Estimation of Relative Risk Aversion with Wealth Heterogeneity

Enkhbaatar Tsenguun
VSE, UBC

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Abstract

In this paper, I estimate the relative risk aversion of households with different positions of wealth by using non-parametric structural estimation method. Relative risk aversion is an important measure of the extent of a household’s reaction towards future uncertainty, and conventionally estimated as being constant across households. However, there can be a significant heterogeneity in risk aversion across households with dissimilar characteristics. I employ a combination of extremum estimation and non-parametric kernel estimation methods to estimate the degree of heterogeneous relative risk aversion varying across households with different wealth positions. Data for my analysis is sourced from the Panel Study of Income Dynamics for the US, and the Survey on Household Income and Wealth for Italy.

1 Introduction

In a representative agent framework, homogeneous relative risk aversion is assumed to exist across diverse households. Relative risk aversion is an important measurement, since it demonstrates the extent of household’s reaction to the future uncertainty involved in its income process. When a household is risk neutral or its relative risk aversion is zero, that household does not react at all to the future uncertainty. However, when a household’s relative risk aversion is high, it tries to insure itself against the future income fluctuations. This means that the higher the relative risk aversion, the higher the precautionary motives of the household. In this empirical paper, I explore the degree of heterogeneity of relative risk aversion across households. I am particularly interested in how a relative risk aversion changes across households with different wealth positions. The main reason for choosing wealth position as the most important characteristic of a household is that the wealth is the main state variable in most macroeconomic models, and it is a cumulative measure of the degree of a household’s earning performance over time. Under the incomplete market environment, households react in various ways to future uncertainties due to their dissimilar degrees of risk aversion. This difference in household reactions against future uncertainties could push them towards choosing different career or investment strategy choices. Furthermore, these unlike choices might posit households in the long run into diverse wealth positions.
In this paper, I assume the CRRA utility function for a household’s utility representation. Using a household’s intertemporal Euler Equation, I identify, utilizing household panel data, a relative risk aversion as parameter of the CRRA utility function. Combining the Extremum Estimation Method with the Non-Parametric Kernel Estimation Method, I estimate a relative risk aversion conditional on a household’s wealth position. In this way, I can plot relative risk aversion across households with different wealth to see how the former is correlated with the wealth position of a household.

I find a significant heterogeneity in relative risk aversion among households with unlike wealth positions. A hump shaped relationship pattern is found at the bottom end of the wealth distribution. However, there is a significant negative correlation between relative risk aversion and wealth of a household, excluding households at the bottom end of the wealth distribution. These findings are in line with those of several other papers such as those of Chiappori et al. (2011), and Paravisini et al. (2013), even though they employ very different methodologies.

One interesting finding is that my result contradicts the conventional heterogeneous agent models. High risk averse individuals own the majority of the total wealth in the stationary equilibrium in the conventional heterogeneous agent models. However, my results suggest wealthy people are much less risk averse than others. I discuss how to extend the conventional models such that they can be in line with the result found in this empirical paper in the Results and Discussion section. The conventional heterogeneous agent models do not work well for capturing the observed wealth distribution, and particularly in the case of US wealth distribution. The extension I consider may actually work well for improving that aspect of the conventional heterogeneous agents model.

The paper proceeds as follows. In the Model and Estimation section, I formulate the underlining model and explain the estimation procedure in detail. In the Data Description and Summary section, I introduce data used in this paper and present its summary. In the Results and Discussion section, I present the result and discuss its implication and comparison with other papers. Furthermore, I discuss the economic interpretation of the result and its relation to the conventional macroeconomic heterogeneous agents model in this section. In the Conclusion section, I present my conclusion and discuss the potential future research topic based on the result.

2 Model and Estimation methodology

2.1 Modelling

Household’s problem at period $t$ is to choose the optimal consumption $c_t$ and the optimal asset holding $a_{t+1}$, given its capital income and the labor income:

$$\max_{\{c_t\}_{t=0}} \sum_{t=0}^{\infty} E_0 \beta^t U(c_t),$$

subject to a constraint $c_t + a_{t+1} = R_t a_t + w_t$, given the initial value of asset $a_0$. Here, $\beta$ is the discount factor, $c_t$ is consumption at period $t$, $a_t$ is asset (wealth) at period $t$, $R_t$ denotes
the gross return on the asset at period \( t \), and \( w_t \) is labor income. The solution to this model is described by the intertemporal Euler equation:

\[
U'(c_t) = E_t\{\beta R_{t+1}U'(c_{t+1})\}.
\]

Here, we assumed that labour supply is inelastic. However, even when we have the elastic labour supply in this model (household chooses its optimal labour supply \( l_t \) endogenously), if we have separability between consumption and labour in a utility function such as \( U(c_t, l_t) = u_1(c_t) + u_2(l_t) \), we will still attain the same intertemporal Euler Equation. In this paper, I am assuming the CRRA utility function,

\[
U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \gamma \in (0, \infty).
\]

In the Representative Agents framework, it is assumed that every agent of the economy has the same \( \gamma \) parameter for his or her CRRA utility function specification. However, there can be a heterogeneous \( \gamma \) for different agents. This means that the agent of the different characteristic \( x \) (\( x \) could be the set of characteristics such as wealth, gender, and age, among others.) could have different \( \gamma \) in his or her CRRA utility function such as \( U(c_t) = \frac{c_t^{1-\gamma(x)}}{1-\gamma(x)} \).

The intertemporal Euler Equation of the agent \( i \) of characteristic \( x \) is given by:

\[
E_t\{\beta R_{t+1}\frac{U'(c_{it+1})}{U'(c_{it})}|X_{it} = x\} = 1,
\]

In our CRRA utility function setting, which becomes:

\[
E_t\{\beta R_{t+1}\left(\frac{c_{it+1}}{c_{it}}\right)^{-\gamma(x)}|X_{it} = x\} = 1
\]

In this paper, I assume that the discount factor \( \beta \) is the same across \( x \) or \( \beta(x) = \beta \). Since I am estimating \( \gamma(x) \) using only the intertemporal Euler Equation, \( \beta \) is not identified or \( \beta(x) \) and \( \gamma(x) \) cannot be estimated simultaneously from one equation. Hence, I fix \( \beta \) across \( x \) and then estimate \( \gamma(x) \). To check the robustness of the results, I choose different values of \( \beta \) selected to cover the values estimated in the past literature. Furthermore, I estimate \( \gamma(x) \) using a non-parametric technique without assuming any particular functional form for \( \gamma(x) \), and the estimation procedure is summarized below. First, I define \( y_{it}(\gamma) \) for given \( \gamma \) as the following:

\[
y_{it}(\gamma) = \beta R_{t+1}\left(\frac{c_{it+1}}{c_{it}}\right)^{-\gamma}
\]

Next, the empirical analogy of the expectations \( E_t\{y_{it}(\gamma)|X_{it} = x\} \) is defined as:

\[
\hat{E}_t\{y_{it}(\gamma)|X_{it} = x\} = \frac{\sum_{t=1}^{T} \sum_{i=1}^{n_t} y_{it}(\gamma)K((X_{it} - x)/h)}{\sum_{t=1}^{T} \sum_{i=1}^{n_t} K((X_{it} - x)/h)}
\]

Note that \( \hat{E}_t \) consistently estimates \( E_t \) as \( T, n \to \infty \) and \( h \to 0 \). Here, \( K() \) is a Kernel function. I use the following Extremum Estimator to estimate \( \gamma(x) \):

\[
\hat{\gamma}(x) = \arg\min_{\gamma \in [0, \infty)} \left(\hat{E}_t\{y_{it}(\gamma)|X_{it} = x\} - 1\right)^2,
\]
In other words, for each \( x \), we find \( \gamma \) that approximately solves the empirical Euler equation.

I use the Epanechnikov Kernel Function, \( K(u) = \frac{3}{4}(1 - u^2)1(|u| \leq 1) \), and find the optimal Kernel bandwidth using \textbf{Cross-Validation}.

### 2.2 Estimation of Standard Error

Let’s assume that \( \gamma_0(x) \) is the true value of relative risk aversion of a household of characteristics \( x \). I am defining the function \( G(\gamma_0(x)) \) as follows:

\[
G(\gamma_0(x)) = E_t\{y_{it}(\gamma_0)|X_{it} = x\} - 1 = 0
\]

\( \hat{\gamma}(x) \) is an estimate which makes the sample version of this function equal to zero.

\[
\hat{G}_n(\hat{\gamma}_n) = \mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} y_{it}(\hat{\gamma}_n)K((X_{it} - x)/h) / \mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} K((X_{it} - x)/h) - 1 = 0
\]

The First Order Optimality Condition for Extremum Estimator in equation (3) corresponds to the Z-Estimator in Equation (5). To derive the asymptotic normality, I first employ a Mean Value Expansion of \( \hat{G}_n(\hat{\gamma}_n) \) around \( \gamma_0\),

\[
0 \approx \sqrt{n}h\hat{G}_n(\hat{\gamma}_n) + \hat{G}_n'(\hat{\gamma}_n)(\gamma_0 - \hat{\gamma}_n) \\ \forall \gamma_n \in (\hat{\gamma}_n, \gamma_0) \Rightarrow \sqrt{n}h(\hat{\gamma}_n - \gamma_0) \approx \frac{1}{-\hat{G}_n'(\hat{\gamma}_n)} \sqrt{n}h(\hat{G}_n(\gamma_0) - G(\gamma_0)), \quad \bar{\gamma}_n \approx \gamma_0.
\]

Here, \( n \) is defined as \( n = \mathbf{\sum}_{t=1}^T n_t \) and \( \hat{G}_n'(\gamma) \) is the first derivative of \( \hat{G}_n(\gamma) \) with respect to \( \gamma \). Now, \( \hat{\gamma}_n \rightarrow \gamma_0 \) means that \( \bar{\gamma}_n \rightarrow \gamma_0 \). With point-wise convergence of \( \hat{G}_n'(\gamma_0) \rightarrow G'(\gamma_0) \) and uniform convergence, \( \hat{G}_n'(\hat{\gamma}_n) \rightarrow G'(\gamma_0) \) holds. Furthermore, according to the Asymptotic Normality of Kernel Estimation,

\[
\sqrt{n}h(\hat{G}_n(\gamma_0) - G(\gamma_0)) = \sqrt{n}h \left( \frac{1}{nh} \mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} y_{it}(\gamma_0)K((X_{it} - x)/h) / \mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} K((X_{it} - x)/h) - 1 \right) \rightarrow d N \left( 0, \frac{R(k)\sigma^2(x)}{f(x)} \right),
\]

where roughness, \( R(k) \), for the Epanechnikov Kernel is \( 3/5 \). \( \sigma^2(x) \) is a conditional variance of the error term, \( e_{it}(\gamma_0) \), and can be estimated as below:

\[
\hat{\sigma}^2_n(x) = \frac{\mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} \hat{e}_{it}^2(\hat{\gamma}_n)K((X_{it} - x)/h)}{\mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} K((X_{it} - x)/h)}.
\]

Furthermore, the Probability Density Function of \( x \), \( f(x) \), is estimated as:

\[
\hat{f}_n(x) = \frac{1}{nh} \mathbf{\sum}_{t=1}^T \mathbf{\sum}_{i=1}^{n_t} K \left( \frac{X_{it} - x}{h} \right).
\]
Combining these, the following asymptotic normality is obtained:

\[ \sqrt{nh(\hat{\gamma}_n - \gamma_0)} \xrightarrow{d} N\left(0, \frac{1}{(G'(\gamma_0))^2} \frac{R(k)\sigma^2(x)}{f(x)}\right) \quad \text{as } n \to \infty \quad (6) \]

The variance of \( \hat{\gamma}_n \), \( \sigma^2_{\hat{\gamma}_n} \), can be estimated as:

\[ \hat{\sigma}^2_{\hat{\gamma}_n} = \frac{1}{(\hat{G}'(\hat{\gamma}_n))^2} \frac{R(k)\hat{\sigma}^2_n(x)}{f_n(x)}. \]

The optimal bandwidth can be constructed as \( C_0n^{-1/5} \). Non-parametric estimate \( \hat{\gamma}_n(x) \) has a asymptotic bias of \( \sqrt{nh^5\kappa_2 B(x)/(G'(\gamma_0))^2} \), and \( B(x) \) does not depend on \( n \) or \( h \). By setting \( h \) as \( C_0n^{-1/5-\epsilon} \), this bias converges to zero in the limit. Here, I set \( \epsilon \) as 0.01. However, this also means a small deviation from the optimal bandwidth.

### 2.3 Connecting two years interval datasets

I derive an intertemporal Euler Equation for the ratio of consumptions at time \( t \) and \( t+2 \), because SHIW and PSID conduct the survey every two years, respectively, since 1991 and 1997.

\[
E_t\left\{ \beta R_{t+1} \left( \frac{C_{t+1,i}}{C_{t,i}} \right)^{-\gamma(x)} \left| X_{it} = x \right. \right\} = \\
E_t\left\{ \beta R_{t+1} \left( \frac{C_{t+1,i}}{C_{t,i}} \right)^{-\gamma(x)} \right| X_{it} = x \right\} E_{t+1} \left\{ \beta R_{t+2} \left( \frac{C_{t+2,i}}{C_{t+1,i}} \right)^{-\gamma(x)} \left| X_{it} = x \right. \right\} = \\
E_t \left\{ \beta R_{t+1} \left( \frac{C_{t+1,i}}{C_{t,i}} \right)^{-\gamma(x)} \beta R_{t+2} \left( \frac{C_{t+2,i}}{C_{t+1,i}} \right)^{-\gamma(x)} \left| X_{it} = x \right. \right\} X_{it} = x \right\},
\]

since

\[ E_{t+1} \left\{ \beta R_{t+2} \left( \frac{C_{t+2,i}}{C_{t+1,i}} \right)^{-\gamma(x)} \left| X_{it} = x \right. \right\} = 1. \]

According to the Law of Iterated Expectations,

\[ E_t \left\{ \beta^2 R_{t+2} R_{t+1} \left( \frac{C_{t+2,i}}{C_{t,i}} \right)^{-\gamma(x)} \left| X_{it} = x \right. \right\} = 1 \quad (7) \]

I use Equation (7), as defined above, rather than Equation (2), since the panel datasets used in this paper were collected every two years. Henceforth,

\[ y_{it}(\gamma) = \beta^2 R_{t+2} R_{t+1} \left( \frac{c_{it+2}}{c_{it}} \right)^{-\gamma}. \]

\(^1\)Bruce E. Hansen, Lecture Notes on Nonparametrics (2009)
3 Data description and summary

3.1 Data description

In this paper, I use two different datasets. The first dataset is the Survey on Household Income and Wealth (SHIW) conducted by the Bank of Italy. SHIW started in 1977 even though it was not complete enough for the analysis of this paper till 1991. Therefore, I use SHIW dataset starting from 1991. Since 1991, SHIW is being conducted every two years. Every wave of the survey has approximately 8,000 households (20,000 individuals), distributed over about 300 Italian municipalities. In the next wave of the survey, approximately a half of the households from the previous wave is kept. I construct the dataset for households who are in the two adjacent waves of the survey by merging these two adjacent waves. Then, merged datasets are pooled across time. SHIW constructs the aggregate wealth and consumption variables for each household. Moreover, I adjust the household wealth and consumption variables to 1998 Euro. As a result, I have the following waves of merged datasets for wealth and $c_{t+2}/c_t$: 1991, 1993, 1998, 2000, 2002, 2004, 2006, 2008, 2010, and 2012, each of which includes approximately 4,000 households.

The second dataset is the Panel Study of Income Dynamics (PSID) directed by the University of Michigan. The survey follows individuals and their descendants over a long period of time. The PSID survey was initiated in 1968, and has been conducted every two years since 1998. Since all households from the previous wave are retained in the next wave, I have constructed the ratio of $c_{t+2}/c_t$ for all households in each wave. However, due to the incompleteness of the data, I use household aggregate wealth data from the following waves: 1983, 1993, 1998, 2000, 2002, 2004, 2006, and 2008, each of which includes approximately 5,000 households (18,000 individuals). Furthermore, household aggregate consumption data is available from PSID since 1998 wave. Therefore, I use the imputed household aggregate consumption data for the waves of 1983 and 1993 estimated in Attanasio and Pistaferri (2014). All variables were adjusted to 1998 USD.

3.2 Data summary

The following table summarizes important statistics:


\[\text{http://psidonline.isr.umich.edu/}\]
The spread of wealth and the ratio of consumption in PSID is larger than that in SHIW. Furthermore, all variables from both surveys are highly right-skewed. PSID wealth data is much more right-skewed compared to SHIW wealth data. On the other hand, SHIW \( c_{t+2}/c_t \) is more right-skewed than PSID \( c_{t+2}/c_t \).

Histograms for the variables are plotted below.

![Empirical Distribution of The Ratio of Consumptions at t and t+2 periods for SHIW](a) \( c_{t+2}/c_t \) ratio from SHIW

![Empirical Distribution of Wealth for SHIW](b) wealth from SHIW

![Empirical Distribution of The Ratio of Consumptions at t and t+2 periods for PSID](c) \( c_{t+2}/c_t \) ratio from PSID

![Empirical Distribution of Wealth for PSID](d) wealth from SHIW

Figure 1: Empirical Distributions for \( c_{t+2}/c_t \) and wealth from SHIW and PSID

The optimal bandwidth obtained from Cross-validation:

- Pooled PSID wealth data: 158,861.43
• Pooled SHIW wealth data: 108,062.48

The optimal bandwidth for the PSID dataset is approximately 46% higher than the one for the SHIW dataset. From the Statistics Summary Table, we can see that the standard deviation for the wealth data from PSID is approximately 40% higher than that of SHIW.

Furthermore, annual real interest rate data from the World Bank databank for the USA and Italy are used as $R_{t+1}$. The inflation rate (CPI) from the World Bank is used to adjust the variables to the 1998 USD and Euro values for US and Italy consumption and wealth data, respectively.

Figure 2: Real Interest Rate and Inflation Rate data for US and Italy
3.3 Parameter specification

I estimated a heterogeneous $\gamma$ for each of 51 equally spaced $\beta$’s on the interval $[0.86, 0.96]$. Because the data used in this paper are annual, the range of $\beta$ chosen in this paper is equivalent to the range of $\beta$ on the interval $[0.965, 0.99]$ for the quarterly data.

Relative Risk Aversion $\gamma(x)$ is estimated in this paper conditioning on wealth of a household. 81 wealth grid points are chosen as conditioning values. These grid points are chosen as the first grid point corresponds to $\text{mean}(X_{it}) - \text{std}(X_{it})$, the 41st grid point corresponds to $\text{mean}(X_{it})$, and the 81st grid point corresponds to $\text{mean}(X_{it}) + \text{std}(X_{it})$. Here, $X_{it}$ is the wealth of household $i$ at year $t$. All other grid points are equally spaced between these three. Thus, the majority of values in the wealth data are treated as conditioning values approximately.

4 Results and Discussion

I find a significant heterogeneity among individuals with different wealth positions. A hump-shaped correlation pattern between $\gamma$ and wealth of a household is found at the bottom of the wealth distribution in the data. First, the Relative Risk Aversion $\gamma$ increases with wealth at the bottom end of the wealth axis. Next, it decreases sharply toward the top end of the wealth axis in the sample.

4.1 Estimation results for the SHIW dataset

The following figure plots the estimated $\gamma$’s over the different values of wealth when $\beta$ is between 0.906 and 0.914.
\( \gamma(x) \) ranges from 0.85 to 0.55 when \( \beta \) is between 0.906 and 0.914. There is a small hump in the curve when wealth is between 20,000 Euro and 120,000 Euro (at the 1998 value). Beginning from 120,000 Euros, there is a steady decline in \( \gamma(x) \) toward the top end of the wealth axis.

The 3D figure below plots the estimated \( \gamma \)'s for all \( \beta \)'s on the interval \([0.86, 0.96]\) over the different values of wealth:

A humped curve at the bottom end of the wealth axis is estimated for every \( \beta \) on the interval \([0.86, 0.96]\). The decline of \( \gamma(x) \) toward the top end of the wealth axis becomes less sharp as \( \beta \) increases. \( \gamma(x) \) ranges from 1.265 to 0.159.

We can see that when \( \beta \) increases, \( \gamma \) decreases. This relationship is proven mathematically in Cozzi (2011). He demonstrates that the expected consumption growth \( c_{t+1}/c_t \) increases in both the discount factor \( \beta \) and the relative risk aversion \( \gamma \).

In this paper, the consumption growth itself is the data used for the estimation. Hence, when \( \beta \) increases, to offset the effect of increasing the expected consumption growth (or the empirical average of the consumption growth; in our analysis, this is fixed), \( \gamma \) must decrease. Higher \( \beta \) means that households aim for future higher consumption, since they place relatively greater value on future utility. This suggests that the expected consumption growth \( E_t\{\frac{c_{t+1}}{c_t} - 1\} \) is higher when \( \beta \) is higher. The higher the degree of risk aversion, as captured by \( \gamma(x) \), the higher are the household’s precautionary saving motives. This drives a decrease in today’s consumption for households and an increase in savings (due to precautionary motives). Hence, the higher the \( \gamma(x) \), the higher is the expected consumption growth rate \( E_t\{\frac{c_{t+1}}{c_t} - 1\} \).
95% Confidence Interval of $\gamma$ when $\beta = 0.91$:

Upper and Lower bound of 95% Confidence Interval for all $\beta$'s:
The standard error for $\gamma(x)$ ranges from 0.0289 to 0.0072. A standard error becomes slightly higher toward both ends of the wealth axis. Fewer observations have positive kernel weights toward both ends of the wealth axis. Hence, variance of $\gamma(x)$ increases toward both ends of the wealth axis.

### 4.2 Estimated results for the PSID dataset

![Non-Parametric Estimation of $\gamma$ for households with different wealth, PSID](chart.png)

Estimates of $\gamma(x)$ range from 1.15 to 0.22 when $\beta$ is between 0.906 and 0.914. There is a hump in the estimated curve when wealth is between -150,000 USD and -70,000 USD (at the 1998 value). However, the hump is much sharper compared with the SHIW result. Beginning from -70,000 USD, there is a steady decline in $\gamma(x)$ toward the top end of the wealth axis. However, compared with the SHIW result, the marginal decline is very high around 150,000 USD. Hence, the relative risk aversion $\gamma(x)$ for more affluent people is much lower than that estimated in SHIW for those of the same wealth. This illustrates there could be a significant heterogeneity in $\gamma(x)$ across different countries.
Estimation of $\gamma$ for all $\beta$'s on the interval $[0.86, 0.96]$: 

![Graph showing estimated $\gamma$ for different $\beta$'s for different wealth points, PSID](image)

A hump at the bottom end of the wealth axis is estimated for every $\beta$ on the interval $[0.86, 0.96]$ for PSID as well. Compared to the SHIW results, the hump is very sharp for all $\beta$'s considered. At around 150,000 USD, there is a much sharper marginal decline in $\gamma(x)$ for all $\beta$'s compared with the SHIW result. Thus, $\gamma(x)$ declines much more severely toward the top end of the wealth axis as compared to the SHIW result. $\gamma(x)$ ranges from 1.374 to 0.055. Furthermore, another difference from SHIW is that when $\beta$ increases, $\gamma(x)$ decreases, but its marginal decrease is much less than a marginal decrease of the $\gamma(x)$'s estimated from SHIW.
95% Confidence Interval of $\gamma$ when $\beta = 0.91$:

Upper and Lower bound of 95% Confidence Interval for all $\beta$'s:

The standard error for $\gamma(x)$ ranges from 0.0759 to 0.0028. A standard error becomes higher toward both ends of the wealth axis because fewer observations will have positive
kernel weights around both ends of the wealth axis. A standard error becomes much higher, particularly at the bottom end of the wealth axis, as compared to the SHIW result. In the SHIW, the standard error at the top end of the wealth axis was higher than the standard error at the bottom end of the wealth axis.

For the most values of wealth, there is a significant negative correlation between Relative Risk Aversion, $\gamma$, and wealth of a household. The hump in the correlation pattern occurs only at the bottom end of the wealth axis. Furthermore, compared to the SHIW, the decline in $\gamma$ when wealth increases is much sharper for the PSID. The highest estimated $\gamma$'s are 1.37 and 1.26 for PSID and SHIW respectively.

### 4.3 Comparison to other papers

The hump shape of the correlation pattern between $\gamma$ and wealth is documented in Nie et al. (2014). They found that Relative Risk Aversion first increases with wealth and then decreases with it over some region of the wealth values. They estimate this relationship using an experimental survey. The negative correlation between Relative Risk Aversion and wealth of a household is documented in Chiappori et al. (2011), and Paravisini et al. (2013). Paravisini et al. (2013) estimates Relative Risk Aversion using data from a person-to-person lending platform in the USA. They show that after controlling for investor specific fixed effect, there is a negative relationship between RRA and the wealth of an investor. These results are in line with my result even though they employed a very different estimation methodology.

Furthermore, Chiappori et al. (2011) indicates that Relative Risk Aversion is constant or does not vary significantly over time for an individual. Moreover, Brunnermeier and Nagel (2006) shows that the relationship between an individual’s wealth and the asset allocation seems best described by constant relative risk aversion. Thus, these papers support using the constant relative risk aversion utility function to express an individual’s preferences (since the Relative Risk Aversion for an individual does not vary significantly over time).

In the aforementioned papers, the relationship between wealth and Relative Risk Aversion is estimated using experimental surveys or linear regression on the cross sectional or the panel data after log-transformation on an intertemporal Euler equation (in this way, Euler equation is transformed into a linear equation). However, in this paper, I employ the non-parametric estimation and the extremum estimation techniques on a non-linear intertemporal Euler equation. Log-transformation is inaccurate as one can not interchange log and the expectations operator. My approach avoids this pitfall since I do not apply any non-linear transformation on the intertemporal Euler equation. Moreover, I use an individual’s consumption and wealth data (realized, not collected in an experiment) from the PSID and the SHIW, not experimental survey data. Compared with the literature, these points are the advantages of my estimation methodology.

I estimated relative risk aversion by conditioning on the wealth of a household for various $\beta$'s. When $\beta$ increases such that $\beta R_{t+1}$ becomes sufficiently close to one, the estimation result is no longer smooth in the conditioning variable wealth. It jumps around and becomes negative for some cases. The objective function which needs to be close to zero under the estimated value of $\gamma$ becomes not close to zero when $\beta R_{t+1}$ becomes sufficiently close to one.
Chamberlain et al. (2000) and Huggett (1993) demonstrate that, when the income process is sufficiently stochastic, $\beta R_{t+1} < 1$ is necessary for the optimal consumption sequence and the asset space to be bounded. In my paper, this result is confirmed in the sense that the objective function which has to be minimized does not behave well when $\beta R_{t+1}$ is sufficiently close to 1.

4.4 Economic interpretation of the result and discussion

In this paper, I have demonstrated that there is a negative correlation between wealth and Relative Risk Aversion, except the very bottom end of the distribution of wealth. However, this result does not clarify the causal relationship between these two.

Low risk averse individuals may pursue the career of entrepreneur. Moreover, high risk averse individuals may pursue the career of worker. The career of entrepreneur might represent an income process of the type, high-risk and high-return. In conjunction with this, the career of worker might represent a low-risk and low-return type income process. Once we have the heterogeneity in the preferences of individuals, there will be a self-selection into various careers by individuals. Cozzi (2011) concluded that the model of incomplete markets and precautionary savings extended to allow for relative risk aversion heterogeneity (the CRRA utility) and allow for self-selection into risky jobs could explain several features of U.S. wealth distribution better compared to the conventional heterogeneity models. One major problem with the conventional heterogeneity models is that they can not generate sufficient inequality in the wealth distribution that is observed in the data. In the U.S.’s observed wealth distribution, the top 1% holds approximately 30% of the total wealth. However, the conventional heterogeneity models generate a very low share of the total wealth for the top quintiles of the distribution. Furthermore, another major problem with the conventional heterogeneity models is that the bottom quintiles of the wealth distribution hold too much wealth compared to the observed share of the total wealth held by the bottom quintiles of the wealth distribution in the data. Cozzi (2011) shows that those flaws are much reduced upon introducing preference heterogeneity and the self-selection of careers. He also indicates that the Gini index calculated from the extended model gets much closer to the Gini index calculated from the real data for U.S.

The problem with the self-selection into careers according to an individual’s degree of risk-aversion is that even though low risk-averse individuals would pursue high-risk and high-return type of career such as entrepreneurship, they might still end up with the very low wealth with positive probability since their career is highly risky. One possible extension to addressing this problem is as follows. Individuals have to choose their career in every period. They can stick to the career they have chosen in the previous period or they can choose another career. If they stick to the career they have chosen in the previous period, the risk involved in that career would become less over time for that individual. In particular, for individuals who are in entrepreneurship career, the lessening of the risk would provide an extra motivation to keep pursuing entrepreneurship career even though, in the early stages of their entrepreneurship career, they might experience a large negative
shock. Individuals who continued to pursue entrepreneurship career for a lengthy period will enjoy a high average return with much less risk compared to individuals who just started entrepreneurship career. Thus, the top quintiles of the equilibrium wealth distribution would hold a much larger share of the total wealth as compared to the conventional heterogeneous agents model because the top quintiles of the wealth distribution would be mostly comprised of individuals who pursued entrepreneurship career over many periods. This will be in line with my result since low risk averse individuals who choose entrepreneurship career would be at the top quintiles of the wealth distribution. Moreover, the wealth distribution generated from this extension will capture the observed wealth distribution much better than the conventional heterogeneous agents model.

Here, I expect that the top end of the wealth distribution is comprised of mostly entrepreneurs and stockholders or investors. Moreover, the humped curve of Relative Risk Aversion at the bottom end of the wealth axis obtained in this paper reveals that the bottom of the wealth distribution still includes some less risk averse individuals compared to the slightly higher quintiles of the wealth distribution since the less risk averse individuals in high-risk and high-return career could end up in very low wealth position with some positive probability, particularly when they are young or inexperienced in the career of entrepreneurship.

The observed share of the total wealth at the bottom quintiles of the wealth distribution is much less than the share generated by the conventional heterogeneity models. One way to address this problem is to introduce the welfare policy targeted at poor individuals. With such a policy present, individuals at the bottom of the wealth distribution will have less incentive to save for precautionary motive. Therefore, the share of the total held by them might be reduced sharply in the heterogeneous agents models.

Furthermore, at the bottom of the wealth distribution, we have showed that there is a hump-shaped relationship between wealth and Relative Risk Aversion. Here, I have assumed that there is no borrowing constraint such that the intertemporal Euler equation holds with equality for poor households. However, if there is a significant borrowing constraint for households at the bottom end of the wealth distribution, the estimated Relative Risk Aversion at the bottom end of the wealth distribution could be different. For the SHIW dataset, only 2% of the households in the data have wealth below zero or possibly could be facing a borrowing constraint. Hence, the borrowing constraint might not generate the big problem toward the estimation based on the SHIW. However, for the PSID dataset, 12% of households in the data have the wealth position below zero. Possibly, the borrowing constraint could pose some problem toward the estimation of Relative Risk Aversion at the bottom end of the wealth distribution from the PSID dataset. Tackling this problem is an open challenge to the future research.

5 Conclusion

In this paper, I have analyzed the relationship between wealth of a household and Relative Risk Aversion. At the bottom end of wealth axis, there is a hump shaped relationship between wealth and Relative Risk Rversion. For the SHIW dataset, this hump is small. However, for the PSID dataset, the hump is very sharp. For the most values of the wealth
of a household, there is a significantly negative relationship between wealth and Relative Risk Aversion. Particularly for the PSID, starting from the middle of the wealth axis, there is a sharp decline in relative risk aversion when wealth increases.

This empirical result contradicts the conventional heterogeneity models in the sense that in those models, high risk averse individuals eventually hold the majority of the total wealth because of their high precautionary saving motive. However, if we add the different career options such as high risk - high return career (such as entrepreneurship) and low risk - low return career (paid employees excluding high-level executives such as CEO etc.) and self sorting into those careers to the conventional heterogeneity agent model, high risk averse individuals will choose low risk - low return career and less risk averse individuals will choose high risk - high return career. In addition to this, if an individual sticks to one career over a longer period, the risk or uncertainty involved in that career might shrink over time. Particularly for entrepreneurs, should they retain the entrepreneur career for longer periods, they might enjoy a high average return for themselves with the lessening of the risk. This extension to the conventional heterogeneity models might generate a significant negative correlation pattern between wealth and relative risk aversion.

Furthermore, several other papers found similar result to this paper, even though they have employed totally different estimation methodologies. Some authors log-linearized an intertemporal Euler equation to run a linear regression. However, after the non-linear transformation, the expectations operator does not hold anymore. In this analysis, I did not employ any non-linear transformation to the intertemporal Euler equation. Moreover, an experimental survey is used in some papers to estimate relative risk aversion. On the other hand, I used the observed consumption and wealth data for households from the PSID and the SHIW panel datasets.

As a future research question topic, I would like to extend the conventional heterogeneity agents model such that the addition of different career options and an experience or tenure effect to the conventional heterogeneity models may generate a negative relationship between wealth and a relative risk aversion. The conventional heterogeneity models do not work well to capture the wealth distribution observed in reality. The extension I consider here could possibly improve that aspect of the heterogeneity agents models.

References


