The System of National Accounts and Alternative Approaches to the Construction of Commercial Property Price Indexes[1]

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Abstract

While fluctuations in commercial property prices have an enormous impact on economic systems, the development of related statistics that can capture these fluctuations is one of the areas that is lagging the furthest behind. The reasons for this are that, in comparison to housing, commercial property has a high level of heterogeneity and there are extremely significant data limitations. Focusing on the Tokyo office market, this study estimated commercial property price indexes using the data available in the property market, and clarified discrepancies in commercial property price indexes based on differences in the method used to create them. Specifically, we estimated a quality adjusted price index with the hedonic price method using property appraisal prices and transaction prices. The international System of National Accounts (SNA) requires a decomposition of property values into price and volume (quantity) components for both the structure and land components of property value. The paper shows how this can be accomplished for Commercial properties.

Key Words

Commercial property price indexes, System of National Accounts, the builder’s model, transaction based indexes, appraisal prices, assessment prices, land and structure price indexes, hedonic regressions, depreciation rates, Stock based Property Price Index.

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1. Introduction

How can a commercial property price index (CPPI) be defined and constructed? And what kind of relationship does the measurement of commercial property's value have to the System of National Accounts and to concerns about national financial sectors? In order to answer such questions, this paper aims to outline the concepts that can be used to define and measure the value of commercial property, and to clarify the relationship of such measurement to the System of National Accounts and to the financial system.

In constructing CPPI's in the National Accounts, we should use transaction prices. However, due to a lack of commercial property sales, it may be necessary to use appraisal based information combined with transaction prices in a mixed approach.

In the case of price indexes, prices transacted on the market are in principle the most fundamental or relevant type of data. However, in the creation of real estate price indexes - especially commercial real estate price indexes - it is not unusual for real estate appraisal values to be used directly in the index construction rather than transaction prices, in part due to the low number of actual transactions.[4] As a prominent example, MSCI-IPD (IPD), a private company based in the U.K. which supplies property return (income return and capital return) indexes for more 30 countries, creates its indexes based on appraisal values.[5] The NCREIF capital returns -- a leading U.S. real estate investment index - are also, like IPD's index, based on appraisal valuations.[6]

In recent years, commercial price indexes based on transaction prices have also been published, such as the U.S. Moody's/RCA Commercial Property Price Index (CPPI) and the MIT/CRE Transaction Based Index (TBI).[7] Beyond that, new indexes based on stock market share prices of companies specialized in the ownership of commercial property equity are beginning to be developed, such as the NAREIT Pure Property Index Series which was launched in 2012 in the United States. Thus, an important point that arises with regard to the creation of commercial real estate price indexes is the question of selecting the type of value indication data, along with the issue of the index calculation method. The question is whether to use transaction price data, to use real estate appraisal value, or to use a different method altogether, such as stock market data.

Commercial property investment, like housing, plays an extremely important part in the System of National Accounts. Investment in buildings as part of new property development must be recorded in the System of National Accounts, and while the economy is growing rapidly, new building investment represents a significant share of total national investment in the System of National Accounts. In addition, since buildings are finite-lived durable goods, it is necessary to correctly measure depreciation

[4] Note however that there are circumstance where transaction price information is more plentiful than appraisal valuation information, notably in countries (such as the U.S.) where IFRS accounting rules are not yet prevalent, such that assets are normally carried on companies' books at historical cost rather than at current "fair value." In the United States, only specialized populations of commercial properties are frequently and professionally appraised.

[5]For details of IPD's real estate investment index, see http://www1.ipd.com/Pages/default.aspx.

[6]NCREIF: (http://www.ncreif.org/ . (Note that the NCREIF Index is based on a little over 7,000 properties, out of a universe of probably some 3,000,000 commercial properties in the U.S.))

[7]http://mitcre.mit.edu/research-publications/cred/transaction-based-index . (Note that the TBI is now produced and published by NCREIF as the "NTBI.")

that occurs with the passage of time. What's more, maintenance is conducted and investments are made in renovations or improvements in order to increase the value. Such investment in maintenance and improvements/renovations also comprises a significant share of investment.

The following issues should be considered in preparing commercial property price indexes:

- We should use transaction prices. However, since transactions may be few in number and only observed sporadically in many markets, we may be forced to consider combining transactions values with appraisal values to construct CPPI's.
- The characteristics that determine the market value of commercial property are extremely heterogeneous and include characteristics both of the property itself and of its site and location. As well, "commercial property" covers a broad range; depending on the source of income, it may refer to offices, retail facilities, investment housing, factories, distribution facilities, hotels, hospitals, care facilities, and land as well as other categories.
- Economic indicators as typified by price indexes of market or transaction prices (such as the CPI) generally track the prices of the same items through time and observe the changes in those prices. But for non-durable goods the "same item" refers to exactly that, a new example of the same good, bought at different points in time. But real estate is not only durable but is unique. No two properties are identical, and when the same property transacts repeatedly at different points in time it is older at the later point in time, and a property's age is one feature determining its value. Indeed, the price of a commercial property can change over time even if depreciation of the structure does not occur, either because some characteristics affecting value can change (e.g., the distance to the nearest subway station may change if a new station is built, or the building's features/structure may change if there are renovations), or because the imputed prices of the property characteristics that determine its value may have changed (e.g., it may be more valuable to be located near an airport or near an internet data center and less valuable to be located near a railway station). Thus, from an SNA (System of National Accounts) perspective, defining what is a change in "price" and what is a change in "quantity" (including "quality") of real estate is non-trivial, and is particularly complicated when dealing with commercial property.
- Current CPPIs compiled largely in the private sector from observing the commercial property market in actual business dealings, may be either transaction based or (more frequently) appraisal based.
- CPPIs produced by the private sector are normally confined to those properties that fall in to the professionally managed investment industry. Smaller non-professionally managed commercial properties, including owner occupied commercial properties, are often not included in these indexes. Even large buildings, if owner-occupied and not in the investment market, are not tracked. As such, the universe of commercial properties is often only partially covered and is biased towards the professionally managed property sector. While CPIs for instance aim to sample the universe of all household transactions, CPPIs produced by the private sector generally refer only to the sub-sector of transactions or properties which fall within the scope of their clients.

With the above general considerations in mind, an outline of the paper is as follows: In section 2, we will consider the differences between stock and flow price indexes for commercial real estate and the relationship of these indexes in the System of National Accounts. In subsequent sections, we will concentrate on the construction of stock indexes. In particular, we will look at methods of index construction that enable one to decompose commercial property sales prices into land and structure components with separate price indexes for both components. Sections 3-6 will work with indexes based on transaction prices for commercial office properties. Section 3 introduces the *builder's model* which enables us to construct separate land and structure price indexes for commercial office properties for Tokyo. The model also allows us to estimate a geometric depreciation rate for these properties. Section 4 extends the section 3 model to allow for geometric depreciation rates that change as the structure ages. Section 5 looks at the estimation of straight line structure depreciation rates and also generalizes this simple model with a single rate to a model that allows for a piecewise linear depreciation schedule. The final models in sections 4 and 5 end up producing very similar price indexes for land as well as similar structure aging functions.

Due to the scarcity of transactions in commercial properties in Tokyo, the land price indexes that result from the final models in sections 4 and 5 are fairly volatile. Thus in section 6, we look at some simple methods for smoothing these volatile series.

Section 7 uses appraisal data and section 8 uses tax assessment data in order to construct commercial property land price indexes. It will be seen that the resulting indexes appear to be too smooth and they significantly lag the turning points in the transaction based indexes that are exhibited in sections 4 and 5.

In section 9, overall property price indexes based on transactions data and on appraisal data are constructed and these overall indexes are compared to a simple average type of index and to a traditional log price hedonic regression overall commercial property price index. It turns out that the traditional hedonic price index is reasonably close to our overall commercial property transactions based index. However, the traditional log price hedonic index cannot provide separate land and structure subindexes. These separate indexes are required in the country's National Balance Sheet Accounts as well in national multifactor productivity accounts (if the country produces these accounts).

Section 10 shows how stock market information on Real Estate Investment Trusts (REITs) can be used to provide additional "transactions" that could be used to increase sample size when running a hedonic regression on sales of commercial properties.

Section 11 concludes.

2. The System of National Accounts and Stock and Flow Prices for Commercial Properties

Commercial Property Price Indexes (CPPPs) play an important role in economic statistics and especially for the System of National Accounts (SNA). Decomposing the value of a commercial property into price and quantity components is important for financial system oversight and important for guiding macroeconomic policy. The SNA Balance Sheet Accounts require information on the price and quantity (or volume) of commercial properties located in the country. Moreover, the SNA also requires a separate decomposition of property value into *separate* price and quantity components for the *structure* on the commercial property and for the *land* that the structure sits on. This information is essential for measuring the Total Factor Productivity (or Multifactor Productivity) of the commercial property sector and hence, it is also essential to measure the productivity of the national economy. Given the recent increase in the value of land in many advanced economies, determining the value, price and quantity of land used by the commercial property sector is important to guide economic policy.

It is useful to develop a general relationship between the value of an asset and the period by period rents that it can generate as it ages.[8] In general, the value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is equal to the discounted future *service flows* that the asset is expected to yield in future periods.

The System of National Accounts also requires information on the flow of services generated by the commercial property sector and on the flow of inputs used by the sector over an accounting period.

We introduce some notation for modeling the output and input flows for the commercial property sector for accounting period t. Consider commercial property n in period t which has I(n) separate units in it which are rented out at the rental price p_{tni} for n = 1,…,N and i = 1,…,I(n). The owner of the property provides various intermediate and primary inputs to the renters such as electricity, heating, air conditioning, security services which have prices w_{tnij} for n = 1,…,N, i = 1,…,I(n) and j = 1,…,J where there are J separate inputs used by the owners of the commercial properties.[9] Price indexes for rented outputs, (i.e., for the p_{tni}) and price indexes for intermediate inputs used in order to produce the rented outputs (i.e., for the w_{tnij}) are required in the System of National accounts. But note that these prices depend not only on the age of the structure on property n and the time period t but also on the particular characteristics of the property and the particular rented unit . The reason for this dependence is that each unit i in property n will generally offer a different mix of amenities associated with the rented space s_{tni}. Thus the rental price of a unit in a commercial property will generally depend on the following *quality adjusting factors*:

- The floor space area of the unit;
- Is electricity provided?
- Is heating or air conditioning provided?
- Are cleaning services provided and how much maintenance is provided?
- Does the rented space offer a view?
- What is the average vacancy rate?
- Is parking provided?
- How close to rapid transit is the property?
- What is the ratio of structure area to the land area of the building?
- How much landscaping is provided?
- Are other amenity areas provided, such as laundry facilities or recreation areas?

The fact that the quality of rented space will depend heavily on what amenities are associated with the rented space and these amenities can vary greatly across properties means that the output and intermediate input prices p_{tni} and w_{tnij} will generally depend on the *characteristics* associated with unit i in property n. The way to deal with these difficult quality adjustment problems is to use *hedonic regression techniques* to adjust observed unit rental prices for quality differences.[10]

[8] See Diewert (2005; 480-485) and Diewert, Fox and Shimizu (2016).

[9] In most cases, the input prices w_{tnij} will not depend on the particular rented unit i in property n.

[10] Hedonic regression analysis dates back to Court (1939) who introduced the term. For more recent expositions of the method and references to the literature, see Triplett (2004) and Diewert (2019).

Similar quality adjustment problems occur when constructing price indexes for *stocks* of commercial properties. Thus for property n in period t, we need the beginning of the period current value of the property, say V_{tn}, and a decomposition of this value into a land value, V_{Ltn}, and a corresponding structure value, V_{Stn}. Furthermore, these two value components need to be further decomposed into price and constant quality quantity components, say $V_{Ltn} = P_{Ltn}Q_{Ltn}$ and $V_{Stn} = P_{Stn}Q_{Stn}$ where P_{Ltn} and P_{Stn} are constant quality prices for land and structures and Q_{Ltn} is the land plot area of property n and Q_{Stn} is floor space area of property n in period t. But the land quality may not remain constant from period to period: new amenities may be built (such as a new subway line that has a station near the property) or new "bads" may occur (such as increased pollution or increased traffic congestion) which will affect the land component of the price of the property. Similarly, the quality of the structure will not remain constant across time periods: depreciation of the structure and renovation expenditures will change the structure component of the price of the property. Thus there are also quality adjustment problems when constructing commercial property price indexes for the values of properties. In the following section, we will show how hedonic regression analysis can be used in order to deal with these quality adjustment problems.

In order to construct a commercial property price index, it is first necessary to obtain information on the *period by period value* of these properties. We conclude this section by noting that there are several methods that could be used to measure the value of a commercial property:

- Use transaction prices for commercial properties.
- Use appraisal values.
- Use property tax assessment values.
- Use estimates of future cash flows generated by the property.
- Use stock market information on Real Estate Investment Trusts (REITs).

The problem with using transactions prices is that commercial properties do not transact very frequently. Moreover, commercial properties are very heterogeneous and hence the selling price of a particular commercial property may not be very relevant for other properties. The problem with using appraisal or assessed values is that they may be somewhat arbitrary. They may be based on market transactions for comparable properties which are in fact, not really comparable. Or appraisals may be based on estimates of future cash flows, which are inherently uncertain and require many somewhat arbitrary assumptions about inflation rates for rents, input prices and interest rates. The use of stock market information is also problematic: REIT investors may rely on published appraisal values to justify their investment decisions and as we have indicated, appraisal values are somewhat subjective. Moreover, there are some technical problems that arise when one attempts to construct property values from stock market values; i.e., information on the outstanding debt of the REIT is required and the current value of the debt may not be easy to construct because it may not be very liquid. Furthermore, a REIT may hold a changing portfolio of buildings which will lead to complications. Finally, the portfolio of REIT properties may not be representative of the entire commercial property market.

To conclude this section, we will look at the pros and cons of using different data sources for commercial property values in more detail.

Research studies on commercial property price indexes have emphasized the problem of data selection when formulating indexes. Traditionally, transaction prices (also called market prices in the literature) have usually been used to estimate price indexes. However, the number of commercial property market transactions is extremely small. Furthermore, even if a sizable number of transaction prices can be obtained, the heterogeneity of the properties is so pronounced that it is difficult to compare like with like and thus the construction of reliable constant quality price indexes becomes very difficult.

Under such circumstances, many commercial property price indexes have been constructed using either appraisal prices from the real estate investment market, or using assessment prices for property tax purposes. The rationale for these price indexes is that, since appraisal prices and assessment prices for property tax purposes are regularly surveyed for the same commercial property, indexes based on these surveys hold most characteristics of the property constant[11], thus greatly reducing the heterogeneity problem as well as generating a wealth of data.

However, while appraisal prices look attractive for the construction of price indexes, they are somewhat subjective; i.e., exactly how are these appraisal prices constructed? Thus these prices lack the objectivity of market selling prices. Such considerations have led to the development of various arguments concerning the precision and accuracy of appraisal and assessment prices when used in measuring price indexes; see Shimizu and Nishimura (2006) on these issues. In particular, the literature on this issue has pointed out that an appraisal based index will typically lag actual turning points in the real estate market.[12] Geltner, Graff and Young (1994) clarified the structure of bias in the NCREIF Property Index, a representative U.S. index based on appraisal prices. In a later study, Geltner and Goetzmann (2000) estimated an index using commercial property transaction prices and demonstrated the magnitude of errors and the degree of smoothing in the NCREIF Property Index. These problems plague not only the NCREIF Property Index, but all indexes based on appraisal prices, including the MSCI-IPD Index.

In the case of appraisal prices for investment properties, a systemic factor of appraiser incentives emerges as an additional problem. This problem differs intrinsically from the lagging and smoothing problems that arise in appraisal based methods. Specifically, the incentive problem involves inducing higher valuations from appraisers in order to bolster investment performance; see Crosby, Lizieri and McAllister (2010) on this point.

In this connection, Bokhari and Geltner (2012) and Geltner and Bokhari (2019) estimated quality adjusted price indexes by running a time dummy hedonic regression using transaction price data. Geltner (1997) also used real estate prices determined by the stock market in order to examine the smoothing effects of the use of appraisal prices. Finally, Geltner, Pollakowski, Horrigan and Case (2010), Shimizu, Diewert, Nishimura and Watanabe (2015), Shimizu (2016) and Diewert and Shimizu (2017) (2019) proposed various estimation methods for commercial property price indexes using REIT data.

[11] Two important characteristics which are not held constant are the age of the structure and the amount of capital expenditures on the property between the survey dates. Changes in these characteristics are an important determinant of the property price.

[12] Another problem with appraisal based indexes is that they tend to be smoother than indexes that are based on market transactions. This can be a problem for real estate investors since the smoothing effect will mask the short term riskiness of real estate investments. However, for statistical agencies, smoothing short term fluctuations will probably not be problematic.

The above paragraphs indicate that it will not be easy to construct a Commercial Property Price Index. However, in the following sections of this paper, we will look at some attempts to construct a Tokyo CCPI using alternative data sources. We will also attempt to construct separate subindexes for the land and structure components of a CCPI.

In the following sections, we will examine three alternative data sources suggested in the literature that enable one to construct land price indexes for commercial properties in Tokyo Special District: (i) sales transactions data (1907 observations) in sections 3 and 4; (ii) appraisal data for Real Estate Investment Trusts (REITs) (1804 observations) in section 5 and (iii) assessed values of land for property taxation purposes (6242 observations) in section 6. We will utilize these three sources of data for commercial properties in Tokyo over 44 quarters covering the period Q1:2005 to Q4:2015 and compare the resulting land prices.

3. The Builder's Model with a Single Geometric Depreciation Rate

The *builder's model* for valuing a commercial property postulates that the value of a commercial property is the sum of two components: the value of the land which the structure sits on plus the value of the commercial structure.

In order to justify the model, consider a property developer who builds a structure on a particular property. The total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S square meters, times the building cost per square meter, β_t during quarter or year t, plus the cost of the land, which will be equal to the cost per square meter, α_t during quarter or year t, times the area of the land site, L. Now think of a sample of properties of the same general type, which have prices or values V_{tn} in period t [13] and structure areas S_{tn} and land areas L_{tn} for n = 1,...,N(t) where N(t) is the number of observations in period t. Assume that these prices are equal to the sum of the land and structure costs plus error terms ε_{tn} which we assume are independently normally distributed with zero means and constant variances. This leads to the following *hedonic regression model* for period t where the α_t and β_t are the parameters to be estimated in the regression:[14]

(1) $V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}$; $t = 1,...,T; n = 1,...,N(t).$

Note that the two characteristics in our simple model are the quantities of land L_{tn} and the quantities of structure floor space S_{tn} associated with property n in period t and the two *constant quality prices* in period t are the price of a square meter of land α_t and the price of a square meter of structure floor space β_t.

[13] The period index t runs from 1 to 44 where period 1 corresponds to Q1 of 2005 and period 44 corresponds to Q4 of 2015.

[14] Other papers that have suggested hedonic regression models that lead to additive decompositions of property values into land and structure components include Clapp (1980; 257-258), Bostic, Longhofer and Redfearn (2007; 184), Diewert (2008) (2010), Koev and Santos Silva (2008), de Haan and Diewert (2011), Francke (2008; 167), Rambaldi, McAllister, Collins and Fletcher (2010), Diewert, Haan and Hendriks (2011) (2015), Diewert and Shimizu (2015a) (2015b) (2016) (2017) (2019), Burnett-Isaacs, Huang and Diewert (2016), Rambaldi, McAllister and Fletcher (2016) and Diewert, Huang and Burnett-Isaacs (2017).

The hedonic regression model defined by (1) applies to new structures. But it is likely that a model that is similar to (1) applies to older structures as well. Older structures will be worth less than newer structures due to the (net) depreciation of the structure. Assuming that we have information on the age of the structure n at time t, say A(t,n), and assuming a geometric (or declining balance) depreciation model, a more realistic hedonic regression model than that defined by (1) above is the following *basic builder's model*:[15]

$$(2)\ V_{tn} = \alpha_t L_{tn} + \beta_t(1-\delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,T;\ n = 1,...,N(t)$$

where the parameter δ reflects the *net geometric depreciation rate* as the structure ages one additional period. Thus if the age of the structure is measured in years, we would expect an annual *net* depreciation rate to be between 2 to 3%.[16] Note that (2) is now a nonlinear regression model whereas (1) was a simple linear regression model.[17] The period t constant quality price of land will be the estimated coefficient for the parameter α_t and the price of a unit of a newly built structure for period t will be the estimate for β_t. The period t quantity of land for commercial property n is L_{tn} and the period t quantity of structure for commercial property n, expressed in equivalent units of a new structure, is $(1-\delta)^{A(t,n)}S_{tn}$ where S_{tn} is the space area of commercial property n in period t.

Note that the above model can be interpreted as a *supply side model* as opposed to the *demand side model* of Muth (1971) and McMillen (2003) since the value of a property with a new structure is equal to the cost of production. Basically, for newly developed properties, we are assuming competitive suppliers of commercial properties so that we are in Rosen's (1974; 44) Case (a), where the hedonic surface identifies the structure of supply. This assumption is justified for the case of newly built offices but it is less well justified for sales of existing commercial properties.[18]

This study compiled the following three types of micro-data relating to commercial properties in the Tokyo office market: (i) the transaction price data compiled by the Japanese Ministry of Land, Infrastructure, Transport and Tourism; (ii) the appraisal prices periodically determined in the Tokyo office REIT market; and (iii) the "official land prices" surveyed by the Japanese Ministry of Land,

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[15] This formulation follows that of Diewert (2008) (2010), de Haan and Diewert (2011), Diewert, de Haan and Hendriks (2015) and Diewert and Shimizu (2015b) (2016) (2017) in assuming property value is the sum of land and structure components but movements in the price of structures are proportional to an exogenous structure price index. This formulation is designed to be useful for national income accountants who require a decomposition of property value into structure and land components. They also need the structure index which in the hedonic regression model to be consistent with the structure price index they use to construct structure capital stocks. Thus the builder's model is particularly suited to national accounts purposes; see Schreyer (2001) (2009), Diewert and Shimizu (2015a) and Diewert, Fox and Shimizu (2016).

[16] This estimate of depreciation is regarded as a net depreciation rate because it is equal to a "true" gross structure depreciation rate less an average renovations appreciation rate. Since we do not have information on renovations and major repairs to a structure, our age variable will only pick up average gross depreciation less average real renovation expenditures.

[17] We used Shazam to perform the nonlinear estimations; see White (2004). Note that (2) is estimated as a single nonlinear regression using the data for all 44 quarters.

[18] For sold properties with older structures on them, we are basically following National Accounting conventions which postulates that property value is equal to the current value of the depreciated structure plus the current value of land; see Schreyer (2001) (2009).

Infrastructure, Transport and Tourism since 1970.[19] Official land prices are based on appraisals that are released on January 1st of each year. In Japan, asset taxes relating to land, such as inheritance taxes and fixed assets taxes, are assessed on the basis of these official land prices. Thus official land prices are considered as assessment data for tax purposes. As official land prices are exclusively based on surveys of land prices, they do not include structure prices.

Using the first two data sources, land price indexes were estimated using the Builder's Model. These land price indexes will be compared with those estimated using official land prices in later section of the paper. Our analysis covers the period from 2005 to 2015.

In estimating builder's model using transaction or appraisal prices in Tokyo, there is a major problem with the hedonic regression model defined by (2): The multicollinearity problem. Experience has shown that it is usually not possible to estimate sensible land and structure prices in a hedonic regression like that defined by (2) due to the multicollinearity between lot size and structure size.[20] Thus in order to deal with the multicollinearity problem, we drew on *exogenous information* on the cost of building new commercial properties from the Japanese Ministry of Land, Infrastructure, Transport and Tourism (MLIT) and we assumed that the price of new structures is equal to an official measure of commercial building costs (per square meter of building structure), p_{St}. Thus we replaced β_t in (2) by p_{St} for t = 1,...,44. This reduced the number of free parameters in the model by 44.

Experience has also shown that it is difficult to estimate the depreciation rate before obtaining quality adjusted land prices. Thus in order to get preliminary land price estimates, we temporarily assumed that the annual geometric depreciation rate δ in equation (2) was equal to 0.025. The resulting regression model became the model defined by (3) below:

$$(3)\quad V_{tn} = \alpha_t L_{tn} + p_{St}(1 - 0.025)^{A(t,n)}S_{tn} + \varepsilon_{tn}\,; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

The final log likelihood for this **Model 1** was −13328.15 and the R^2 was 0.4003.[21]

In order to take into account possible neighbourhood effects on the price of land, we introduced *ward dummy variables*, $D_{W,tnj}$, into the hedonic regression (3). There are 23 wards in Tokyo special district. We made 23 ward or locational dummy variables.[22] These 23 dummy variables were defined as follows: for t = 1,...,44; n = 1,...,N(t); j = 1,...,23:

$$(4)\quad D_{W,tnj} \equiv 1 \text{ if observation n in period t is in ward j of Tokyo;}$$

[19] For more details on the data and the regressions used in this study, see Diewert and Shimizu (2019).

[20] See Schwann (1998) and Diewert, de Haan and Hendriks (2011) and (2015) on the multicollinearity problem.

[21] Our R^2 concept is the square of the correlation coefficient between the dependent variable and the predicted dependent variable.

[22] The 23 wards (with the number of observations in brackets) are as follows: 1: Chiyoda (191), 2: Chuo (231), 3: Minato (205), 4: Shinjuku (203), 5: Bunkyo (97), 6: Taito (122), 7: Sumida (74), 8: Koto (49), 9: Shinagawa (69), 10: Meguro (28), 11: Ota (64), 12: Setagaya (67), 13: Shibuya (140), 14: Nakano (39), 15: Suginami (39), 16: Toshima (80), 17: Kita (30), 18: Arakawa (42), 19: Itabashi (35), 20: Nerima (40), 21: Adachi (19), 22: Katsushika (18), 23:Edogawa (25).

$\equiv 0$ if observation n in period t is not in ward j of Tokyo.

We modified the model defined by (3) to allow the *level* of land prices to differ across the Wards. The new nonlinear regression model is the following one:[23]

(5) $V_{tn} = \alpha_t(\sum_{j=1}^{23} \omega_j D_{W,tnj})L_{tn} + p_{St}(1 - 0.025)^{A(t,n)}S_{tn} + \varepsilon_{tn}$; $t = 1,...,44; n = 1,...,N(t)$.

Not all of the land time dummy variable coefficients (the α_t) and the ward dummy variable coefficients (the ω_j) can be identified. Thus we imposed the following normalization on our coefficients:

(6) $\alpha_1 = 1$.

The final log likelihood for the model defined by (5) and (6) was −12956.60 and the R^2 was 0.5925. Thus there was a large increase in the R^2 and a huge increase in the log likelihood of 371.55 over the previous model. However, many of the wards had only a small number of observations and thus it is unlikely that our estimated ω_j for these wards are very accurate.

In order to deal with the problem of too few observations in many wards, we used the results of the above model to group the 23 wards into 4 Combined Wards based on their estimated ω_j coefficients. The *Group 1 high priced wards* were 1,2,3 and 13 (their estimated ω_j coefficients were greater than 1), the *Group 2 medium high priced wards* were 4,5,6,9 and 14 ($0.6 < \omega_j \leq 1$), the *Group 3 medium low priced wards* were 7,8,10,12,15 and 16 ($0.4 < \omega_j \leq 0.6$), and the *Group 4 low priced wards* were 11,17,18,19 20,21,22 and 23 ($\omega_j \leq 0.4$).[24] We reran the nonlinear regression model defined by (5) and (6) using just the 4 Combined Wards (call this **Model 2**) and the resulting log likelihood was −12974.31 and the R^2 was 0.5850. Thus combining the original wards into grouped wards resulted in a small loss of fit and a decrease in log likelihood of 17.71 when we decreased the number of ward parameters by 19. We regarded this loss of fit as an acceptable tradeoff.

In our next model, we introduced some nonlinearities into the pricing of the land area for each property. The land plot areas in our sample of properties ran from 100 to 790 meters squared. Up to this point, we have assumed that land plots in the same grouped ward sell at a constant price per m^2 of lot area. However, it is likely that there is some nonlinearity in this pricing schedule; for example, it is likely that very large lots sell at an average price that is below the average price of medium sized lots. In order to capture this nonlinearity, we initially divided up our 1907 observations into 7 groups of observations based on their lot size. The Group 1 properties had lots less than 150 m^2, the Group 2 properties had lots greater than or equal to 150 m^2 and less than 200 m^2, the Group 3 properties had lots greater than or equal to 200 m^2 and less than 300 m^2, ... and the Group 7 properties had lots greater than or equal to 600 m^2. However, there were very few observations in Groups 4 to 7 so we added together these groups to

[23] From this point on, our nonlinear regression models are nested; i.e., we use the coefficient estimates from the previous model as starting values for the subsequent model. Using this nesting procedure is essential to obtaining sensible results from our nonlinear regressions. The nonlinear regressions were estimated using Shazam; see White (2004).

[24] The estimated combined ward relative land price parameters turned out to be: $\omega_1 = 1.3003$; $\omega_2 = 0.75089$; $\omega_3 = 0.49573$ and $\omega_4 = 0.25551$. The sample probabilities of an observation falling in the combined wards were 0.402, 0.278, 0.177 and 0.143 respectively.

form Group 4.[25] For each observation n in period t, we defined the 4 *land dummy variables*, $D_{L,tnk}$, for k = 1,...,4 as follows:

(7) $D_{L,tnk} \equiv 1$ if observation tn has land area that belongs to group k;
$\equiv 0$ if observation tn has land area that does not belong to group k.

These dummy variables are used in the definition of the following piecewise linear function of L_{tn}, $f_L(L_{tn})$, defined as follows:

(8) $f_L(L_{tn},\lambda) \equiv D_{L,tn1}\lambda_1 L_{tn} + D_{L,tn2}[\lambda_1 L_1+\lambda_2(L_{tn}-L_1)] + D_{L,tn3}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_{tn}-L_2)]$
$+ D_{L,tn4}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_3-L_2) +\lambda_4(L_{tn}-L_3)]$

where $\lambda \equiv [\lambda_1, \lambda_2, \lambda_3, \lambda_4]$ and the λ_k are unknown parameters and $L_1 \equiv 150$, $L_2 \equiv 200$ and $L_3 \equiv 300$. The function $f_L(L_{tn})$ defines a *relative valuation function for the land area of a commercial property* as a function of the plot area.

The new nonlinear regression model is the following one:

(9) $V_{tn} = \alpha_t(\sum_{j=1}^4 \omega_j D_{W,tnj})f_L(L_{tn},\lambda) + p_{St}(1-\delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}$; t = 1,...,44; n = 1,...,N(t).

Comparing the models defined by equations (5)[26] and (9), it can be seen that we have added an additional 4 *land plot size parameters*, $\lambda_1,...,\lambda_4$, to the model defined by (5) (with only 4 ward dummy variables). However, looking at (9), it can be seen that the 44 land time parameters (the α_t), the 4 ward parameters (the ω_j) and the 4 land plot size parameters (the λ_k) cannot all be identified. Thus we imposed the following identification normalizations on the parameters for **Model 3** defined by (9):

(10) $\alpha_1 \equiv 1$; $\lambda_1 \equiv 1$.

Note that if we set all of the λ_k equal to unity, Model 3 collapses down to Model 2. The final log likelihood for Model 3 was an improvement of 59.65 over the final LL for Model 2 (for adding 3 new marginal price of land parameters) which is a highly significant increase. The R^2 increased to 0.6116 from the previous model R^2 of 0.5850. The parameter estimates turned out to be $\lambda_2 = 1.4297$, $\lambda_3 = 1.2772$ and $\lambda_4 = 0.2973$. For small land plot areas less than 150 m^2, the (relative) marginal price of land was equal to 1 per m^2. As lot sizes increase from 150 m^2 to 200 m^2, the (relative) marginal price of land increased to $\lambda_2 = 1.4297$ per m^2. For the next 100 m^2 of lot size, the (relative) marginal price of land decreased to $\lambda_3 = 1.2772$ per m^2. For lot sizes greater than 200 m^2, the (relative) marginal price of land decreased to 0.2973 per m^2. Thus the average cost of land per m^2 initially increased and then tends to decrease as lot size becomes large.

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[25] The sample probabilities of an observation falling in the 7 initial land size groups were: 0.291, 0.234, 0.229, 0.130, 0.050, 0.034 and 0.033.

[26] We compare (9) to the modified equation (5) where we have only 4 combined ward dummy variables in the modified (5) rather than the original 23 ward dummy variables.

For property n in period t, we set the *price* of land for this property equal to $PL_{tn} = \alpha_t^*$, the estimated parameter value for α_t for t = 2,3,…,44 and we set $\alpha_1^* \equiv 1$. The corresponding constant quality *quantity* for property n in period t is $QL_{tn} \equiv (\sum_{j=1}^4 \omega_j^* D_{W,tnj}) f_L(L_{tn},\lambda^*)$ for t = 1,…,44 and n = 1,2,…,N(t). For property n in period t, the *price* and *quantity* of constant quality structure is set equal to p_{St} and $(1 - \delta^*)^{A(t,n)} S_{tn}$ respectively for t = 1,…,44 and n = 1,2,…,N(t) where δ^* is the estimated depreciation rate. Since all properties in period t have the same price of land α_t^* and the same price for the structure p_S^t, the *overall period t price indexes for land and structures*, PL^t and PS^t, are set equal to α_t^* and p_{St}/p_{S1} respectively for t = 1,…,44. The same definitions are used to define the aggregate price indexes for land and structures ($PL^t \equiv \alpha_t^*$ and $PS^t \equiv p_{St}/p_{S1}$)[27] for all of the hedonic regression models in this section and the subsequent two sections.

The *footprint* of a building is the area of the land that directly supports the structure. An approximation to the footprint land for property n in period t is the total structure area S_{tn} divided by the total number of stories in the structure H_{tn}. If we subtract footprint land from the total land area, TL_{tn}, we get *excess land* [28], EL_{tn}, defined as follows:

$$(11)\ EL_{tn} \equiv L_{tn} - (S_{tn}/H_{tn})\ ; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

In our sample, excess land ranged from 1.083 m^2 to 562.58 m^2. We grouped our observations into 5 categories, depending on the amount of excess land that pertained to each observation. Group 1 consists of observations tn where1: $EL_{tn} < 50$; 2: observations such that $50 \leq EL_{tn} < 100$; 3: $100 \leq EL_{tn} < 150$; 4: $150 \leq EL_{tn} < 300$; 5: $EL_{tn} \geq 300$.[29] Now define the excess land dummy variables, $D_{EL,tnm}$, as follows: for t = 1,...,44; n = 1,...,N(t); m = 1,...,5:

$$(12)\ D_{EL,tnm} \equiv 1 \text{ if observation n in period t is in excess land group m;}$$
$$\equiv 0 \text{ if observation n in period t is not in excess land group m.}$$

We will use the above dummy variables as adjustment factors to the price of land. As will be seen, in general, the more excess land a property possessed, the lower was the average per meter squared value of land for that property.[30]

The new **Model 4** excess land nonlinear regression model is the following one:

$$(13)\ V_{tn} = \alpha_t(\sum_{j=1}^4 \omega_j D_{W,tnj})(\sum_{m=1}^5 \chi_m D_{EL,tnm}) f_L(L_{tn},\lambda) + p_{St}(1 - \delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}\ ;$$

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[27] Thus PS^t is a normalization of the official construction price series p_{St} so that $PS^t = 1$ when t = 1. The series PS^t is plotted in Figure 2 below.

[28] This is land that is usable for purposes other than the direct support of the structure on the land plot. Excess land was first introduced as an explanatory variable in a property hedonic regression model for Tokyo condominium sales by Diewert and Shimizu (2016; 305).

[29] The sample probabilities of an observation falling in the 4 excess land size groups were: 0.352, 0.343, 0.149, 0.114 and 0.041.

[30] The excess land characteristic was also used by Diewert and Shimizu (2016) and Burnett-Isaacs, Huang and Diewert (2016) in their studies of condominium prices. The same phenomenon was observed in these studies: the more excess land that a high rise property had, the lower was the per meter land price.

$t = 1,...,44; n = 1,...,N(t).$

However, looking at the model defined by (9) and (13), it can be seen that the 44 land price parameters (the α_t), the 4 combined ward parameters (the ω_j), the 4 land plot size parameters (the λ_k) and the 5 excess land parameters (the χ_m) cannot all be identified. Thus we imposed the following identifying normalizations on these parameters:

(14) $\alpha_1 \equiv 1; \lambda_1 \equiv 1; \chi_1 \equiv 1.$

Note that if we set all of the χ_m equal to unity, Model 4 collapses down to Model 3. The final log likelihood for Model 4 was an improvement of 23.99 over the final LL for Model 3 (for adding 4 new excess land parameters) which is a significant increase. The R^2 increased to 0.6207 from the previous model R^2 of 0.6116. The χ_m parameter estimates turned out to be $\chi_2 = 0.9173$, $\chi_3 = 0.7540$, $\chi_4 = 0.7234$ and $\chi_5 = 0.8611$. Thus excess land does reduce the average per meter price of land.

The nonlinear estimating equations for **Model 5** are exactly the same as those defined by equations (13) above except that we estimated the geometric depreciation rate δ instead of assuming that it was equal to 0.025. The final LL increase for Model 5 (for adding one new parameter) was 50.58 which was highly significant. However, the estimated δ turned out to be 0.00165 with a standard error of 0.00152, which seemed low. The R^2 for this model was 0.6399.

It is likely that the height of the building affects the quality of the structure. In our sample of commercial property prices, the height of the building (the H variable) ranged from 3 stories to 14 stories. Thus initially, we had 12 building height categories. Define the building height dummy variables, $D_{H,tnh}$, as follows: for $t = 1,...,44$; $n = 1,...,N(t)$; $h = 3,...,14$:

(15) $D_{H,tnh} \equiv 1$ if observation n in period t is a building which has height h;
$\equiv 0$ if observation n in period t is not a building which has height h.

Due to the small number of observations in the last 5 height categories, we combined these dummy variables into a single height category that included all buildings of height 10 to 14 stories; i.e., the new $D_{H,tn10}$ was defined as $\Sigma_{h=10}^{14} D_{H,tnh}$. The new **Model 6** nonlinear regression model is the following one:

(16) $$V_{tn} = \alpha_t(\sum_{j=1}^{4}\omega_j D_{W,tn,j})(\sum_{m=1}^{5}\chi_m D_{EL,tnm})(\sum_{h=3}^{10}\mu_h D_{H,tnh})f_L(L_{tn},\lambda) + p_{St}(1-\delta)^{A(t,n)}(\sum_{h=3}^{10}\phi_h D_{H,tnh})S_{tn} + \varepsilon_{tn}\,; \quad t = 1,...,44; n = 1,...,N(t).$$

In addition to the normalizations (14), we also imposed the normalization $\phi_3 \equiv 1$. Note that if we set all of the μ_h equal to unity, the new model collapses down to Model 5.

The final log likelihood for the new model was −12,649.26, a big improvement of 190.83 over the final log likelihood for Model 5 (for adding 7 new height parameters). The R^2 increased to 0.7036 from the previous model R^2 of 0.6207. The ϕ_4 to ϕ_{10} parameter estimates turned out to be 1.2071, 1.4599, 1.5720, 1.5114, 2.0950, 2.3062, 2.5437 respectively. Recall that ϕ_3 is set equal to 1. It can be seen that the structure value of a property increased (with one exception) as the height of the building increased. The estimated geometric depreciation rate for this model was $\delta = 0.0212$ (with a standard error of 0.0020). This is a reasonable estimate for a wear and tear (net) depreciation rate.

Recall that we used building height as a quality adjustment factor for the structure portion of the property value. In our next model, we use building height as a possible quality adjustment factor for the land component of the property. Consider two adjacent commercial office properties with the same lot size and building footprint but Property A has a 10 story tower while property B has a 4 story modest office building. In theory, the land plot for each property should be valued at its best potential use. But the local market may not be able to support two high rise buildings in the same area. Hence the land component of Property B may not be valued at the same level as that of Property A, due to an accident of history. Moreover, placing a high rise building on Property B may lead to a decline in the land value of Property A due to an impairment of views (or sunlight) from the higher stories of Property A. In any case, we will introduce one new building height parameter μ that reflects possible changes in land value due to the height H of the building on the property. Thus **Model 7** is defined as the following nonlinear regression model:

$$(17)\ V_{tn} = \alpha_t(\textstyle\sum_{j=1}^{4}\omega_j D_{W,tnj})(\sum_{m=1}^{5}\chi_m D_{EL,tnm})(1+\mu(H_{tn}-3))f_L(L_{tn},\lambda) + p_{St}(1-\delta)^{A(t,n)}(\sum_{h=3}^{10}\phi_h D_{H,tnh})S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

For identification purposes, we imposed the following restrictions on the parameters in (17):

$$(18)\ \alpha_1 \equiv 1;\ \lambda_1 \equiv 1;\ \chi_1 \equiv 1;\ \phi_3 \equiv 1.$$

The final log likelihood for Model 7 was – 12640.40, an improvement of 8.86 log likelihood points over the final log likelihood for Model 6 (for adding one new parameter μ). The R^2 increased to 0.7063 from the previous model R^2 of 0.7036. The estimated depreciation rate δ was 3.41% with a standard error of 0.0077. The estimated $\phi_4,...,\phi_{10}$ were equal to 1.11, 1.31, 1.32, 1.11, 1.83, 2.01 and 2.12 (recall that ϕ_3 was set equal to 1). The estimate for μ was 0.1135 with a standard error of 0.0339. Thus as the building height increased by one story, the land value appears to increase by approximately 11%. Thus some of the extra cash flow generated by an extra story for the structure appears to leak over into the land value of the property.[31]

This completes our description of our preliminary hedonic regression models for Tokyo office buildings. In the following section, we will extend these preliminary models by estimating more complex depreciation schedules.

4. The Builder's Model with Multiple Geometric Depreciation Rates

In the following model, we allowed the geometric depreciation rates to differ after each 10 year interval (except for the last interval).[32] We divided up our 1907 observations into 5 groups of observations

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[31] It should be pointed out that our estimate for μ in our final model is 0.0602 instead of 0.1135.

[32] The analysis in this section and the subsequent section follows the approach taken by Diewert, Huang and Burnett-Isaacs (2017). Geltner and Bokhari (2019) estimate a much more flexible model of commercial property depreciation using US transaction data by allowing an age dummy variable for each age of building. This methodological approach generates a combined land and structure depreciation rate whereas our approach will generate depreciation rates that apply only to the structure portion of property value.

based on the age of the structure at the time of the sale. The Group 1 properties had structures with structure age less than 10 years, the Group 2 properties had structure ages greater than or equal to 10 years but less than 20 years, the Group 3 properties had structure ages greater than or equal to 20 years but less than 30 years, the Group 4 properties had structure ages greater than or equal to 30 years but less than 40 years and the Group 5 properties had structure ages greater than or equal to 40 years.[33] For each observation n in period t, we define the 5 *age dummy variables*, $D_{A,tni}$, for i = 1,...,5 as follows:

(19) $D_{A,tni} \equiv 1$ if observation tn has structure age that belongs to age group i;
$\equiv 0$ if observation tn has structure age that does not belong to age group i.

These age dummy variables are used in the definition of the following *aging function*, $g_A(A_{tn},\delta)$, defined as follows where $\delta \equiv [\delta_1,\delta_2\ \delta_3,\delta_4]$:[34]

(20) $$g_A(A_{tn},\delta) \equiv D_{A,tn1}(1-\delta_1)^{A(t,n)} + D_{A,tn2}(1-\delta_1)^{10}(1-\delta_2)^{(A(t,n)-10)} + D_{A,tn3}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{(A(t,n)-20)} + D_{A,tn4}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{10}(1-\delta_4)^{(A(t,n)-30)} + D_{A,tn5}(1-\delta_1)^{10}(1-\delta_2)^{10}(1-\delta_3)^{10}(1-\delta_4)^{10}(1-\delta_5)^{(A(t,n)-40)} .$$

Thus the annual geometric depreciation rates are allowed to change at the end of each decade that the structure survives.

The new **Model 8** nonlinear regression model is the following one:

(21) $$V_{tn} = \alpha_t(\sum_{j=1}^{4}\omega_j D_{W,tnj})(\sum_{m=1}^{5}\chi_m D_{EL,tnm})(1+\mu(H_{tn}-3))f_L(L_{tn},\lambda) + p_{St}\, g_A(A_{tn},\delta)(\Sigma_{h=3}^{10}\ \phi_h\, D_{H,tnh})S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

We imposed the normalizations $\alpha_1 \equiv 1$, $\lambda_1 \equiv 1$, $\chi_1 \equiv 1$ and $\phi_3 \equiv 1$. Note that Model 8 collapses down to Model 7 if $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta$. Thus the number of unknown parameters in Model 8 increased by 4 over the number of parameters in Model 7. The final log likelihood for Model 8 was −12631.21, an improvement of 9.19 over the final log likelihood for Model 7 (for adding 4 additional parameters). The R^2 increased to 0.7091 from the previous model R^2 of 0.7063. The estimated depreciation rates (with standard errors in brackets) were as follows: $\delta_1 = 0.0487$ (0.0111), $\delta_2 = 0.0270$ (0.0097), $\delta_3 = 0.0096$ (0.0106), $\delta_4 = 0.0403$ (0.0154), $\delta_5 = -0.0319$ (0.0185).[35] Thus properties with structures which are over 40 years old tended to have a negative depreciation rate; i.e., the value of the structure tends to *increase* by 3.19% per year.

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[33] There were only 28 properties which had age greater than 50 years so these properties were combined with the age 40 to 50 properties.

[34] A_{tn} is the same as A(t,n). The aging function $g_A(A_{tn})$ quality adjusts a building of age A_{tn} into a comparable number of units of a new building.

[35] Recall that these depreciation rates are net depreciation rates. As surviving structures approach their middle age, renovations become important and thus a decline in the net depreciation rate is plausible. The pattern of depreciation rates is similar to the comparable geometric depreciation rates that were observed for Richmond (a suburb of Vancouver, Canada) detached houses by Diewert, Huang and Burnett-Isaacs (2017).

There are two additional explanatory variables in our data set that may affect the price of land. Recall that DS was defined as the distance to the nearest subway station and TT as the subway running time in minutes to the Tokyo station from the nearest station. DS ranges from 0 to 1,500 meters while TT ranges from 1 to 48 minutes. Typically, as DS and TT increase, land value decreases.[36] **Model 9** introduces these new variables into the previous nonlinear regression model (21) in the following manner:

$$(22)\ V_{tn} = \alpha_t(\sum_{j=1}^{4}\omega_j D_{W,tnj})(\sum_{m=1}^{5}\chi_m D_{EL,tnm})(1+\mu(H_{tn}-3))(1+\eta(DS_{tn}-0))(1+\theta(TT_{tn}-1))\times f_L(L_{tn},\lambda) + p_{St}\, g_A(A_{tn},\delta)(\Sigma_{h=3}^{10}\, \phi_h\, D_{H,tnh})S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

Thus two new parameters, η and θ, are introduced. If these new parameters are both equal to 0, then Model 9 collapses down to Model 8.

The final log likelihood for Model 9 was −12614.70, an improvement of 16.51 over the final log likelihood for Model 8 (for adding 2 additional parameters). The R^2 increased to 0.7142 from the previous model R^2 of 0.7091. The estimated walking distance parameter was $\eta = -0.00023$ (0.000066), which indicates that commercial property land value does tend to decrease as the walking distance to the nearest subway station increases. However, the estimated travel time to Tokyo Central Station parameter was $\theta = 0.0209$ (0.0053) which indicates that land value increases on average as the travel time to the central station increases, a relationship which was not anticipated.

Recall that α_1 was set equal to 1. The sequence of coefficients $\alpha_1, \alpha_2, ..., \alpha_{44}$ comprise our estimated quarterly commercial office building price index for the land component of property value. This land price index is quite volatile due to the sparseness of commercial property sales and the heterogeneity of the properties. In the following section, we will show how this volatile land price index can be smoothed in a fairly simple fashion.

Turning to the other estimated coefficients, the ward relative land price parameters, ω_1-ω_4, decline (substantially) in magnitude as we move from the first more expensive composite ward to the less expensive composite wards. The marginal value of land parameters, λ_1 (set equal to 1), λ_2, λ_3 and λ_4, exhibit the same inverted U pattern that emerged in Model 3 (and persisted through all of the subsequent models). The excess land parameters, χ_1 (set equal to 1), χ_2, χ_3, χ_4 and χ_5, show that excess land is generally valued less than footprint land but the decline in land value as excess land increases is not monotonic. The building height land parameter $\mu = 0.0602$ is no longer as large as it was in Model 5 but an extra story of building height still adds 6% to the land value of the structure which is a significant premium for extra building stories. The walking distance to the nearest subway station parameter $\eta = -0.00023$ seems small but it tells us if the property is 1000 meters away from the nearest station, then the land value of the property is expected to fall by 23% compared to a nearby property. The travel time to Tokyo station parameter $\theta = 0.0209$ has a counterintuitive sign; it is possible that this variable is correlated with other land price determining characteristics and hence is not reliably determined. The height parameters, $\phi_3 = 1$ and ϕ_4-ϕ_{10}, are very significant determinants of structure value; the value of the structure increases almost monotonically as the number of stories increases. Finally, the decade by decade estimated geometric depreciation rates, δ_1-δ_5, show much the same pattern as was shown by the results for the previous model. Overall, the results of Model 9 seem to be reasonable.

[36] See Diewert and Shimizu (2015b) where these relationships also held for Tokyo detached houses.

5. The Builder's Model with Multiple Straight Line Depreciation Rates

Thus far, we have assumed that geometric depreciation models can best describe our data. In this section, we check the robustness of our approach in the previous section by assuming alternative depreciation models.

Recall that the structure aging (or survival) function for Model 9, $g_A(A_{tn},\delta)$, was defined by (20) above. In this section, we switch from a geometric model of depreciation to a *straight line* or *linear depreciation model*. Thus for **Model 10**, we define the aging function as follows:

$$(23)\ g_A(A_{tn},\delta) \equiv (1-\delta A_{tn})$$

where δ is the straight line depreciation rate. Our new nonlinear regression model is the same as the previous model defined by equations (22) except that the function g_A is defined by (23). The starting parameter values were taken to be the final parameter values from Model 7 except that the initial δ was set equal to 0.01 and the initial values for the parameters η and θ were set equal to 0.

The final log likelihood for Model 10 was −12635.83 and the R^2 was 0.7078. The estimated straight line depreciation rate was $\delta = 0.01357$ (0.00127). This model generated reasonable parameter estimates and the imputed value of the structure component of property value was positive for all observation.[37]

The straight line model of depreciation is not very flexible. Thus following the approach used by Diewert and Shimizu (2015b), we implemented a *piece-wise linear depreciation model*. Recall definitions (19) above which defined the 5 age dummy variables, $D_{A,tni}$, for i = 1,...,5. We use these age dummy variables to define the piece-wise linear aging function, $g_A(A_{tn},\delta)$, as follows:

$$\begin{aligned}(24)\ g_A(A_{tn},\delta) \equiv\ & D_{A,tn1}(1-\delta_1 A_{tn}) + D_{A,tn2}(1-10\delta_1-\delta_2(A_{tn}-10)) \\ & + D_{A,tn3}(1-10\delta_1-10\delta_2-\delta_3(A_{tn}-20)) + D_{A,tn4}(1-10\delta_1-10\delta_2-10\delta_3-\delta_4(A_{tn}-30)) \\ & + D_{A,tn5}(1-10\delta_1-10\delta_2-10\delta_3-10\delta_4-\delta_5(A_{tn}-40))\end{aligned}$$

where δ is now defined as $\delta \equiv [\delta_1,\delta_2,\delta_3,\delta_4,\delta_5]$. The **Model 11** nonlinear regression model is the same as the model defined by equations (22) except that the function g_A is defined by (24). The starting parameter values were taken to be the final parameter values from Model 10 except that the new depreciation parameters $\delta_1,...,\delta_5$ were all set equal to the final straight line depreciation rate δ estimated in Model 10. If all 5 δ_i are set equal to a common δ, then Model 11 collapses down to Model 10.

The final log likelihood for Model 11 was −12614.35, which was an increase in log likelihood of 21.48 over the Model 10 log likelihood. The R^2 for Model 11 was 0.7143.[38] The estimated piecewise linear

[37] This does not always happen for straight line depreciation models; i.e., for properties with very old structures, the imputed value of the structure can become negative if the estimated depreciation rate is large enough. This phenomenon cannot occur with geometric depreciation models, which is an advantage of assuming this form of depreciation.

[38] Recall that the log likelihood for the comparable geometric model of depreciation, Model 9, was −12614.70 and the R^2 for Model 9 was 0.7142. Thus the descriptive power of both models is virtually identical.

depreciation rates (with standard errors in brackets) were as follows: δ_1 = 0.0393 (0.0057), δ_2 = 0.0125 (0.0049), δ_3 = 0.0302 (0.0041),[39] δ_4 = 0.0159 (0.0054), δ_5 = –0.0135 (0.0074). Thus as was the case with the multiple geometric depreciation rate model, properties with structures which are over 40 years old tended to *increase* in value by 1.35% per year.

Comparing the estimated coefficients, the parameter estimates for Models 9 and 11 were very similar except that there were some differences in the estimated depreciation rates δ_1 to δ_5. However, it turns out that the ageing functions generated by these alternative depreciation models approximate each other reasonably well. Thus both depreciation models describe the underlying data to the same degree of approximation.

The determination of depreciation schedules for commercial office buildings is important for tax purposes, for investors and for the estimation of commercial office structure stocks, which in turn feed into the computation of the Multifactor Productivity of the Commercial Office Sector. The methodology explained above should be helpful to national income accountants and tax offices who require estimates for depreciation rates.

As was mentioned in section 3 above, all of the above models generate land and structure price indexes which are equal to the sequence of estimated α_t^* estimates for Models 1-11 for the land indexes PL_t and are equal to the sequence of official new building cost series p_{St}. Thus the structure price series remain constant across models but the land price series will vary across the various models. In the following section, we will chart the land price series for Model 11.

6. Smoothing the Land Price Series

In Figure 1 below, it can be seen that our Model 11 estimated land price series, $PL_{MLIT}^t \equiv \alpha_t^*$, is somewhat volatile. This is due to the fact that commercial properties are very heterogeneous and we have relatively few transactions per quarter. Thus the raw series PL_{MLIT}^t does not accurately represent the *trend* in commercial land prices in Tokyo; the raw series requires some smoothing in order to model the trends in land prices.[40] Patrick (2017) found the same problem for Irish house price sales and we will follow his example and smooth the raw series.[41]

We used the LOWESS nonparametric smoother in Shazam[42] in order to construct a preliminary smoothed land price series, PL_S^t, using the Model 11 land price series, PL_{MLIT}^t, as the input series.[43] We

[39] Recall that these depreciation rates are net depreciation rates. As surviving structures approach their middle age, renovations become important and thus a decline in the net depreciation rate is plausible. The pattern of depreciation rates is similar to the comparable geometric depreciation rates that were observed for Richmond (a suburb of Vancouver, Canada) detached houses by Diewert, Huang and Burnett-Isaacs (2017).

[40] The volatility in the raw series could be real phenomenon in that land prices are inherently volatile. If this is the case, it would be useful for statistical offices to publish the unsmoothed series as well as the smoothed series. As noted by Geltner, Miller, Clayton and Eichholtz (2014), property investors would find unsmoothed property price indexes useful in order to evaluate the riskiness of property investments. On the other hand, the volatility may be due to the heterogeneity of commercial properties (and the scarcity of market transactions). Thus there are important price determining characteristics of these properties that we have not taken into account in our regressions and this leads to volatility in our indexes.

[41] Patrick initially smoothed his series by taking a three month rolling average of the raw index prices for Ireland. He found that the resulting index was still too volatile to publish and he ended up using a double exponential smoothing procedure.

[42] The method is due to Cleveland (1979).

used the cross-validation criterion to choose the smoothing parameter which turned out to be f = 0.12. The Model 11 land prices PL_{MLIT}^t and their smoothed counterparts PL_S^t are plotted in Figure 1 below.

The jagged black line in Figure 1 represents the unsmoothed land price index PL_{MLIT} that we estimated from Model 11 while the lowest line represents the LOWESS nonparametric smoothed series PL_S that was generated using Shazam. It can be seen that while PL_S is reasonably smooth, it is not quite centered; i.e., it is consistently below the raw series. Thus we considered some alternative methods for smoothing the raw series.

Henderson (1916) was the first to realize that various moving average smoothers could be related to rolling window least squares regressions that would exactly reproduce a polynomial curve. Thus we apply his idea to derive the moving average weights that would be equivalent to fitting a linear function to 5 consecutive quarters of a time series, which we represent by the vector $Y^T \equiv [y_1,...,y_5]$ where Y^T denotes the transpose of a vector Y. Define the 5 dimensional column vectors X_1 and X_2 as $X_1 \equiv [1,1,1,1,1]^T$ and $X_2 \equiv [-2,-1,0,1,2]^T$. Define the 5 by 2 dimensional X matrix as $X \equiv [X_1,X_2]$. Denote the linear smooth of the vector Y by Y^*. Then least squares theory tells us that $Y^* = X(X^TX)^{-1}X^TY$. Thus the 5 rows of the 5 by 5 projection matrix $X(X^TX)^{-1}X^T$ give us the weights that can be used to convert the raw Y series into the smoothed Y^* series. For our particular example, the 5 rows of the projection matrix are as follows: Row 1 = (1/10)[6,4,2,0,–2]; Row 2 = (1/10)[4,3,2,1,0]; Row 3 = (1/5)[1,1,1,1,1]; Row 4 = (1/10)[0,1,2,3,4]; Row 5 = (1/10)[–2,0,2,4,6]. Note that Row 3 tells us that the third component of the smoothed vector Y^* is equal to $y_3^* = (1/5)(y_1+y_2+y_3+y_4+y_5)$. a simple equally weighted moving average of the raw data for 5 periods. Thus the way this smoothing method could be applied in practice to 44 consecutive quarters of PL_{MLIT} data is as follows. The first 3 components of the smoothed series are set equal to the inner products of the first 3 rows of the projection matrix $X(X^TX)^{-1}X^T$ times the first 5 components of the PL_{MLIT} series. This would generate the first 3 components of the smoothed series, PL_L^t for t = 1,2,3. For t = 3,4,...,42, define $PL_L^t \equiv (1/5)[PL_{MLIT}^{t-2} + PL_{MLIT}^{t-1} + PL_{MLIT}^{t} + PL_{MLIT}^{t+1} + PL_{MLIT}^{t+2}]$. Thus for all observations t except for the first two and last two observations, the smoothed series PL_L^t would be defined as the simple centered moving average of 5 consecutive PL_{MLIT} observations with equal weights. The final two observations would be defined as the inner products of Rows 4 and 5 of $X(X^TX)^{-1}X^T$ with the last 5 observations in the PL_{MLIT} series. In practice, as the data of a subsequent period became available, the last two observations in the existing series would be revised but after receiving the data of two subsequent periods, there would be no further revisions; i.e., the final smoothed value of an observation would be set equal to the centered 5 period moving average of the raw data.

[43] The initial smoothed series was divided by the Quarter 1 value so that the resulting normalized series equalled 1 in Quarter 1. Recall that Quarter 1 is the first quarter in 2005 and Quarter 44 is the last quarter in 2015.

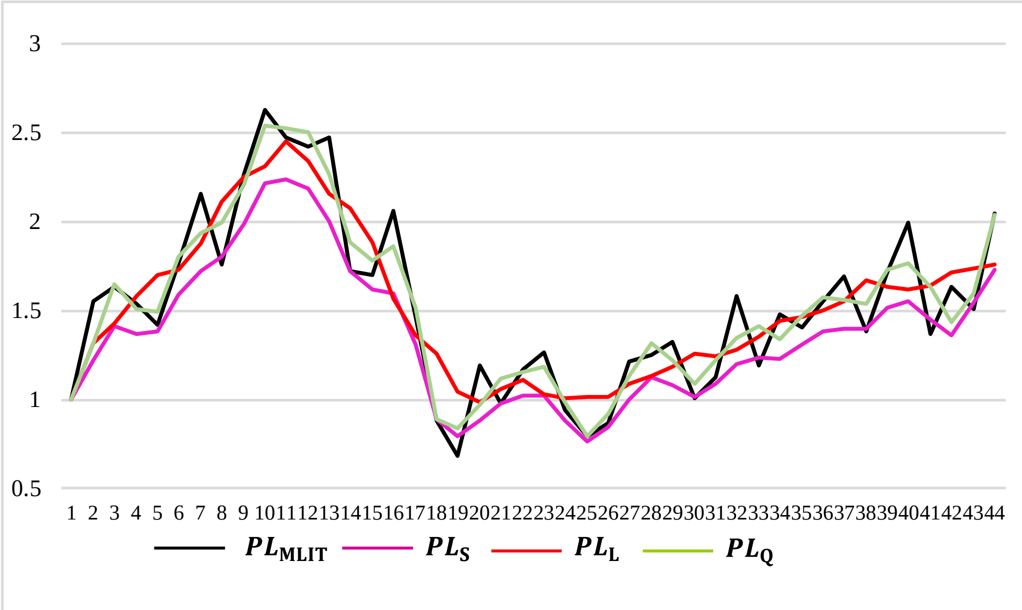

Figure 1: MLIT Land Prices, Lowess Smoothed Prices and Linear and Quadratic Smoothed Prices

We implemented the above procedure but the above algorithm does not ensure that the value of the smoothed series in the first quarter of the sample is equal to 1 and so the generated series had to be divided by a constant to ensure that the first observation in the smoothed series is equal to unity. We found that this division caused the smoothed series to lie below the raw series for the most part.[44] Patrick (2017; 25-26) found that a similar problem occurred with his initial simple moving average smoothing method. He solved the problem by setting the smoothed values equal to the actual values for the first two observations when he applied his second smoothing method. We solved the centering problem in a similar manner: we set the initial value of the smooth equal to the corresponding raw number (so that $PL_L^1 \equiv PL_{MLIT}^1$) and we set the second value of the smooth equal to the average of the first and third observations in the raw series (so that $PL_L^2 \equiv (1/2)[PL_{MLIT}^1 + PL_{MLIT}^3]$). For the Quarter 3 value of the smooth, we used the simple 5 term centered moving average so that $PL_L^3 \equiv (1/5)[PL_{MLIT}^1 + PL_{MLIT}^2 + PL_{MLIT}^3 + PL_{MLIT}^4 + PL_{MLIT}^5]$ and we carried on using this moving average until Quarters 43 and 44 where we used Rows 4 and 5 of the matrix $X(X^TX)^{-1}X^T$ defined above for our moving average weights. The resulting smoothed series PL_L^t plotted in Figure 1 above. It can be seen that it does a good job of smoothing the initial PL_{MLIT}^t series.

We also applied the same least squares methodology to a rolling window 5 term quadratic regression model. Define the 5 dimensional column vectors X_1 and X_2 as before and define $X_3 \equiv [4,1,0,1,4]^T$. Define the 5 by 3 dimensional X matrix as $X \equiv [X_1,X_2,X_3]$. Denote the quadratic smooth of the vector Y by Y^{**}. Again least squares theory tells us that $Y^{**} = X(X^TX)^{-1}X^TY$. The 5 rows of the new 5 by 5

[44] A similar problem of a lack of centering occurred when we implemented the LOWESS smoothing procedure; i.e., we had to divide by a constant to make the first component of the smoothed series equal to one. As a result, the Lowess smooth tended to lie below the raw series as can be seen in Figure 2.

projection matrix $X(X^TX)^{-1}X^T$ give us the weights that can be used to convert the raw Y series into the smoothed Y^{**} series. The 5 rows of the new projection matrix are as follows: Row 1 = (1/35)[31,9,–3,–5,3]; Row 2 = (1/35)[9,13,12,6,–5]; Row 3 = (1/35)[−3,12,17,12,–3]; Row 4 = (1/35)[−5,6,12,13,9]; Row 5 = (1/35)[3,–5,–3,9,31]. Now repeat the steps that were used to construct the linear smooth PL_L^t to construct a preliminary quadratic smooth PL_Q^t, except that the new 5 by 5 projection matrix $X(X^TX)^{-1}X^T$ replaces the previous one. A final PL_Q^t series was constructed by replacing the first 2 values in the smoothed series by the same initial 2 values that we used to construct the final versions of PL_L^1 and PL_L^2. The resulting smoothed series PL_Q^t plotted in Figure 1 above. It can be seen that PL_Q^t is not nearly as smooth as the linear smoothed series PL_L^t but of course, it is a lot closer to the unadjusted series PL_{MLIT}^t. For our particular data set, we would recommend the linear smoother over the quadratic smoother.[45]

7. The Use of Appraisal Prices as the Data Source in the Builder's Model

We turn now to the construction of land prices using commercial property appraisal data.

As was indicated in the introduction, we have quarterly appraisal data for 41 commercial office REIT office buildings located in Tokyo for the 44 quarters starting at Q1:2005 and ending at Q4:2015, which of course, is the same period that was covered by the MLIT selling price data. We will implement the builder's model for this data set in this section.

The builder's model using appraisal data is somewhat different from the builder's model using selling price data. The panel nature of the REIT data means that we can use a single property specific dummy variable as a variable that concentrates all of the location attributes of the property into a single property dummy variable; i.e., we do not have to worry about how close to a subway line the property is or how many stories the building has or how much excess land is associated with the property. The single property specific dummy variable will take all of these characteristics into account.

There are 41 separate properties in our REIT data set. For each of our 44 quarters, we assume that the 41 properties appear in the appraised property value for property n in period t, V_{tn}, in the same order. Our initial regression model is the following one where the variables have the same definitions as in equations (5) above except that ω_n is now the *property n sample average land price* (per m^2) rather than a Ward n relative price of land:

$$(25)\ V_{tn} = \sum_{n=1}^{41} \omega_n L_{tn} + p_{St}(1-0.025)^{A(t,n)} S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,\ldots,44;\ n = 1,\ldots,41.$$

Thus in **Model 1** above, there are no quarter t land price parameters in this very simple model with 41 unknown property average land price ω_n parameters to estimate. Note that the geometric (net) depreciation rate in the model defined by (25) was assumed to be 2.5% per year.

The final log likelihood for this model was −14968.77 and the R^2 was 0.9426. Thus the 41 property average price of land parameters ω_n explain a large part of the variation in the data.

[45] A quadratic Henderson type smoother would be much smoother if we lengthened the window. But a longer window would imply a longer revision period before the series would be finalized. Since the linear smoother with window length 5 seems to do a nice job of smoothing, we would not recommend moving to a longer window length for this particular application.

In **Model 2**, we introduce quarterly land prices α_t into the above model. The new nonlinear regression model is the following one:

(26) $V_{tn} = \sum_{n=1}^{41} \alpha_t\omega_n L_{tn} + p_{St}(1 - 0.025)^{A(t,n)}S_{tn} + \varepsilon_{tn}$; t = 1,...,44; n = 1,...,41.

Not all of the quarterly land price parameters (the α_t) and the average property price parameters (the ω_n) can be identified. Thus we impose the following normalization on our coefficients:

(27) $\alpha_1 = 1$.

We used the final parameter values for the ω_n from Model 1 as starting coefficient values for Model 2 (with all α_t initially set equal to 1).[46] The final log likelihood for Model 2 was −13999.00, a huge improvement of 969.77 for adding 43 new parameters. The R^2 was 0.9804. Thus the 41 property average price parameters ω_n and the 43 quarterly average land price parameters α_t explain most of the variation in the data.

Model 3 is the following nonlinear regression model:

(28) $V_{tn} = \sum_{n=1}^{41}\alpha_t\omega_n L_{tn} + p_{St}(1 - \delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}$; t = 1,...,44; n = 1,...,41

where δ is the annual geometric (net) depreciation rate. The normalization (27) is also imposed. Thus Model 3 is the same as Model 2 except that we now estimate the single geometric depreciation rate δ.

We used the final parameter values for the α_t and ω_n from Model 2 as starting coefficient values for Model 3 (with δ initially set equal to 0.025). The final log likelihood for this model was −13993.47, and increase of 5.53 for one additional parameter, and the R^2 was 0.9806. The estimated geometric (net) depreciation rate was $\delta = 0.01353$.[47] Recall that α_1 was set equal to 1. The sequence of land price (per m^2) α_t, for t =1,2,...,44 is our estimated sequence of quarterly Tokyo land prices, PL_{REIT}^t, which appears in Figure 2 below.

The implied standard errors on the quarterly land price coefficients, the α_t, were fairly large whereas they were fairly small for the property coefficients, the ω_n. This means that our estimated land price indexes, $PL_{REIT}^t = \alpha_t$, were not reliably determined. Note also that our estimated geometric depreciation rate δ is only 1.35% per year which is much lower than our estimated depreciation rate from Model 7 in Section 3 above which was 3.41% per year. One factor which may help to explain this divergence in

[46] The reader may well wonder why we estimated the ω_n in Model 1 rather than first estimating the α_t in Model 1. When this alternative strategy was implemented, we found that the resulting Model 2 did not converge to the "right" parameter values; i.e., the resulting R^2 was very low. This is the reason for following our nested estimation methodology where each successive model uses the final coefficient values from the previous model. It is not possible to simply estimate our final models in one step and obtain sensible results.

[47] We also estimated the straight line depreciation model counterpart to Model 3. The resulting estimated straight line depreciation rate δ was equal to 0.01317 (t statistic = 45.73). The R^2 for this model was 0.9806 and the final log likelihood was −13989.83. The resulting land price series was very similar to the land price series generated by Model 3 above.

estimates of wear and tear depreciation is that appraisers take into account capital expenditures on the properties. However, our current data base did not have information on capital expenditures and it is likely that not having capital expenditures as an explanatory factor affected our estimates for the depreciation rate. In our earliest study of land prices using REIT data for Tokyo, Diewert and Shimizu (2017), we adjusted our nonlinear regressions for capital expenditures and found that the resulting estimated quarterly wear and tear geometric depreciation rate was 0.005 which implied an annual (single) geometric depreciation rate of about 2%.[48]

In the following section, we will estimate our final land price series for Tokyo commercial office buildings using official estimates for the land values of commercial properties for taxation purposes.

8. The Use of Land Tax Assessment Values as the Data Source

In this section, we will use the Official Land Price (OLP) data described in section 2 above. We have 6242 annual assessed values for the land components of commercial properties in Tokyo covering the 11 years 2005-2015. We will label these years as t = 1,2,...,11. The assessed land value for property n in year t is denoted as V_{tn}.[49] We have information on which Ward each property is located and the ward dummy variables $D_{W,tnj}$ are defined by definitions (4) above. The land plot area of property n in year t is denoted by L_{tn} and the subway variables DS_{tn} and TT_{tn} are defined as in previous section above. The number of observations in year t is N(t).

Our initial regression model is the following one where we regress property land value on the ward dummy variables times the land plot area:

$$(29)\ V_{tn} = (\sum_{j=1}^{23} \omega_j D_{W,tnj})L_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,11;\ n = 1,...,N(t).$$

Thus in **Model 1** above, there are no year t land price parameters in this very simple model and ω_j is an estimate of the average land price (per m^2) in Ward j for j = 1,...,23. The final log likelihood for this model was −67073.91 and the R^2 was 0.3647.

In **Model 2**, we introduce annual land prices α_t into the above model. The new nonlinear regression model is the following one:

$$(30)\ V_{tn} = \alpha_t(\sum_{j=1}^{23} \omega_j D_{W,tnj})L_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,11;\ n = 1,...,N(t).$$

[48] In the multiple geometric depreciation rate model estimated by Diewert and Shimizu (2017), the various rates averaged out to an annual rate of 2.6% per year. Our earlier study covered the 22 quarters starting at Q1 of 2007 and ending at Q2 of 2012. The correlation coefficient between the price of land series in this model in Diewert and Shimizu (2017) and the above Model 3 price of land series for the overlapping 22 quarters is 0.9901 so these two studies using REIT appraisal data show much the same trends in Tokyo commercial property land prices even though the estimated wear and tear depreciation rates are different. Note that in addition to wear and tear depreciation, depreciation due to the early demolition of a structure before it reaches "normal" retirement age should be taken into account. Our current study does not estimate this extra component of depreciation. However, Diewert and Shimizu (2017) estimated *demolition depreciation* for Tokyo commercial office buildings at 1.2% per year.

[49] The units of measurement used in this section are in 100,000 yen.

Not all of the 11 annual land price parameters (the α_t) and the 23 Ward average property relative price parameters (the ω_n) can be identified. Thus we impose the normalization $\alpha_1 = 1$.

We used the final parameter values for the ω_n from Model 1 as starting coefficient values for Model 2 (with all α_t initially set equal to 1). The final log likelihood for Model 2 was −67022.90, an increase of 51.01 for adding 43 new parameters. The R^2 was 0.3748.

In our next model, we allowed the price of land to vary as the lot size increased. We divided up our 6242 observations into 5 groups of observations based on their lot size. The Group 1 properties had lots less than 100 m^2, the Group 2 properties had lots greater than or equal to 100 m^2 and less than 150 m^2, the Group 3 properties had lots greater than or equal to 150 m^2 and less than 200 m^2, the Group 4 properties had lots greater than or equal to 200 m^2 and less than 300 m^2 and the Group 5 properties had lots greater than or equal to 300 m^2.[50] For each observation n in period t, we define the 5 *land dummy variables*, $D_{L,tnk}$, for k = 1,...,5 as follows:

$$(31)\ D_{L,tnk} \equiv 1 \text{ if observation tn has land area that belongs to group k;}$$
$$\equiv 0 \text{ if observation tn has land area that does not belong to group k.}$$

Define the constants L_1-L_4 as 100, 150, 200 and 300 respectively. These constants and the dummy variables defined by (31) are used in the definition of the following piecewise linear function of L_{tn}, $f(L_{tn})$:

$$(32)\ f(L_{tn}) \equiv D_{L,tn1}\lambda_1 L_{tn} + D_{L,tn2}[\lambda_1 L_1+\lambda_2(L_{tn}-L_1)] + D_{L,tn3}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_{tn}-L_2)]$$
$$+ D_{L,tn4}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_3-L_2)+\lambda_4(L_{tn}-L_3)]$$
$$+ D_{L,tn5}[\lambda_1 L_1+\lambda_2(L_2-L_1)+\lambda_3(L_3-L_2)\ )+\lambda_4(L_4-L_3)+\lambda_5(L_{tn}-L_4)].$$

Model 3 was defined as the following nonlinear regression model:

$$(33)\ V_{tn} = \alpha_t(\textstyle\sum_{j=1}^{23} \omega_j D_{W,tn,j})f(L_{tn}) + \varepsilon_{tn}\ ; \qquad t = 1,...,11;\ n = 1,...,N(t).$$

We imposed the normalizations $\alpha_1 = 1$ and $\lambda_1 = 1$ so that all of the remaining parameters in (33) could be identified. These normalizations were also imposed in Model 4 below.

We used the final parameter values for the α_t and ω_j from Model 2 as starting coefficient values for Model 3 (with all λ_k initially set equal to 1). Thus Model 3 adds the 4 new marginal prices of land, λ_2, λ_3, λ_4 and λ_5 to Model 2. The final log likelihood for Model 3 was −66044.02, an increase of 978.88 for adding 4 new parameters. The R^2 was 0.4668.

Our final land price model added the subway variables to Model 3. Thus **Model 4** was defined as the following nonlinear regression model:[51]

[50] The sample probabilities of an observation falling in the 5 land size groups were: 0.171, 0.285, 0.175, 0.178 and 0.191.

[51] The minimum value for the distance to the nearest subway station DS_{tn} is 50 meters and the minimum value for the subway running time from the nearest station to the central Tokyo subway station was 4 minutes.

(34) $V_{tn} = \alpha_t(\sum_{j=1}^{23} \omega_j D_{W,tnj})\ (1+\eta(DS_{tn}-50))(1+\theta(TT_{tn}-4))f(L_{tn}) + \varepsilon_{tn}$; $t = 1,...,11;\ n = 1,...,N(t)$.

Thus Model 4 has added two new subway parameters, η and θ, to Model 3. We used the final parameter values for the α_t, ω_j and λ_k from Model 3 as starting coefficient values for Model 4 (with η and θ initially set equal to 0). The final log likelihood for Model 4 was −65584.56, an increase of 459.46 for adding 2 new parameters. The R^2 was 0.5401. The α_t sequence of estimated parameters (along with $\alpha_1 \equiv 1$) forms an annual (quality adjusted) Official Land Price series. For comparison purposes, we repeat each α_t four times and convert the annual Official Land Price series into the quarterly Official Land Price series, PL_{OLP}^t. It will be listed and compared with our final transactions based MLIT land price series PL_{MLIT}^t and its linear smooth PL_L^t along with our final REIT based land price series PL_{REIT}^t in the following section.

The standard errors on the estimated annual land prices α_t were fairly small; they were fairly large for the REIT based quarterly land price series, PL_{REIT}^t. The estimated λ_2, λ_3, λ_4, and λ_5 were 0.7011, −0.3331, 0.3568 and 0.1440 respectively. Except for λ_3, it can be seen that the λ_k monotonically decrease as k increases; this indicates that the marginal price of land decreases (for the most part) as the land plot size increases. The estimates for the subway parameters were $\eta^* = -0.000740$ and $\theta^* = -0.022807$. These estimates have the expected negative sign and are reasonable in magnitude. Since we do not have additional information on the height or size of the buildings, we cannot add more explanatory variables to the Model 4 regression.

The 4 land price series, PL_{MLIT}^t (the transaction price based series), PL_L^t (our preferred smoothed version of PL_{MLIT}^t), PL_{REIT}^t (the appraisal based series) and PL_{OLP}^t (the tax assessment based series), along with the official (normalized) construction price series PS^t plotted in Figure 2.

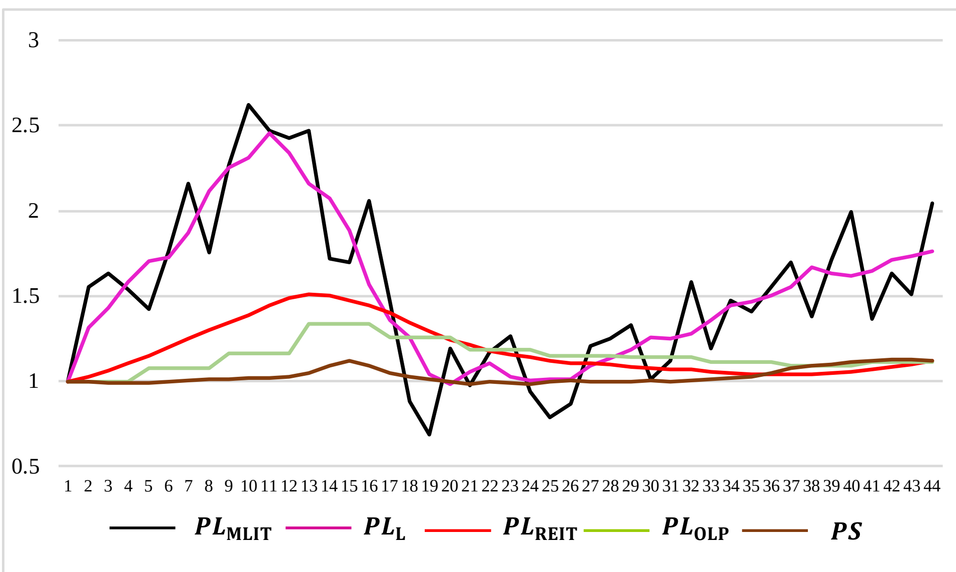

Figure 2: Alternative Land Price Series and the Price of Structures

It can be seen that the land price series based on transactions data, PL_{MLIT}^t and its linear smooth, PL_L^t, paint a very different picture of land price movements as compared to the series based on appraisal values for commercial land in Tokyo, PL_{REIT}^t, and the series based on property tax assessed values, PL_{OLP}^t. As was noted in previous section above, appraisal prices tend to lag behind the movements in transaction prices and they also smooth the sales data. The same phenomenon evidently applies to assessed value prices. Figure 2 shows that the appraisal and assessed value based price indexes for commercial land fluctuate far less than the index based actual transactions prices. However, it can be seen that the appraisal and assessed value series do tend to move in the same direction as the transactions prices but with a lag. The Figure also shows the problem with the transactions based series: its quarter to quarter fluctuations are massive. But it also can be seen that the linear smoothed series PL_L^t (which is essentially a centered five quarter moving average of the unsmoothed series PL_{MLIT}^t) captures the trend in transactions prices quite well. This series can be finalized after a two quarter delay. Our preferred land price series is the linear smoothed transaction series PL_L^t.

In the following section, we will use the MLIT and REIT data to construct alternative commercial property price indexes; i.e., we will aggregate the land and structure price data into overall property price indexes and compare these indexes with other indexes which are simpler to construct.

9. Overall Commercial Property Price Indexes

Recall that in section 3, the MLIT value of property n in quarter t was defined as V_{tn} in period t and the corresponding property land and structure areas were defined as S_{tn} and L_{tn} for n = 1,...,N(t) and t = 1,...,44. In the property price literature, a frequently used index of overall property prices is the period average of the individual property values V_{tn} divided by the corresponding structure areas S_{tn}. Thus define the (preliminary) quarter t mean property price P_{MEANP}^t as follows:

$$(35)\quad P_{MEANP}^t \equiv (1/N(t))\Sigma_{n=1}^{N(t)} V_{tn}/S_{tn}\ ; \qquad t = 1,...,44.$$

The final mean property price index for quarter t, P_{MEAN}^t, is defined as the corresponding preliminary index P_{MEANP}^t divided by P_{MEANP}^1; i.e., we normalize the series defined by (35) to equal 1 in quarter 1.

As could be expected, the mean property price series P_{MEAN}^t is rather volatile and so in order to capture the trends in Tokyo commercial property prices, it is necessary to smooth this series. We used the same linear smoothing procedure that was explained in previous section above to construct the smoothed land price series PL_L^t. Thus we set the initial value of the smoothed mean series, P_{MEANS}^1, equal to the corresponding unsmoothed value P_{MEAN}^1. We set the quarter 2 value of the smooth equal to the average of the first and third observations in the raw series (so that $P_{MEANS}^2 \equiv (1/2)[P_{MEAN}^1 + P_{MEAN}^3]$). For the Quarter 3 value of the smooth, we used the simple 5 term centered moving average so that $P_{MEANS}^3 \equiv (1/5)[P_{MEAN}^1 + P_{MEAN}^2 + P_{MEAN}^3 + P_{MEAN}^4 + P_{MEAN}^5]$ and we carried on using this 5 term centered moving average until Quarters 43 and 44 where we used Rows 4 and 5 of the matrix $X(X^TX)^{-1}X^T$ defined in Section 7 for our Henderson linear regression smoother. The resulting smoothed mean price series, P_{MEANS}^t, plotted in Figure 3 below. We note that the average value of the unsmoothed series P_{MEAN}^t is 1.1644 while the average value of the corresponding smoothed series P_{MEANS}^t is 1.1614.

We can use the predicted values from the Model 11 regression explained in previous section above in order to construct the imputed value of land sold during quarter t. This quarter t value of land is defined as follows:

(36) $V_L^t \equiv \alpha_t\Sigma_{n=1}^{N(t)} (\sum_{j=1}^{4}\omega_j D_{W,tn,j})(\sum_{m=1}^{5}\chi_m D_{EL,tnm})(1+\mu(H_{tn}-3))(1+\eta(DS_{tn}-0))\times$
$(1+\theta(TT_{tn}-1))f_L(L_{tn})$; t = 1,...,44.

In a similar fashion, we can use the predicted values from the Model 11 regression in order to define the imputed value of structures sold during quarter t, V_S^t, as follows:

(37) $V_S^t \equiv p_{St}\Sigma_{n=1}^{N(t)} g_A(A_{tn})(\Sigma_{h=3}^{10} \phi_h D_{H,tnh})S_{tn}$ t = 1,...,44.

The *quality adjusted quarter t quantities of land and of structures*, Q_L^t and Q_S^t, are defined as follows:

(38) $Q_L^t \equiv V_L^t/PL_{MLIT}^t$; $Q_S^t \equiv V_S^t/PS^t$; t = 1,...,44.

With the prices and quantities of land and structures defined for each quarter, we calculated Fisher (1922) property price indexes, which are listed as P_{FMLIT}^t plotted on Figure 3 below.[52]

From viewing Figure 3, it can be seen that the Fisher property price indexes using MLIT data, P_{FMLIT}^t, are quite volatile (due of course to the volatility of the MLIT land price component indexes, PL_{MLIT}^t). The Henderson linear regression smooth of the unsmoothed land price series PL_{MLIT}^t was calculated as PL_L^t. We use this smoothed land price series along with the new land quantities defined as $Q_L^t \equiv V_L^t/PL_L^t$ in order to define the smoothed Fisher property price index, P_{FMLITS}^t, which plotted on Figure 3. This series is our preferred measure of overall commercial property prices for Tokyo.

Recall Model 3 in Section 7 above that used REIT data to implement a version of the builder's model. We can use the predicted values from the Model 3 regression in order to construct the imputed value of land sold during quarter t. This quarter t value of land is defined as follows:

(39) $V_L^t \equiv \sum_{n=1}^{41} \alpha_t\omega_n L_{tn}$; t = 1,...,44.

In a similar fashion, we can use the predicted values from the Model 3 REIT regression in order to define the impute value of structures sold during quarter t, V_S^t, as follows:

(40) $V_S^t \equiv \sum_{n=1}^{41} p_{St}(1-\delta)^{A(t,n)}S_{tn}$ t = 1,...,44.

The (REIT data based) quality adjusted land price for quarter t is the α_t which appears in (39) and is calculated as PL_{REIT}^t. The price of structures is $PS^t = p_{St}S_{tn}$ where p_{St} is the official construction price per m^2 in period t. The corresponding period t quantities of land and structure are defined as follows:

(41) $Q_L^t \equiv V_L^t/PL_{REIT}^t$; $Q_S^t \equiv V_S^t/PS^t$; t = 1,...,44.

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[52] The Laspeyres and Paasche price indexes for quarter t are defined as $P_L^t \equiv [PL_{MLIT}^t Q_L^1 + PS_t Q_S^1]/[PL_{MLIT}^1 Q_L^1 + PS_1 Q_S^1]$ and $P_P^t \equiv [PL_{MLIT}^t Q_L^t + PS_t Q_S^t]/[PL_{MLIT}^1 Q_L^t + PS_1 Q_S^t]$ respectively. The quarter t Fisher index is defined as $P_{FMLIT}^t \equiv [P_L^t P_P^t]^{1/2}$ for t = 1,...,44. See Fisher (1922) for additional materials on these indexes. The Fisher index has strong economic and axiomatic justifications; see Diewert (1976) (1992). We also calculated chained Fisher property price indexes using the same data but these indexes were virtually the same as the Fisher fixed base indexes.

The overall REIT based property price index for quarter t is defined as the Fisher index P_{FREIT}^t using the above prices and quantities for land and structures as the basic building blocks. The REIT based overall property price series P_{FREIT}^t plotted in Figure 3. It can be seen that this series is not volatile and does not require any smoothing.

Our final property price index will be generated by a traditional log price time dummy hedonic regression using the MLIT data.[53]

We use the same notation and definitions of variables as was used in previous sections. Define the natural logarithms of V_{tn}, L_{tn} and S_{tn} as LV_{tn}, LL_{tn} and LS_{tn} for t = 1,...,44 and n = 1,...,N(t). The *log price time dummy hedonic regression model* is the following linear regression model:

(42) $LV_{tn} = \beta_t + \sum_{j=2}^{4}\omega_j D_{W,tnj} + \gamma A_{tn} + \lambda LL_{tn} + \mu LS_{tn} + \Sigma_{h=4}^{10} \phi_h D_{H,tnh} + \eta DS_{tn}$
$+ \theta TT_{tn} + \varepsilon_{tn}$; $\quad t = 1,...,44; n = 1,...,N(t).$

The 4 combined ward dummy variables $D_{W,tnj}$ were defined below definitions (4) and the discussion around Model 3 in Section 3. The building height dummy variables, $D_{H,tnh}$, were defined by (22) in Section 3. However, due to the small number of observations in the heights equal to 10-14 stories, all buildings in this range were aggregated into the height 10 stories category. As usual, A_{tn} is the age of building n sold in quarter t and DS_{tn} and TT_{tn} are the two subway variables pertaining to building n in quarter t. The 44 time dummy variable coefficients are $\beta_1,...,\beta_{44}$. Note that the dummy variable for the first combined ward, $D_{W,tn1}$, is not included in the linear regression defined by (42) in order to prevent multicollinearity. Similarly, the dummy variable for building height equal to 3 was also excluded from the regression to prevent multicollinearity. There are 59 unknown parameters in the regression. The R^2 for this regression was 0.7593. This is higher than our Model 9 and Model 11 R^2 using the same data, which were 0.7091 and 0.7143 respectively.

The standard errors for the time coefficients β_t^* were fairly large (in the 0.13 to 0.15 range). Define the unnormalized land price for quarter t, α_t^*, as the exponential of β_t^*; i.e., $\alpha_t^* \equiv \exp(\beta_t^*)$ for t = 1,...,44. The log price hedonic regression property price for quarter t, P_{LPHED}^t is defined as α_t^*/α_1^* for t = 1,...,44. This traditional hedonic regression model property price index P_{LPHED}^t graphed in Figure 3.

The estimated λ and μ parameters were $\lambda^* = 0.5296$ and $\mu^* = 0.4939$ and hence, they almost sum to unity. Thus a generic commercial property sold in quarter t at price P with land and structure areas L and S respectively has a price that is approximately proportional to the Cobb-Douglas function $\alpha_t L^\lambda S^\mu$ which has returns to scale that are approximately equal to $\lambda^* + \mu^* \approx 1$. The estimated ω_j followed the same pattern that was estimated in Models 9 and 11 in section 3; the composite Ward 1 was the most expensive ward, Ward 2 the next most expensive, Ward 3 less expensive again and Ward 4 had the lowest level of property prices. The height dummy variables exhibited the same trends that were observed in our MLIT builder's models: the higher the height of the structure, the higher was the price of the property. Finally, the distance from the nearest subway station parameter η was significantly negative indicating that property value falls as the distance increases. The subway travel time parameter

[53] Recent developments in estimating traditional log price hedonic regression property models are reviewed by Hill, Scholz, Shimizu and Steurer (2018) and Silver (2018).

θ had an unexpected positive sign but was not significantly different from 0. Finally, it is possible to convert the estimated age coefficient γ^* into an estimate for a geometric rate of structure depreciation, δ. The formula for this conversion is $\delta \equiv 1 - e^{\gamma/\beta}$.[54] When this conversion formula was utilized, we found that the estimated δ^* was 0.01945; i.e., the traditional hedonic regression model generated an implied annual geometric depreciation rate equal to 1.945% per year, which is a reasonable estimate.

Viewing Figure 3, it can be seen that the time dummy hedonic regression model implied property price index P_{LPHED}^t is just as volatile as the corresponding builder's model property price index P_{FMLIT}^t. Thus we applied our modified Henderson linear smoothing operator to P_{LPHED}^t which produces the smoothed series, P_{LPHEDS}^t, which is also plotted in Figure 3.

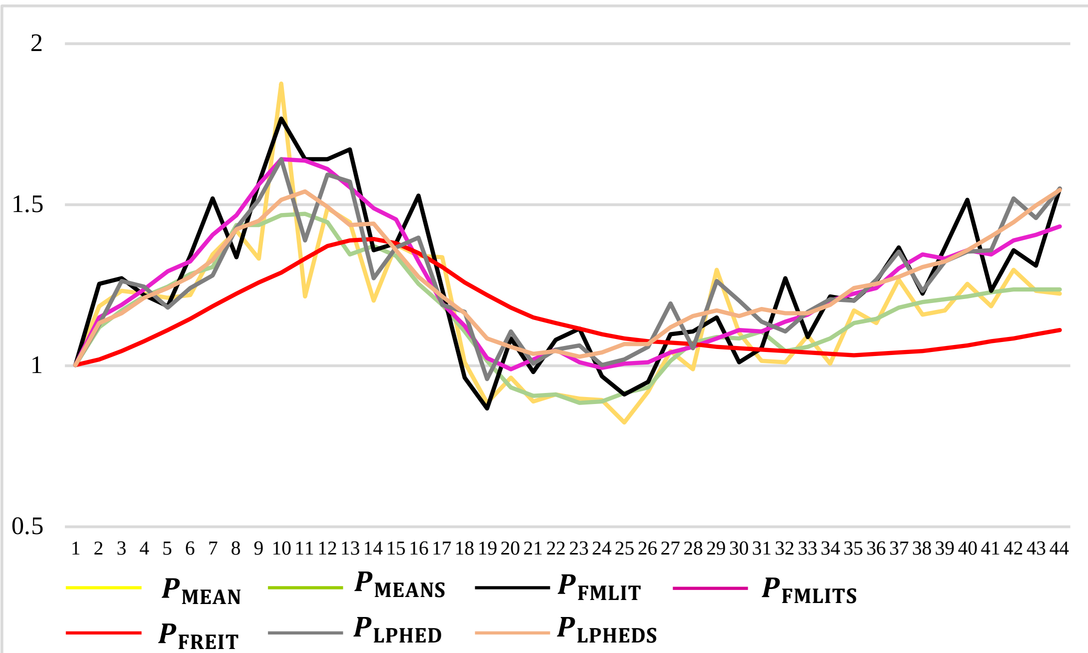

Figure 3: Alternative Commercial Property Price Indexes Using MLIT and REIT Data

The two top jagged lines in Figure 3 are the Fisher property price index using the builder's model, P_{FMLIT}^t, and the log price time dummy hedonic regression property price index, P_{LPHED}^t. Both of these series use the MLIT sales transaction data. Their linear smooths are P_{FMLITS}^t and P_{LPHEDS}^t. It can be seen that these two smoothed series approximate each other reasonably well.[55] What is somewhat surprising

[54] See McMillen (2003; 289-290), Shimizu, Nishimura and Watanabe (2010; 795) and Diewert, Huang and Burnett-Isaacs (2017; 24) for derivations of this formula.

[55] Diewert (2010) noticed that the Fisher property price index generated by the builder's model frequently approximated the traditional log price time dummy property price index using the same data. However, the key to close approximation is that the time dummy model must generate a reasonable implied structure depreciation rate, which is the case for our particular data set.

is that the smoothed mean index P_{MEANS}^t (which uses the same transactions data) approximates the two smoothed hedonic indexes to some degree but the series gradually diverge due to the fact that an index based on average prices per m^2 cannot take depreciation into account.[56] The hills and valleys in the P_{MEANS}^t series are less pronounced than the corresponding fluctuations in the P_{FMLITS}^t and P_{LPHEDS}^t series but the turning points are the same. Finally, it can be seen that the Fisher property price series that is based on appraised values of properties, P_{FREIT}^t, does not provide a good approximation to the two smoothed series based on transactions, the P_{FMLITS}^t and P_{LPHEDS}^t series. The fluctuations in P_{FREIT}^t are too small and the turning points in this series lag well behind our preferred series.

10. Commercial Property Price Indexes Based on Stock Market Data

As was seen in the previous section, the use of appraisal or tax assessment data in constructing a commercial property price index can result in an index which has been excessively smoothed and has lagged turning points. Thus the use of the resulting indexes can render early warning signals for monetary policy ineffective and distort SNA national land asset value estimates.

Under these circumstances, there are advantages to using the transaction price. However, the problems of property heterogeneity and the scarcity of sales of commercial properties lead to difficulties in constructing transaction based indexes. Furthermore, many countries do not collect data on transaction prices. Hence, estimating price indexes using the *stock price* of real estate investment trusts (REIT) has been proposed as a fourth data source, following the transaction price, appraisal price, and assessment price methods.

The stock market method of property valuation can be applied to REITs that have only a single structure in their real estate portfolio. Suppose there are N such single asset REITs. At any point of time in period t, the value of the property in the REIT n, V_{tn}, will be equal to the stock market value of REIT n, say V_{Stn}, plus the value of outstanding debt at that time, V_{Dtn}. Now simply apply the model that was explained in section 7 above using the stock market values $V_{tn} = V_{Stn} + V_{Dtn}$ in place of the corresponding appraised values. However, Model 3 defined by equations (28) does require information on the land plot area, the floor space area of the structure and an exogenous cost of construction index.

The advantage in using stock market data in place of transactions data is that stock market data may be more plentiful than data on sales of commercial properties. But there are some disadvantages as well:

- Stock market data may be more volatile than sales data;
- Stock market data may not be representative of the entire market.

The first difficulty can be overcome by using the smoothing methods discussed in the paper (or alternative methods). The second difficulty can be overcome if stock market data is used to supplement transactions data.

11. Conclusion

[56] If the age structure of the quarterly sales of properties remains reasonably constant, then this neglect of depreciation will probably not be a factor.

Appraisal based commercial property price indexes have been published for many years, focusing on Japan, the U.S., and the U.K. As noted in the paper, these indexes tend to diverge from actual market conditions due to the smoothing and turning points problems. Thus in recent years, commercial property price indexes based on transactions have been developed and are published in the U.S and Japan.

However, in many countries (including Japan), many difficulties accompany the estimation of indexes based on transaction prices due to the lack of transactions. In addition, compared to housing, commercial properties have a high level of heterogeneity, so quality adjustment must be performed.

In addressing problems such as this lack of data and rigorous quality adjustment, one may refer to past experience and efforts that have been made in the practical property appraisal. Property appraisal prices are determined based on the sales comparison approach, using comparables for similar transactions in the vicinity of the property being appraised. For housing price indexes, this approach leads to these appraisal type price indexes being estimated essentially by performing quality adjustment through the use of transaction prices.

For commercial properties, on the other hand, since there is lack of transaction comparables as well as a high level of heterogeneity, it is difficult to construct appraisal based indexes on the sales comparison approach. As a result, commercial property appraisals are generally determined using present values, based on a method known as the capitalization method. This means that the difficulty level of estimating commercial property price indexes using transaction prices is extremely high compared to housing.

In this paper, based on past experience in the practical property appraisal, in addition to a price index using property appraisal prices and price index using transaction prices, we explored the possibility of estimating a price index based on the Builder's Model proposed by Diewert and Shimizu (2019). Specifically, focusing on the Tokyo area, we estimated a transaction based CPPI, along with an appraisal-based price index, using transaction prices and published J-REIT data with the same characteristics as data possessed by NCREIF in the U.S., MSCI-IPD in the U.K., etc. The following provides an overview of the analysis and results obtained.

Here are our main conclusions in comparison with transaction based CPPI using Builder's model, appraisal based index, assessment price index:

- It is possible to construct a quarterly transactions based commercial property price index that can be decomposed into land and structure components.
- The main characteristics of the properties that are required in order to implement our approach are: (i) the property location (or neighbourhood); (ii) the floor space area of the structure on the property; (iii) the area of the land plot; (iv) the age of the structure and (v) the height of the building. We also require an appropriate exogenous commercial property construction cost index that gives the average cost of construction per square meter for each period in the sample.
- The land price index that our hedonic regression model generates may be too volatile and hence may need to be smoothed. We found that a slightly modified five quarter moving average of the raw land price indexes did an adequate job of smoothing. This means that the final land price index could be produced with a two quarter lag.

- We found that a smoothed version of a traditional log price time dummy hedonic regression model produced an acceptable approximation to our preferred smoothed builder's model overall price index.
- We also found that a very simple overall price index which is proportional to the quarterly arithmetic average of each property price divided by the corresponding structure area provided a rough approximation to our preferred price index. This model cannot take depreciation into account and hence will in general have an downward bias but it has the advantage of requiring information on only a single property characteristic (the structure floor space area) in order to be implemented.
- The price indexes that were based on appraisal and assessed value information were not satisfactory approximations to the transactions based indexes. The turning points in these series lagged our preferred series and the appraisal based series smoothed the data based series to an unacceptable degree.[57]

Numerous problems still remain. In the realm of commercial properties, there are many other structures with diverse uses, e.g. commercial establishments, hotels, and warehousing & distribution facilities. In such markets, it is to be expected that transactions prices are even more scarce, and properties, even more heterogeneous, when compared to the office market. Furthermore, certain quantities of transaction price data and appraisal prices from the real investment market are available for use in large cities such as Tokyo. However, it is highly probable that sufficient data will be hard to come by in regional cities.

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[57] These points are well known in the real estate literature; see Chapter 25 in Geltner, Miller, Clayton and Eichholtz (2014).

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