Nirei (2016), and Jones and Kim (2018).

investments, the right side of (11) may be just an approximation to the corresponding U.S. distribution, though Toda (2014), Jones (2015), and Nirei and Aoki (2016) offers reasons to expect most processes that generate high incomes to have Pareto distributions resembling (11). Taking (11) to apply in the range $\left[\frac{\bar{y}}{(1+m)},\infty\right]$, and imposing that $y(\theta)=\theta$, it follows that (10) implies:

$$(12)\ \psi=\Theta_{\bar{y}}+p\frac{\gamma}{(1-\alpha)}\bar{y}^{1-\alpha}\phi\left\{1-\frac{1}{(1+k)^{1-\alpha}}+\frac{k}{(1-m)^{1-\alpha}}-\frac{k}{(1+k)^{1-\alpha}}-\frac{(1-\phi)}{\phi}\left[\frac{1}{(1-m)^{1-\alpha}}-1\right]\right\},$$

in which $\Theta_{\bar{y}}$ is the aggregate value of $y(\theta)$ in the population with $\theta\geq\bar{y}$.

Imposing (7) and simplifying produces:

$$(13)\qquad \psi=\Theta_{\bar{y}}+p\frac{\gamma}{(1-\alpha)}\bar{y}^{1-\alpha}\left[1-(1+k)^{\alpha}\phi-(1-\phi-k\phi)^{\alpha}(1-\phi)^{1-\alpha}\right].$$

The expression on the right side of (13) can be further simplified by noting that $y(\theta)=\theta$, from which it follows that:

$$(14)\qquad \Theta_{\bar{y}}=\int_{\theta_2}^{\infty}\theta\frac{\gamma}{\theta^{1+\alpha}}d\theta=\frac{\gamma}{(\alpha-1)}\bar{y}^{1-\alpha}.$$

Combining (13) and (14),

$$(15)\qquad \psi=\Theta_{\bar{y}}(1+p\Delta),$$

with

$$(16)\qquad \Delta\equiv(1+k)^{\alpha}\phi+(1-\phi-k\phi)^{\alpha}(1-\phi)^{1-\alpha}-1.$$

Several features of ψ are evident from (15) and (16), including that if $p=0$, $k=0$, or $\phi=0$, then $\psi=\Theta_{\bar{y}}$: under any of these circumstances, there is no income uncertainty associated with noncorporate business activity, and the income distribution is determined by the distribution of θ. If $k > 0$, $\phi > 0$, and $p > 0$, so that there is meaningful noncorporate business activity, then (15) and (16) imply that ψ is increasing in p and k (proofs appear in Appendix B), reflecting that Δ is both positive and increasing in *k*. Furthermore, Δ is increasing in α, as also demonstrated in Appendix B. Greater levels of noncorporate business activity increase aggregate high incomes, since the wealthy population consists disproportionately of those whose risky business ventures succeed, as in the model analyzed in section 3. Greater noncorporate business riskiness as reflected in higher values of *k* (and accompanying implied greater values of *m*) similarly increases aggregate high incomes, and for the same reasons. And income disparities have greater effects on income concentration at higher values of α that reflect the narrowing of the income distribution at high levels.

Table 1 presents values of Δ for different values of *k* and ϕ, and with $\alpha = 1.67$, which Jones (2015) notes is the consensus value among empirical studies of the U.S. economy. For a given value of *p*, higher values of *k* are associated with greater concentrations of high incomes, as are higher values of ϕ. If $k = 3$ and $\phi = 0.2$, so that a noncorporate investor has a 20 percent chance of quadrupling his or her income, then the table entry 1.10 implies that the aggregate income of the population with incomes above $\bar{y}$ is $\Theta_{\bar{y}}(1+1.1p)$. If in this circumstance ten percent of the population undertakes noncorporate business activity, then the concentration of high income is 11 percent higher than it would be otherwise. The choices of *k* and ϕ in Table 1 are constrained by the implied relationship between gains and losses, as expressed in the requirement given by (8). A representative parameter combination for highly risky investments in the U.S. economy might be $k = 5$ and $\phi = 0.1$, with noncorporate business investors having ten percent chances of large (500 percent) returns, and associated 90 percent chances of losing half their incomes. Under those circumstances, the aggregate income of the population with incomes above $\bar{y}$ would be $\Theta_{\bar{y}}(1+1.23p)$. A representative parameter combination for less risky investments might be $k = 2$ and $\phi = 0.2$, in which case the aggregate income of the population with incomes above $\bar{y}$ would be $\Theta_{\bar{y}}(1+0.5p)$.

In order to evaluate the effect of changes in the corporate tax rate it is necessary to differentiate (10) with respect to the corporate tax rate, denoted τ. Taking changes in τ not to affect the variable components of returns to noncorporate investments (i.e., *k* and *m*), this differentiation yields:

$$\frac{d\psi}{d\tau} = \frac{d\psi^{*}}{d\tau} + \frac{d\psi^{**}}{d\tau} + \frac{d\psi^{***}}{d\tau}, \tag{17}$$

in which:

$$\frac{d\psi^{*}}{d\tau} = \int_{\theta_2}^{\infty} \frac{dy}{d\tau} dF(\theta) - (1-\phi) p \int_{\theta_2}^{\theta_3} \frac{dy}{d\tau} dF(\theta) + p\phi \int_{\theta_1}^{\theta_2} \frac{dy}{d\tau} dF(\theta) + kp\phi \int_{\theta_1}^{\theta_3} \frac{dy}{d\tau} dF(\theta) \tag{18a}$$

(18b)

$$\frac{d\psi^{**}}{d\tau} = -p(1-\phi)\frac{d\theta_3}{d\tau}\left(\frac{\bar{y}}{1-m}\right)^{1-\alpha} dF(\theta_3) - (1-p)\frac{d\theta_2}{d\tau}\bar{y}^{1-\alpha} dF(\theta_2) - p\phi\frac{d\theta_1}{d\tau}\left(\frac{\bar{y}}{1+k}\right)^{1-\alpha} dF(\theta_1)$$

$$\frac{d\psi^{***}}{d\tau} = \frac{dp}{d\tau}\Theta_{\bar{y}}\Delta. \tag{18c}$$

Equation (17) has three components, the first of which, $\frac{d\psi^{*}}{d\tau}$ as expressed in (18a), takes a form that is close to standard in the analysis of corporate tax incidence, incorporating the effects of corporate tax rate changes on labor and capital returns. There are some nonstandard elements in $\frac{d\psi^{*}}{d\tau}$, reflecting the uncertain aspects of noncorporate investment returns, but by and large the effects of corporate tax changes on high income concentrations as captured in (18a) depends on its effect on pretax factor returns. It is possible to simplify matters by assuming that higher corporate taxes reduce all higher incomes proportionately, so that $\frac{dy}{d\tau} = -\lambda\theta$. Making this substitution, and applying (11), (14) and (15), (18a) then implies:

$$\frac{d\psi^{*}}{d\tau} = -\lambda\psi. \tag{19a}$$

The second component of equation (17), $\frac{d\psi^{**}}{d\tau}$, arises because tax changes affect the critical values of θ that determine whose income equals or exceeds $\bar{y}$. Equation (18b) can be simplified by noting that $\frac{d\theta_i}{d\tau}+\frac{dy(\theta_i)}{d\tau}=0$, which implies $\frac{d\theta_i}{d\tau}=-\frac{dy(\theta_i)}{d\tau}$. Again setting $\frac{dy(\theta_i)}{d\tau}=-\lambda\theta_i$, and applying (11), (14) and (15), (18b) becomes:

(19b) $$\frac{d\psi^{**}}{d\tau}=-\lambda(\alpha-1)\psi.$$

The third component of the right side of (17), $\frac{d\psi^{***}}{d\tau}$, is not standard: it is the effect of a change in noncorporate investment on the concentration of high incomes. Notably, if $\frac{dp}{d\tau}=0$ then, from (18c), $\frac{d\psi^{***}}{d\tau}=0$. And since $\Delta>0$, $\frac{d\psi^{***}}{d\tau}$ takes the same sign as $dp/d\tau$: higher corporate taxes that encourage noncorporate business activity thereby increase aggregate high incomes. Furthermore, a positive value of $dp/d\tau$ makes $\frac{d\psi^{***}}{d\tau}$ increasing in *k*, so that greater noncorporate business riskiness is associated with a bigger effect of noncorporate activity on the concentration of high incomes.

The effect of tax-change-induced greater noncorporate business activity on the concentration of high incomes is given by the product of $\frac{dp}{d\tau}\Theta_{\bar{y}}$ and the appropriate entry in Table 1. Taking 1.23 to be a representative high-risk Table 1 entry (corresponding to $k=5$ and $\phi=0.1$), it follows that $\frac{d\psi^{***}}{d\tau}=1.23\frac{dp}{d\tau}\Theta_{\bar{y}}$. If a ten percent higher corporate income tax rate is associated with risky noncorporate investments being undertaken by an additional three percent of the population, then this mechanism increases the concentration of high incomes by 37 percent of what would have been predicted assuming that all rich individuals invested in corporations. Instead using a lower-risk representative entry from Table 1 (0.50, corresponding to $k = 2$ and $\phi = 0.2$), the tax change by increasing noncorporate investment increases the concentration of high incomes by 15 percent of these predicted levels.

It is also useful to express the magnitudes of these effects relative to realized total high incomes. Since $\psi = \Theta_{\bar{y}}\left(1+p\Delta\right)$, (18c) can be expressed as:

(19c) $$\frac{d\psi}{d\tau}^{***} = \frac{dp}{d\tau}\frac{1}{\left[\frac{1}{\Delta}+p\right]}\psi .$$

Applying equation (19c) to the earlier example of high-risk noncorporate investment ($k = 5$ and $\phi = 0.1$), and using parameter values $\frac{dp}{d\tau} = 0.3$ and $p = 0.22$, it follows that $\frac{d\psi}{d\tau}^{***} = \left(0.29\right)\psi$. Instead applying $\frac{dp}{d\tau} = 0.3$ and $p = 0.30$ to the representative case of lower-risk noncorporate investment ($k = 2$ and $\phi = 0.2$) produces $\frac{d\psi}{d\tau}^{***} = \left(0.13\right)\psi$. In both cases, the greater noncorporate business activity produced by higher corporate tax rates significantly increases the concentration of high incomes.

Equations (19a), (19b), and (19c) together imply that the net effect of corporate tax changes on aggregate high incomes is:

(20) $$\frac{d\psi}{d\tau} = \left\{\frac{dp}{d\tau}\frac{1}{\left[\frac{1}{\Delta}+p\right]} - \alpha\lambda\right\}\psi .$$

As captured in (20), the net effect of higher corporate taxes on aggregate incomes received by high-income individuals depends on a comparison of two effects, the first of which arises from greater noncorporate business activity, and the second of which is more standard. In the representative cases of high-risk and low-risk noncorporate investment, a ten percent higher corporate tax rate increases aggregate high incomes by 1.3-2.9 percent. If, in a standard incidence calculation with no investment return uncertainty, a ten percent higher corporate income tax rate would reduce typical top incomes by 1.5 percent,[8] then given that $\alpha = 1.67$, this standard aspect of higher corporate taxes would reduce aggregate high incomes by 2.5 percent. Consequently, in these examples, the two forces are of roughly the same magnitudes, suggesting that the distributional effect of tax-induced greater noncorporate business activity largely offsets, and even may exceed, the distributional effect of after-tax price changes.

4.2. Population with high incomes.

The components of equation (17) capture the effects of corporate tax changes on the aggregate income earned by those whose incomes are $\bar{y}$ or greater. This is not exactly the same as the effect of corporate tax changes on the concentration of income earned by, say, the top one percent of the income distribution, since corporate tax changes also affect the number of people whose incomes exceed $\bar{y}$. In order to evaluate the effect of the corporate tax on the concentration of income it is necessary to adjust for these changing populations. And it is of independent interest to know the effect of tax changes on numbers of high-income individuals.

The population of individuals with incomes exceeding $\bar{y}$ is:

(21) $$n = \int_{\theta_3}^{\infty} dF(\theta) + (1-p)\int_{\theta_2}^{\theta_3} dF(\theta) + p\phi\int_{\theta_1}^{\theta_3} dF(\theta).$$

Differentiating (21) produces:

(22) $$\frac{dn}{d\tau} = \frac{dn}{d\tau}^{*} + \frac{dn}{d\tau}^{**},$$

in which:

(23a) $$\frac{dn}{d\tau}^{*} = -p(1-\phi)\frac{d\theta_3}{d\tau}dF(\theta_3) - (1-p)\frac{d\theta_2}{d\tau}dF(\theta_2) - p\phi\frac{d\theta_1}{d\tau}dF(\theta_1)$$

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[8] The U.S. Treasury assigns 82 percent of the corporate tax burden to capital owners, which together with other aspects of its methodology implies that the burden of 43 percent of corporate taxes are attributed to individuals in the top one percent. This method is inconsistent with prevailing models and evidence, so a more reasonable figure might be half its magnitude, or 22 percent. In 2016 the top one percent of the U.S. income distribution had \$1,465b of income after federal taxes (IRS Statistics of Income), so if a ten percent higher corporate tax rate corresponds to an additional burden of \$100b, the \$22b attributable to the top one percent would represent 1.5 percent of its after-tax income.

(23b) $$\frac{dn^{**}}{d\tau} = \frac{dp}{d\tau}\left[\phi\int_{\theta_1}^{\theta_3} dF(\theta) - \int_{\theta_2}^{\theta_3} dF(\theta)\right].$$

Equation (23a) can be simplified by noting that $\frac{d\theta_i}{d\tau} + \frac{dy(\theta_i)}{d\tau} = 0$, which implies $\frac{d\theta_i}{d\tau} = -\frac{dy(\theta_i)}{d\tau}$. Setting $\frac{dy(\theta_i)}{d\tau} = -\lambda\theta_i$, so that a higher corporate tax rate reduces all high incomes by the same proportion λ, and applying (7), (11), (14), and (15), (23a) becomes:

(24a) $$\frac{dn^{*}}{d\tau} = -\lambda\frac{(\alpha-1)}{\overline{y}}\psi.$$

Similarly evaluating (23b) using (7), (11), (14), and (15) yields:

(24b) $$\frac{dn^{**}}{d\tau} = \frac{dp}{d\tau}\frac{\Theta_{\overline{y}}}{\overline{y}}\frac{(\alpha-1)}{\alpha}\Delta.$$

Equations (24a) and (24b) characterize the two countervailing effects of higher corporate tax rates on numbers of high-income individuals. To the extent that higher corporate taxes reduce high incomes, they reduce the density of individuals above high-income thresholds, as reflected in (24a). Equation (24b) captures the effect of higher corporate taxes in encouraging noncorporate business activity and thereby increasing the numbers of successful high-income investors, which works in the opposite direction. The sign of $dn/d\tau$ depends on the relative magnitudes of these two effects, which in turn depend on λ and $dp/d\tau$. If higher corporate tax rates sharply reduce top incomes, then the numbers of high-income individuals are also likely to decline; whereas if higher corporate tax rates significantly increase risky noncorporate business activity, then the population of high-income individuals is for that reason likely to rise.

One way to gauge the net effect of corporate taxes on numbers of high-income individuals is to combine (18c), (24a) and (24b) to obtain:

(25) $$\frac{dn}{d\tau} = \frac{(\alpha-1)}{\alpha\overline{y}}\left[\frac{d\psi^{***}}{d\tau} - \alpha\lambda\psi\right] = \frac{(\alpha-1)}{\alpha\overline{y}}\frac{d\psi}{d\tau}.$$

It is clear from (25) that $\frac{dn}{d\tau}$ has the same sign as $\frac{d\psi}{d\tau}$: if a higher corporate tax rate reduces aggregate income earned by high-income individuals, then it also reduces their numbers. Indeed, reducing the number of high-income individuals is part of the reason why higher corporate tax rates might reduce the aggregate income received by this top group. The effect may also go the other way: with a sufficiently strong relative effect of corporate taxes on risky noncorporate activity, aggregate top income, and also the population of high-income individuals, can increase at higher tax rates.

4.3. Concentration of high incomes.

Letting Ω denote total income earned by a fixed percentage of the income distribution corresponding to incomes of at least $\bar{y}$ prior to the tax change, it follows that

(26)
$$\frac{d\Omega}{d\tau} = \frac{d\psi}{d\tau} - \bar{y}\frac{dn}{d\tau}.$$

Combining (18c), (19a), (19b), (25) and (26) produces:

(27)
$$\frac{d\Omega}{d\tau} = \alpha\lambda\psi + \frac{dp}{d\tau}\frac{1}{\alpha}\Theta_{\bar{y}}\Delta.$$

And applying the same logic used to derive (19c), (27) implies:

(28)
$$\frac{d\Omega}{d\tau} = \left\{\frac{dp}{d\tau}\frac{1}{\alpha\left[\frac{1}{\Delta}+p\right]} - \lambda\right\}\psi.$$

Equation (28) indicates that the effect of a higher corporate tax rate on aggregate income of the top given percentage of the income distribution is the by now familiar net product of two competing forces: greater noncorporate business activity and changing after-tax factor returns. Equations (20) and (28) together imply that $\frac{d\Omega}{d\tau} = \frac{1}{\alpha}\frac{d\psi}{d\tau}$: the effect of a corporate tax change on aggregate income held by a given top percentage of the income distribution is smaller in magnitude than the effect of the same tax change on aggregate income held by those at or above the prior income threshhold for that percentage of the income distribution. Corporate tax changes affect aggregate high incomes in part by changing the population of high-income individuals, and adjusting for this population change dampens the resulting tax effect. Since for reasonable parameter values the two components of (28) are of comparable magnitudes, it follows that changes in the corporate tax rate may have little effect on the concentration of top incomes.

It is useful to consider the effects of tax-induced income dispersion over ranges of possible parameter values. Table 2 displays values of the first term on the right side of equation (28) - $\frac{dp}{d\tau}\frac{1}{\alpha\left[\frac{1}{\Delta}+p\right]}$ - for various values of ϕ, k, α, and p, in every case based on an assumed level of $dp/d\tau = 0.30$. The Table 2 entries are uniformly increasing in ϕ, k, and α, notably almost doubling as α increases from 1.5 to 1.9. The values in Table 2 decline as p rises, but do so very gradually. Taking $\alpha = 1.67$ and $p = 0.25$ to be baseline levels, the entries in Table 2 vary between 0.76 and 2.16, though these depend critically on population values of ϕ and k, about which little is known. As noted earlier, the relevant value of λ for equation (28) is arguably in the neighborhood of 1.5, so the Table 2 entries are generally of the same order of magnitude. This suggests that the income dispersion effects of higher corporate tax rates may roughly offset the standard factor return effects in influencing the concentration of high incomes.

5. Characteristics of Noncorporate Business Activity

The analysis in section 4 relies on a model of business characteristics that while rather stylized may nonetheless capture important elements of business practice in the United States and elsewhere. This section considers evidence from the United States of the factors that influence noncorporate business activity and the extent to which top income earners receive returns from unincorporated businesses.

There is extensive evidence that labor and capital returns to unincorporated business activities in the United States are considerably riskier than other economic alternatives. For example, Davis et al. (2007) document the significantly greater volatility and dispersion of growth rates of privately held firms than those of publicly held firms. In their analysis of tax return data, DeBacker, Panousi, and Ramnath (2018) find that the idiosyncratic volatility of business returns (in the form of partnership, proprietorship, and S corporation income) is 3-4 times greater than the volatility of wage and salary income for similar individuals. This echoes earlier findings of Hamilton (2000), who reports that self-employed workers have lower mean earnings and much more dispersed outcomes than those who work for others. Similarly, Moskowitz and Vissing-Jorgensen (2002) find that investors in firms that are not publicly traded tend to concentrate very large fractions (averaging 70 percent) of their investment capital in single firms with returns that are highly uncertain and on average no greater than those available from publicly traded alternatives. This evidence and others of modest average returns and significant economic risk prompt observers to infer that nonpecuniary factors such as the benefits of working for one's self, and the social status accorded to business owners, are important determinants of entrepreneurial and other risky noncorproate business activity.[9]

Tax policy also appears to influence levels of noncorporate business activity. Auerbach and Slemrod (1997) note that the greater numbers and incomes of S corporations in the United States after 1986 coincide with more favorable tax treatment relative to corporations in the Tax Reform Act of 1986.[10] Using annual U.S. time series data for 1960-1986, MacKie-Mason and Gordon (1997) find that a ten percent higher corporate tax rates are associated with 2.8 percent greater income shares of noncorporate business. Goolsbee (1998) reports average tax effects that are less than half as large in a time series analysis of annual U.S. data from 1900-1939, though notes that these tax effects are considerably larger for firms with positive taxable income. Prisinzano and Pearce (2018) update and refine the MacKie-Mason and Gordon analysis using U.S. data from 1960-2012, finding somewhat larger tax effect magnitudes: a ten percent higher corporate tax rate is associated with a 3.4 percent greater share of income going to noncorporate business. Goolsbee (2004) analyzes the organizational forms of retail trade firms across U.S.

[9] See, for example, Hamilton (2000), Moskowitz and Vissing-Jorgenson (2002), Roussanov (2010), and Hurst and Pugsley (2011).

states in 1992, finding much larger tax effects: ten percent higher corporate tax rates are associated with 15 percent greater employment shares, and ten percent greater payroll and sales shares, in noncorporate firms. And Barro and Wheaton (2019) use different methods to analyze annual U.S. data for 1968-2013, reporting coefficients that imply that ten percent higher corporate tax rates are associated with rougly three percent greater noncorporate shares of business assets.

The high returns associated with successful large undiversified noncorporate business investments make some entrepreneurs very wealthy, and as a result, top U.S. incomes consist disproportionately returns from unincorporated business activities. Smith et al. (2019) find that 69 percent of those in the top one percent of the U.S. income distribution receive income from noncorporate businesses. Of course some of this income simply represents passive investment, but the study reports that 39 percent of the incomes of the top one percent of U.S. taxpayers consist of returns from noncorporate firms in which they are active participants; this fraction rises to 44 percent of the incomes of the top 0.1 percent.[11] Smith et al. also report that U.S. noncorporate firms typically have one to three owners for whom the business generates large shares of their total incomes. Quadrini (2000) likewise finds business owners and managers to be overrepresented in top U.S. income and wealth groups. Another hint of the impact of the potential impact of entrepreneurial business activity on the distribution of income appears in Aghion et al. (2019), which reports that local areas of the United States with high rates of innovation and patenting have greater income inequality and greater income-based social mobility.

The model used in section 4 carries implications for observed shares of entrepreneurial income among top earners. Using ψ_{NC} to denote the portion of aggregate high income earned by individuals engaging in noncorporate business activities, it follows from the model that

$$\psi_{NC} = p\int_{\theta_3}^{\infty} y(\theta)\,dF(\theta) + p\phi\int_{\theta_1}^{\theta_3} (1+k)\,y(\theta)\,dF(\theta). \tag{29}$$

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[10] Cooper et al. (2016) document the secular rise in noncorporate business income generally since 1986, and its concentration in the top one percent of the income distribution.

[11] Bryan (2017) presents consistent evidence for U.S. taxpayers with incomes above $200,000 in 2014. Bricker et al. (2016) and Guvenen and Kaplan (2017) offer cautionary notes about interpreting tax and survey data on the incomes of top earners.

Evaluating (29) yields:

(30) $$\psi_{NC} = p\Theta_{\bar{y}}\left(1+\Delta\right),$$

And applying (15) produces

(31) $$\frac{\psi_{NC}}{\psi} = \frac{p\left(1+\Delta\right)}{1+p\Delta}.$$

The Smith (2019) study suggests that the ψ_{NC}/ψ ratio for the top one percent of the U.S. income distribution is roughly 0.39. Applying this figure to the earlier example of high-risk noncorporate investment ($k=5$ and $\phi=0.1$), and using $\frac{dp}{d\tau}=0.3$, it follows that $p=0.22$. Instead applying $\frac{dp}{d\tau}=0.3$ to the representative case of lower-risk noncorporate investment ($k=2$ and $\phi=0.2$) produces $p=0.30$. Both cases correspond to situations in which higher income individuals have sizeable proclivities to participate in risky noncorporate business ventures.

The model in section 4 has features that generally correspond to the economic experiences of top income earners in the United States, but that nonetheless contain counterfactual elements. For example, the model assumes that returns to corporate investments are certain, which while clearly not true, is convenient for model tractibility. The model's specification of the risk associated with noncorporate business activity is therefore properly interpreted as reflecting differences between the risks of noncorporate and corporate investments. The model assumes that the returns (*k* and *m*) to risky noncorporate business activity are not themselves functions of the corporate tax rate, which need not be the case. The model takes households to be risk-neutral, whereas a more realistic specification with risk-averse households and capital market equilibrium would imply that expected noncorporate business returns must be more favorable than captured by equation (7), and higher corporate tax rates therefore more strongly associated with increased income concentration. And the simplifying assumption that changes in the corporate tax rate reduce all high incomes proportionately is very unlikely strictly to hold true, since corporate tax changes are close to zero-sum redistributions and therefore cannot raise or lower all incomes in the economy in the same proportions. While reasonable modifications to the model are apt to produce different estimates of the distributional effects of induced noncorporate activity due to changes in the corporate tax, the resulting estimates might be larger or smaller than produced by calculations such as (28).

6. Conclusion

This paper's finding that higher corporate tax rates may not much reduce income inequality, or even increase it, bears only indirectly on tax incidence calculations as conventionally performed. The standard government incidence calculation considers the effects of taxation on the money-metric welfares of different groups of individuals based on how they are situated prior to the tax change. For this purpose, it is sufficient to use the envelope condition to evaluate the welfare effects of small tax changes; and the envelope condition implies that the change in an individual's welfare can be analyzed assuming that their behavior is unaffected by the tax change, even though in fact their behavior is affected.

In evaluating tax-induced changes in the distribution of income the envelope condition is insufficient, and the standard tax incidence approach somewhat misapplied. Behavioral responses matter for income distribution even if they do not for individual welfare. Two individuals who are induced by the tax system to invest in small noncorporate businesses, applying equal ex ante wealth and labor supply, and with similar business prospects, may have very different ex post economic outcomes. The standard incidence calculation would treat them identically, whereas an analysis of the effects of taxation on the distribution of income by necessity must treat them very differently.

Would it be appropriate for those who produce official tax burden distributional estimates to adjust their calculations to account for the effects of corporate taxation on the risk patterns of noncorporate (and presumably corporate) returns? That depends on the objectives of distributional analysis. If the purpose of distributional analysis is to judge the potential consequences of tax reforms for income groups defined prior to reform, then the current methods of distributing tax burdens need not be adjusted for the uncertainty of ex post investment returns.

If, however, the purpose of distributional analysis is not to identify consequences for well-defined current groups, but instead to evaluate the social effects of tax reforms arising from their effects on the economy's distribution of income, then it is difficult to see how the induced investment outcome uncertainty can be ignored.

The analysis in this paper suggests that the nature of business organization, and the resulting risks that business owners face, significantly affects the distribution of income in the economy. Corporate taxes, in changing business organization, thereby change the level of idiosyncratic risk in the economy and therefore the distribution of income. It follows that other policies that affect business organization, such as regulatory rules that govern the formation of different types of businesses, and the taxation of profits earned by unincorporated firms, likewise affect the distribution of income. Furthermore, tax and other policies that directly influence risk-taking, such as the extent to which the tax system permits deductions for loss-making firms, should also influence the income distribution.

Corporations are entities that pool risks and returns among multiple owners. In spreading risks in this way, they reduce the dispersion of economic outcomes, and thereby produce more equal distributions of realized incomes than would be the case if the same business risks were not pooled and instead each held by individual owners. If all businesses in the United States were required to be publicly traded corporations with widely diversified ownership then the U.S. income distribution would be considerably more equal than it is today. Higher rates of corporate taxation move the economy in the direction of having fewer corporations and therefore less of the risk sharing that they provide. There is a substantial sense in which risky outcomes of economic activities writ large, including some undertaken long ago, are fundamental to significant portions of economic inequality – so in understanding the distributional effects of corporate taxation and other government policies, it is important to incorporate the extent to which these policies encourage or discourage ventures with risky economic returns.

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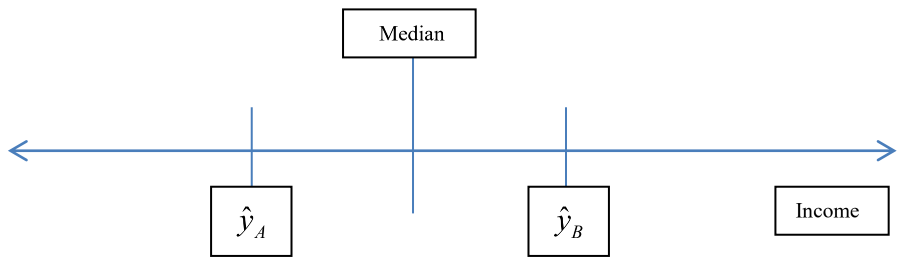
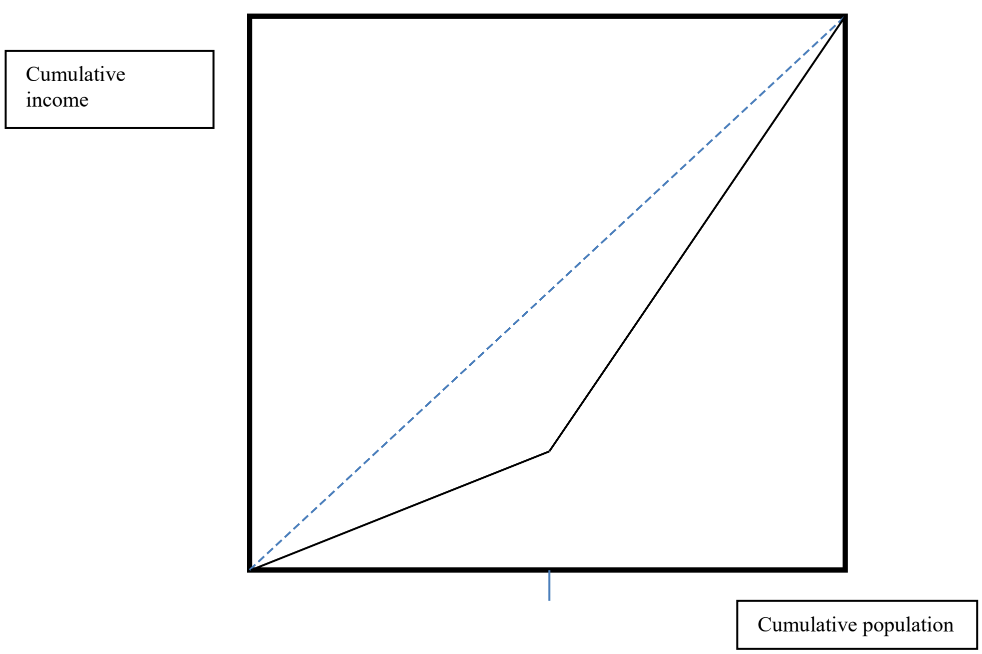
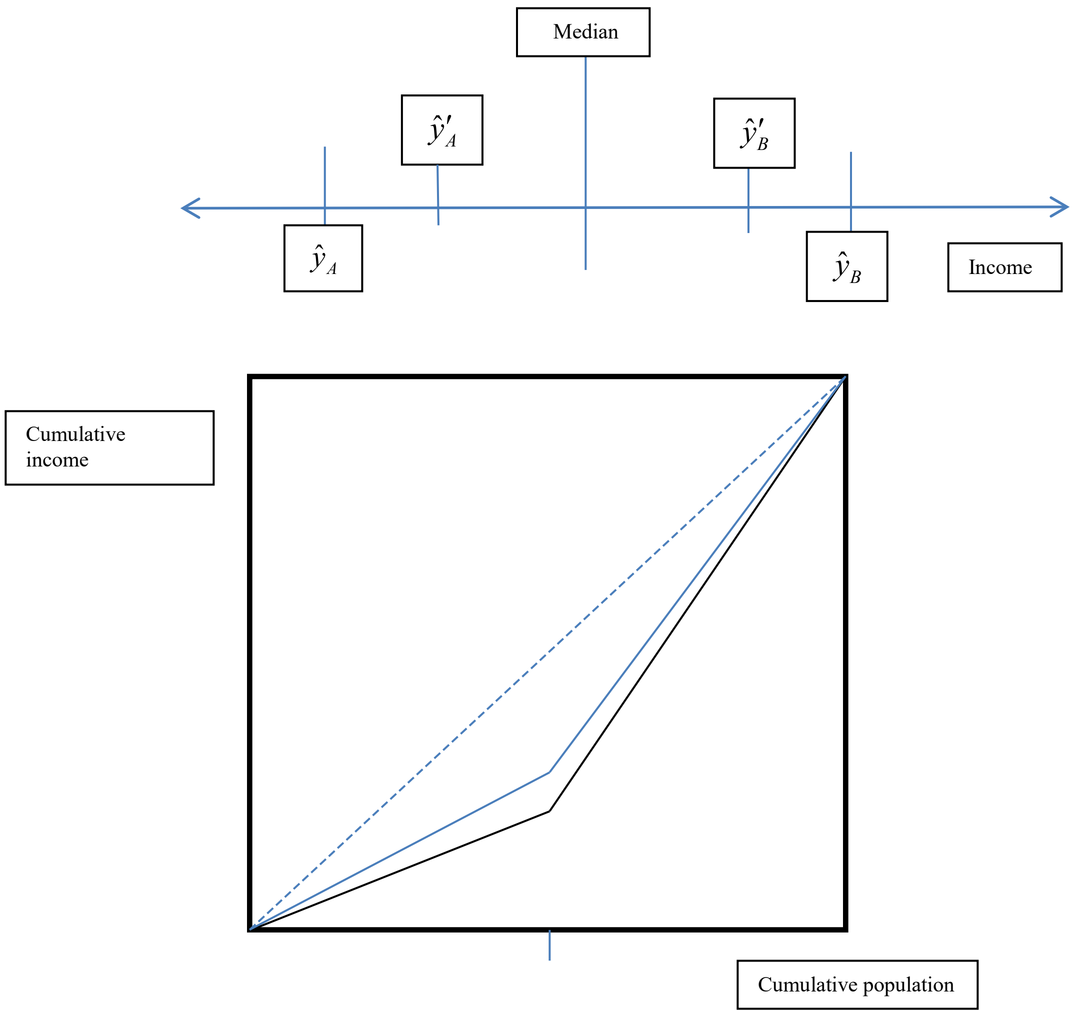
Figure 1: Income distribution prior to change in corporate taxation.





Note: the top part of Figure 1 depicts the distribution of after-tax income prior to a change in the corporate tax rate; the bottom part presents the corresponding Lorenz curve. In this example there are just two types of individuals, *A* and *B*, with individuals of type *A* having lower incomes than those of type *B*.

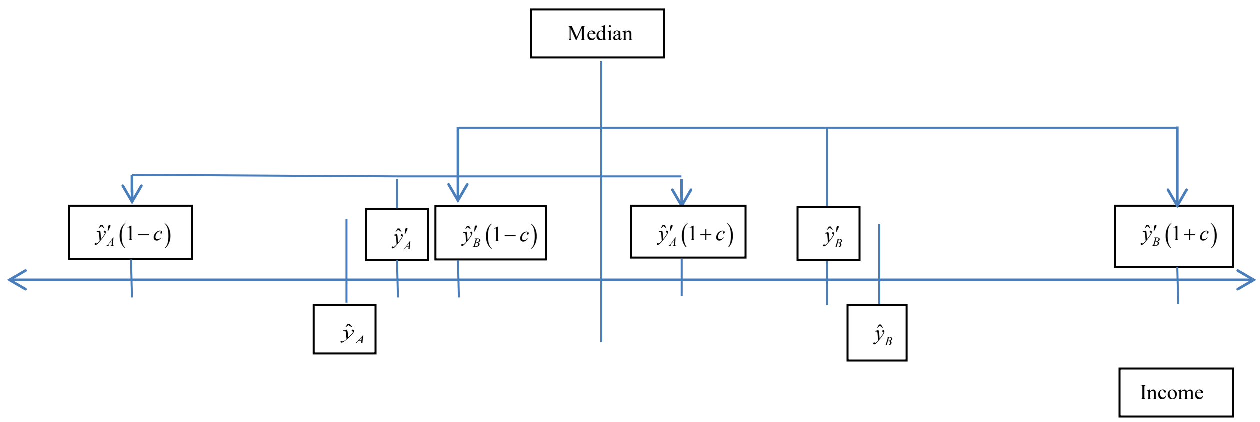
Figure 2: Distribution of expected incomes after a higher corporate tax, the burden of which is entirely on capital owners.



Note: the top part of Figure 2 depicts the distribution of income before and after the introduction of a higher corporate tax rate. There are just two types of individuals, *A* and *B*, with individuals of type *A* having lower incomes than those of type *B*, and having relatively heavier labor endowments than capital endowments. Since in the example the burden of corporate taxes falls entirely on capital, a higher corporate tax (and accompanying lower labor tax) increases type *A*'s after-tax income from $\hat{y}_A$ to $\hat{y}'_A$, and analogously reduces type *B*'s after-tax income from $\hat{y}_B$ to $\hat{y}'_B$, reducing income inequality.

The bottom part of Figure 2 presents the corresponding Lorenz curves, the solid black line corresponding to the income distribution prior to the corporate tax change, and the solid blue line corresponding to the (more equal) income distribution with a higher corporate tax.

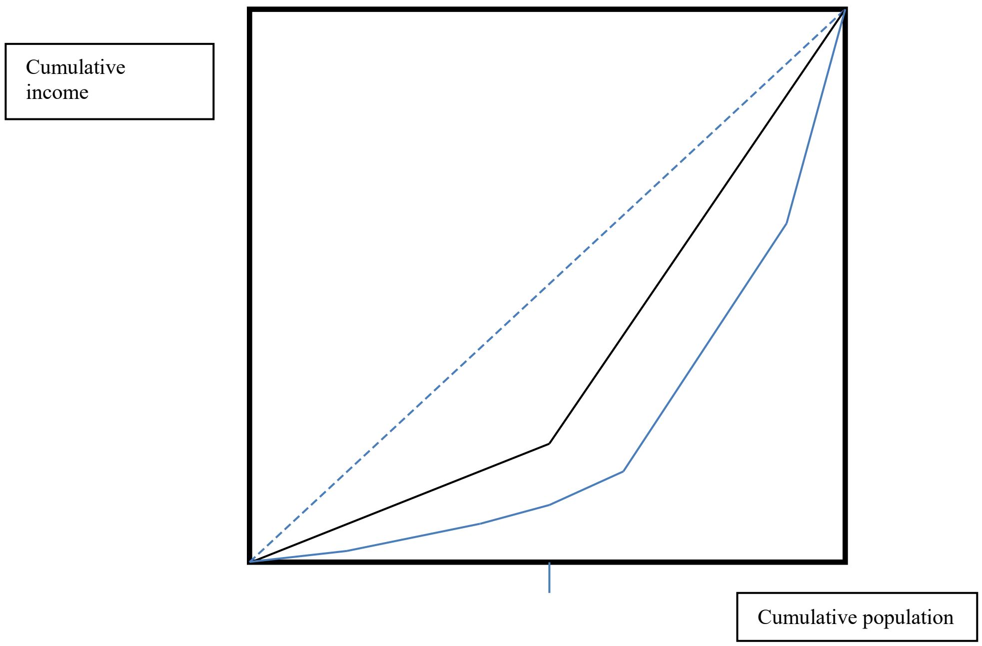
Figure 3: Effects of uncertainty on incomes of types *A* and *B*.



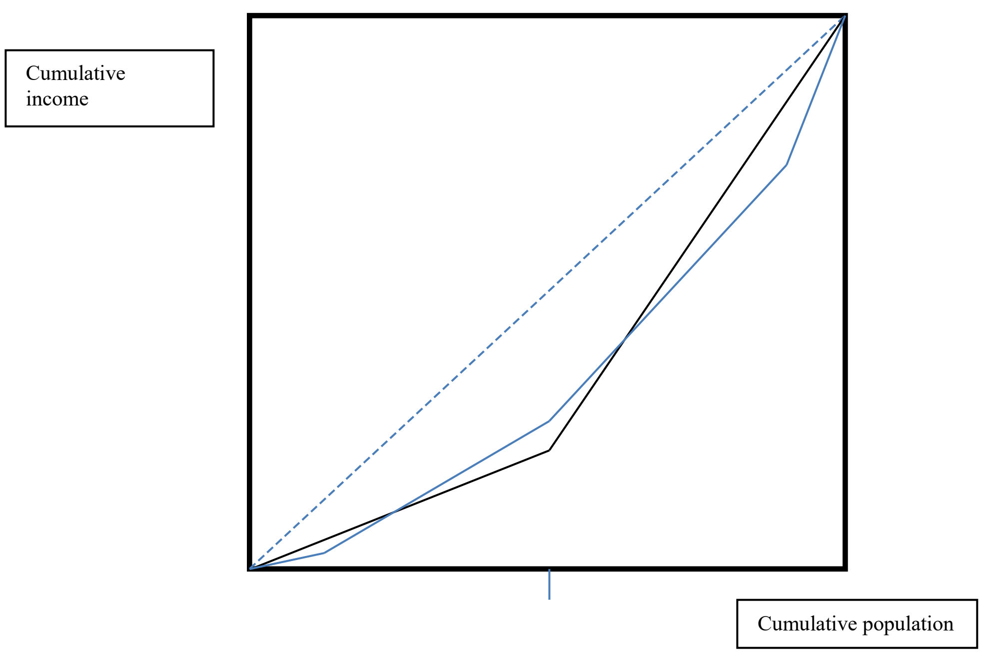
Note: Figure 3 depicts the distribution of realized incomes following the randomizations accompanying noncorporate investment after the introduction of a higher corporate tax rate. There are just two types of individuals, *A* and *B*, with individuals of type *A* having lower incomes than those of type *B*, and type *A*s having relatively heavier labor endowments than capital endowments. Since in the example the burden of corporate taxes falls entirely on capital, a higher corporate tax (and accompanying lower labor tax) increases type *A*'s expected after-tax income from $\hat{y}_A$ to $\hat{y}'_A$, and analogously reduces type *B*'s expected after-tax income from $\hat{y}_B$ to $\hat{y}'_B$. The higher corporate tax rate stimulates noncorporate investment with risky outcomes, as a result of which a portion of type *A* individuals have realized incomes of $\hat{y}'_A(1-c)$, and an equal portion have realized incomes of $\hat{y}'_A(1+c)$. Similarly, a portion of type *B* individuals have realized incomes of $\hat{y}'_B(1-c)$, and an equal portion have realized incomes of $\hat{y}'_B(1+c)$, after the introduction of a higher corporate tax rate.

Figure 4: Possible Lorenz curves after higher corporate tax rate.

Top panel: New distribution is more unequal in the sense of second order stochastic dominance.



Bottom panel: Neither distribution stochastically dominates the other.



Note to Figure 4: diagrams in the the top and bottom panels depict possible Lorenz curves following an increase in the corporate tax rate. The solid black lines correspond to the after-tax income distribution prior to the corporate tax change, and the solid blue lines corresponding to post-change income distributions that incorporate the random outcomes depicted in Figure 3. The diagram in the top panel describes an outcome in which the after-tax-change income distribution is more unequal than the pre-tax-change income distribution, in the sense of second order stochastic dominance. The diagram in the bottom panel describes an outcome in which neither distribution dominates the other in a second order stochastic sense, so it is less clear whether the post-tax-change income distribution is more or less equal than the pre-tax-change income distribution.

Table 1

Noncorporate Business Riskiness and the Concentration of High Incomes

ϕ																
0.3		0.23	0.51	0.91												
0.2		0.13	0.29	0.50	0.78	1.10	1.49	1.94								
0.1		0.06	0.13	0.22	0.33	0.47	0.63	0.81	1.01	1.23		2.30	2.95	3.68		
		1	1.5	2	2.5	3	3.5	4	4.5	5		7	8	9		***k***

Note to Table 1: The table presents values of $\left[(1+k)^{\alpha}\phi+(1-\phi-k\phi)^{\alpha}(1-\phi)^{1-\alpha}-1\right]$, denoted Δ in the text, with $\alpha=1.67$, the parameter *k* taking values indicated on the horizontal axis, and ϕ taking values indicated on the vertical axis. An entry such as 2.30 (for $k=7$ and $\phi=0.1$) indicates that the aggregate income of the population with incomes above $\overline{y}$ equals $\Theta_{\overline{y}}(1+2.3p)$, in which $\Theta_{\overline{y}}$ is the aggregate income of the population with incomes above $\overline{y}$ in the absence of noncorporate investment, and *p* is the fraction of the population making noncorporate investments.

Table 2
Effects of Ten Percent Higher Corporate Tax Rates on Income Concentration

		p						
	α			1.5	1.6	1.67	1.75	1.9
$\phi = 0.2$		0.15		0.61	0.75	0.84	0.95	1.16
$k = 2$		0.20		0.61	0.73	0.82	0.93	1.12
		0.25		0.60	0.72	0.80	0.90	1.08
		0.30		0.59	0.71	0.79	0.88	1.05
		0.35		0.58	0.69	0.77	0.86	1.01
$\phi = 0.2$		0.15		1.27	1.52	1.70	1.90	2.28
$k = 3$		0.20		1.23	1.46	1.62	1.80	2.13
		0.25		1.19	1.41	1.55	1.71	1.99
		0.30		1.16	1.36	1.49	1.63	1.87
		0.35		1.12	1.31	1.43	1.56	1.77
$\phi = 0.1$		0.15		0.56	0.69	0.79	0.91	1.14
$k = 3$		0.20		0.55	0.68	0.77	0.88	1.10
		0.25		0.54	0.67	0.76	0.86	1.06
		0.30		0.53	0.65	0.74	0.84	1.03
		0.35		0.53	0.64	0.72	0.82	1.00
$\phi = 0.1$		0.15		0.91	1.13	1.29	1.48	1.85
$k = 4$		0.20		0.89	1.10	1.25	1.42	1.85
		0.25		0.87	1.07	1.21	1.36	1.66
		0.30		0.86	1.04	1.17	1.31	1.57
		0.35		0.84	1.01	1.13	1.26	1.50
$\phi = 0.1$		0.15		1.33	1.64	1.86	2.12	2.61
$k = 5$		0.20		1.28	1.57	1.77	2.00	2.41
		0.25		1.24	1.50	1.69	1.89	2.24
		0.30		1.21	1.45	1.61	1.79	2.09
		0.35		1.17	1.39	1.54	1.70	1.96
$\phi = 0.1$		0.15		1.78	2.18	2.46	2.78	3.37
$k = 6$		0.20		1.70	2.06	2.30	2.57	3.05
		0.25		1.63	1.95	2.16	2.39	2.78
		0.30		1.57	1.85	2.04	2.24	2.56
		0.35		1.51	1.77	1.93	2.10	2.36

Note to Table 2: The table presents the percentage effects of a ten percent higher corporate tax rate on the concentration of high incomes due to induced greater income dispersion from noncorporate investments. The table entries correspond to the first term on the right side of equation (28) for $dp/d\tau = 0.3$. The net effect of a ten percent higher corporate tax rate then equals the difference between the entry in Table 2 and λ, where λ is the effect of the corporate tax on expected incomes. Thus for example, if $\phi = 0.1$, $k = 5$, and $p = 0.20$, then the greater income dispersion caused by a ten percent higher corporate tax rate increases the concentration of high incomes by 1.77 percent. If the relevant value of λ is smaller in magnitude than 1.77 percent, it would then follow that a higher corporate tax rate increases the concentration of high incomes.

Appendix A

This appendix considers the simple model of section 3. In the absence of noncorporate investment, and treating the returns to corporate investment as certain, matters are quite straightforward. Denoting individual *A*'s income as $\hat{y}_A$, and individual *B*'s as $\hat{y}_B$, it follows that

(A1) $$\hat{y}_A = w(1-t)L_A + r(1-\tau)K_A$$

(A2) $$\hat{y}_B = w(1-t)L_B + r(1-\tau)K_B,$$

in which w is the pretax return to labor, t the tax rate on labor income, r the pretax return to corporate investment, and τ the tax rate on corporate income. Government tax revenue is denoted *R*, for which

(A3) $$R = tw(L_A + L_B) + \tau r(K_A + K_B).$$

Substituting (A3) into (A1) and (A2) produces:

(A4) $$\hat{y}_A = wL_A + rK_A - R\frac{L_A}{(L_A + L_B)} + \tau r(K_A + K_B)\left[\frac{L_A}{(L_A + L_B)} - \frac{K_A}{(K_A + K_B)}\right]$$

(A5) $$\hat{y}_B = wL_B + rK_B - R\frac{L_B}{(L_A + L_B)} - \tau r(K_A + K_B)\left[\frac{L_A}{(L_A + L_B)} - \frac{K_A}{(K_A + K_B)}\right].$$

In an environment with fixed factor supplies, tax changes that do not affect total government revenue also will not affect pretax factor returns.[12] Taking government revenue *R* to be a fixed requirement, it follows from (A4) that a higher corporate tax rate (and accompanying lower labor income tax rate) increases *A*'s after-tax income if *A*'s factor endowment is relatively more labor-intensive than the economy as a whole – and a higher corporate tax rate reduces *B*'s after-tax income if *A*'s factor endowment is relatively more labor-intensive than *B*'s. Hence if type *A*s

[12] See, for example, Kotlikoff and Summers (1987) and Fullerton and Metcalf (2002), who note that the same incidence result can also arise with endogenous corporate-noncorporate substitution if corporate and noncorporate activities have identical production functions (and therefore equal capital intensities) and the elasticity of

have lower incomes than type *B*s, and individuals of type *A* have relatively more labor-intensive factor endowments, it follows that, in the absence of noncorporate investment, higher corporate tax rates narrow the income gap between types *A* and *B*.

capital/labor substitution in the production sector equals the elasticity of substitution in demand between corporate and noncorporate goods.

Appendix B

The following expression for ψ is the joint implication of equations (15) and (16) in the paper:

(B1) $$\psi = \Theta_{\bar{y}}\left\{1 + p\left[(1+k)^{\alpha}\phi + (1-\phi-k\phi)^{\alpha}(1-\phi)^{1-\alpha} - 1\right]\right\}.$$

The purpose of the appendix is to show that if $k > 0$, $\phi > 0$, and $p > 0$, then $\partial\psi/\partial k > 0$, $\partial\psi/\partial p > 0$, and $\partial\Delta/\partial\alpha > 0$.

Differentiating (B1) with respect to *k* produces:

(B2) $$\frac{\partial\psi}{\partial k} = \Theta_{\bar{y}}\,p\phi\alpha\left[(1+k)^{\alpha-1} - \left(\frac{1-\phi-k\phi}{1-\phi}\right)^{\alpha-1}\right].$$

From expression (8) in the paper, $1 > \left(\frac{1-\phi-k\phi}{1-\phi}\right) \geq 0$; and since $\alpha > 1$, it follows that $1 > \left(\frac{1-\phi-k\phi}{1-\phi}\right)^{\alpha-1} \geq 0$. Given that $(1+k)^{\alpha-1} > 1$, the term in brackets on the right side of (B2) is positive. And since $\Theta_{\bar{y}} > 0$, $p > 0$, $\phi > 0$, and $\alpha > 0$, then $\partial\psi/\partial k > 0$.

If $k = 0$ then (B1) implies that $\psi = \Theta_{\bar{y}}$. Since $\partial\psi/\partial k > 0$, the bracketed term on the right side of (B1) must be positive for all values of $k > 0$. Consequently, $\partial\psi/\partial p > 0$.

Finally, differentiating (B1) with respect to α produces:

(B3) $$\frac{\partial\Delta}{\partial\alpha} = \phi(1+k)^{\alpha}\ln(1+k) - (1-\phi-k\phi)^{\alpha}(1-\phi)^{1-\alpha}\left[\ln(1-\phi) - \ln(1-\phi-k\phi)\right].$$

From the fundamental theorem of the calculus,

(B4) $$\left[\ln(1-\phi) - \ln(1-\phi-k\phi)\right] = \int_{k\phi}^{0}\frac{\partial\ln(1-\phi-s)}{\partial s}ds = \int_{0}^{k\phi}\frac{1}{(1-\phi-s)}ds < \frac{k\phi}{(1-\phi-k\phi)}.$$

It follows from (B3) and (B4) that

(B5) $$\frac{\partial\Delta}{\partial\alpha} > \phi\left[\left(1+k\right)^{\alpha}\ln\left(1+k\right) - k\left(\frac{1-\phi-k\phi}{1-\phi}\right)^{\alpha-1}\right].$$

Separate application of the fundamental theorem of the calculus implies that

(B6) $$\ln\left(1+k\right) = \ln\left(1+k\right) - \ln\left(1\right) = \int_0^k \frac{\partial \ln\left(1+s\right)}{\partial s} ds = \int_0^k \frac{1}{\left(1+s\right)} ds > \frac{k}{1+k}.$$

Then (B5) and (B6) together imply that:

(B7) $$\frac{\partial\Delta}{\partial\alpha} > \phi k\left[\left(1+k\right)^{\alpha-1} - \left(\frac{1-\phi-k\phi}{1-\phi}\right)^{\alpha-1}\right].$$

The term in brackets on the right side of (B7) is the same as the term in brackets on the right side of (B2), and is therefore likewise positive, so $\partial\Delta/\partial\alpha > 0$.