The Capital Gain Lock-In Effect and Seasoned Equity Offerings*

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Abstract

We present a general equilibrium model to explain the drop in the price of a company’s shares when it conducts an SEO. Our model is based on the capital gain lock-in effect and how this effect changes when a company issues more shares. We show, based on the lock-in model developed in Klein (1998), that the extent to which investors are locked-in is reduced when an SEO occurs. Since investors are less locked-in, the equilibrium price declines. We provide numerical examples to demonstrate our theoretical results.

JEL Codes: G11, G12, H22

Keywords: Capital gains tax, Lock-in, General equilibrium model, SEO

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1. Introduction

The unusual price movements of public equity before and after a seasoned equity offering (“SEO”) raise an important challenge for academics. The empirical literature on this issue is vast: a large number of authors report significant negative returns around the announcement and offer dates of SEOs as well as afterwards (Bilinski, Liu and Strong (2012), Kryzanowski, Lazrak and Rakita (2010), Aggarwal and Zhao (2008), Asquith and Mullins (1986), Masulis and Korwar (1986) and Mikkelson and Partch (1986), among others).

Models based on asymmetric information have been used to explain this phenomenon (e.g., Akerlof 1970), Lucas and McDonald (1990), Myers (1984), and Myers and Majluf (1984)). Other theories are based on cognitive bias and persistent mispricing ideologies (e.g. Baker and Wurgler (2002), Daniel, Hirshleifer and Subrahmanyam (1998) and Loughran and Ritter (1995)).

Some researchers focus on risk-based explanations. Aggarwal and Zhao (2008) find that the negative offer day return is positively related to the change in volatility around SEO issuance. Korajczyk, Lucas and McDonald (1991), and Mikkelson and Partch (1988) argue that the drop of price post-SEO from pre-SEO can be attributed to the reduced uncertainty that the issue will go ahead.

Changes in the demand and supply of shares around an SEO have also been argued to be responsible. Asquith and Mullins (1986) claim an imbalance between demand and
supply may negatively affect the equity price. Lease, Masulis, and Page (1991) argue that the price drop during the issue day is due to an imbalance between buy and sell orders since most buy orders are diverted to the primary market on the issue day, but sell orders are directed to the secondary market.

In this paper, we provide an additional theory to explain this price phenomenon around SEOs. Our theory is also based on an analysis of changes in the supply and demand of shares around an SEO but it considers the effect of capital gains taxation on these changes. The literature has long recognized that capital gains taxation induces a lock-in effect on investor behavior (Auerbach (1991), Balcer and Judd (1987), Dai, Maydew, Shackelford, and Zhang (2008), Kovenock and Rothschild (1987) and Landsman and Shackelford (1995), among others). Klein (1998) shows in a general equilibrium model that prices are higher for stocks in which investors are locked-in because of accrued capital gains. Klein (2001) finds good empirical support for this theory.

We build on the lock-in model of Klein (1998) and investigate how an investor’s lock-in changes by the increased supply of new shares when an SEO occurs. The effect on investor behavior is analogous to the effect of institutional procedures for short selling as analyzed in Kwan (1997). We show that the extent to which investors are locked-in decreases with the increased supply of shares provided by the SEO. Thus the price of a stock which, through prior price gains, has created large accrued capital gains for its shareholders, will experience a larger decline in price because of the SEO as compared to stocks which have not created large gains for their shareholders. Numerical examples
considering investors with different initial endowments and cost bases demonstrate the results of our model.

This paper contributes to the theoretical literature on the reasons for anomalous price behavior around SEOs. To the author’s knowledge this paper is the first study to draw the link between SEO price behavior and the capital gain lock-in effect. This paper also contributes to the literature on capital gain lock-in (Dai, Maydew, Shackleford and Zhang (2008), Ayers, Lefanowicz and Robinson (2003), Blouin, Raedy and Shackelford (2003), Klein (1998, 1999, 2001 and 2004), Reese (1998), Landsman and Shackelford (1995) and Constantinides (1984)) by showing that capital gains taxes influence the optimal behavior of tax sensitive investors differently from pre-SEO to post-SEO.

We start by presenting a simplified version of the Klein (1998) lock-in model in Section 2 of this paper. We use this model in Section 3 to explain how this lock-in term can drop from pre-SEO to post-SEO. In Section 4 we provide some numerical examples to support our theoretical work.
2. Recap of the Lock-In Model

Klein (1998) develops a discrete-time general equilibrium model of asset pricing under the assumption that capital gains are taxed only upon realization and that short sales, either outright or against the box, are prohibited.\(^1\) Under these assumptions, investors may be unable, without paying capital gains taxes, to reduce an overly large exposure to the risk in a security in which they have large accrued capital gain. As a result, the equilibrium price of a security in which investors have large accrued capital gains is higher than it would be if there were no accrued capital gains.

This section presents an abridged version of the Klein (1998) model which is then used in Section 3 to show the equilibrium price reaction to an SEO. In general, the assumptions and notation are similar to the Klein (1998) model.

Assume there are \(I\) investors in a pure exchange economy and \(K\) companies which have issued shares. Each investor \(i\) is endowed with wealth in the form of heterogeneous non-negative shareholdings \(H_{ik}\) of the stocks and \(H_{io}\) shares of a riskless asset. Investors

\(^1\) Rule 10b-21, adopted by the National Association of Securities Dealers (NASD) on August 25, 1988, prohibited short sellers from covering short positions established after the filing of registration statements or Form 1-A with securities purchased from an underwriter, broker, or dealer participating in the offering. In order to create an exception for transactions that are unlikely to have a market impact, in March 1997, SEC adopted Rule 105, Regulation M to replace Rule 10b-21. Like Rule 10b-21, Rule 105 prohibits short positions established during the restricted period from being covered with securities obtained from an underwriter, broker, or dealer who is participating in an offering. However, Rule 105 only restricts short sales made five trading days before the offering’s pricing rather than the potentially much longer restricted period of Rule 10b-21, which commenced with the filing of a registration statement or Form 1-A (Charoenwong, Ding and Wang (2013)).

\(^2\) The short sales constraint also restricts investors from deferring gains by short selling perfect substitute securities. Klein (2004) explains that these strategies by short-selling a perfect substitute may not be successful because the perfect substitute needs to replicate not only the systemic risk factor but also the unsystematic risk of the security. Also, the perfect substitute may have already accrued capital gains.
decide at time 1 what their holdings, $S_{ik}$, should be and buy or sell their endowment holdings as needed. Investors also decide their optimal holdings of the riskfree asset, $S_{f0}$, at time 1.

Endowments have per share cost bases, $B_{ik}$, which are heterogeneous across stocks as well as investors. Investors know the cost bases of the shares in their endowment portfolios, but not their capital gains since the latter depends on equilibrium prices which are endogenous to the model. Realized capital gains are taxed at investor specific capital gains tax rates, $T_i$.\(^3\)

The equilibrium price after all trading is done is $P_k$. The amount of capital gains tax payable at $t = 1$ can be denoted as $(P_k - B_k)\alpha_{ik}T_i$, where the variable $\alpha_{ik}$ represents the amount of $i$’s position in security $k$ that is sold at $t = 1$. If $i$ buys more stock, no capital gains are being realized and thus $\alpha_{ik} = 0$. If $i$ sells stock, capital gains are being realized and $\alpha_{ik}$ equals the number of shares sold (i.e., $\alpha_{ik} = H_{ik} - S_{ik}$).

Consumption at $t = 1$ is the amount left over from the endowment portfolio to be carried into $t = 2$ and the payment of capital gains taxes on gains realized at $t = 1$. Thus, the time 1 budget constraint is:

$$c_{i,t=1} = E_i - S_{f0} - P S_i - (P - B_i)\alpha_{i}T_i$$  \(1\)

\(^3\) Some institutional investors may not be taxed on capital gains but that will not eliminate the capital gain lock-in effect if other investors are.
Here $E_i$ is the pre-tax value of the endowment portfolio, i.e., $E_i = H_0 + P H_i$. The bold $P$ is a vector of prices of the $K$ risky securities. Throughout this paper the use of bold and absence of a $k$-subscript for a variable denotes a vector of the corresponding variable for all securities $k = 1$ to $K$. We also omit, for notational simplicity, all transpose signs.

Time 2 is defined in the usual way as a later point in time at which all stocks pay a liquidating dividend and the model ends. For simplicity, the derivation in this paper assumes stocks do not pay any other dividends\(^4\). The liquidating dividends is defined as $D_k$, and investor pays capital gains tax on the difference between the liquidating dividend and the price at $t = 1$ (i.e., $(D_k - P_k)S_{ik}T_i$). Any gains on the endowment portfolio that were not realized at $t = 1$ will also be taxed at $t = 2$ (i.e., $(P_k - B_k)(H_{ik} - \alpha_{ik})T_i$). The riskfree investment earns a gross return of $R = 1 + r_f$ where $r_f$ is the riskfree rate. Again for simplicity, we assume there is no tax on interest income. Thus the $t = 2$ budget constraint is:

$$c_{i,2} = S_i R + D_i S_i - (D_i - P_i) S_i - (P_i - B_i) (H_i - \alpha_i) T_i$$

(2)

Assuming utility is concave and time separable, each investor maximizes

$$U_{i,2}(c_{i,2}) + E[U_{i,2}(c_{i,2})]$$

\(^4\) This is in contrast to the assumptions in Klein (1998). Note one can also assume that all unpaid dividends are paid together at time 2 as liquidating dividends.
to solve for optimal holdings $S_{ik}$ and $S_{io}$ subject to the budget constraints and the short selling constraint $S_{ik} \geq 0$ for $k = 1, \ldots K$.

As in Klein (1998), the first order condition of investor $i$ for a given stock $k$ can be written as follows:

$$P_k = R^{-1}(\delta_{ik} + P_k + (E[D_k] - P_k)(1 - T_i) - \theta_i S_i \Omega_{ik}) \quad (3)$$

where

$$\delta_{ik} = w_{ik}(T_i(P_k - B_{ik})(R - 1)) + \lambda_{ik} \quad (5)$$

$\lambda_{ik}$ represents the Lagrange multiplier on the short sales constraint and $i_k$ indicates a vector of zeroes with 1 in position $k$.

Equation (3) specifies the relationship that must hold between $P_k$ and the expected after-tax discounted cash flows on that stock ($D_k$) if investor $i$ is unable to increase utility by changing the number of shares, $S_{ik}$, which are held in stock $k$. This relationship depends on an adjustment for risk $\theta_i S_i \Omega_{ik}$ which in turn depends on $i$'s holdings in all of the $k$ securities, $S_i$, as well as $i$'s risk tolerance, $\theta_i$, and the variance-covariance matrix of the $K$ risky securities, $\Omega$.

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As in Klein (1998) we assume only the first two moments of the return distribution are relevant for investors. Alternatively, our results can be considered as representing the first two terms of a Taylor series approximation. $\alpha_{ik}$ is assumed to be smooth so that it is differentiable as a function of $S_{ik}$. More explanation on this can be found in Klein (1998, pg. 1538).
Equation (3) also depends on $\delta_{ik}$ which is the individual deferral term or lock-in term as defined in Klein (1998). From Equation (5), this lock-in term depends on the size of the capital gain ($P_k - B_k$) on the position, the investor’s capital gains tax rate $T_i$, a timing factor representing the net present value of deferring the capital gains tax liability for one period ($R - 1$), and an unobservable weight ($w_{ik}$). If it is optimal for the investor to sell some of stock $k$, $w_{ik} = 1$. If instead it is optimal for the investor to buy some of stock $k$, $w_{ik} = 0$. Thus, the lock-in term, $\delta_{ik}$, equals zero if $i$ buys $k$ at time $t = 1$, and is equal to the net present value of the benefit of deferring the accrued capital gain if $i$ sells $k$ at $t = 1$. If the investor is bound by the short sales constraint, the individual deferral term takes the rescaled value of the Lagrange multiplier, $\lambda_{ik}$, that is divided by $E[U'_{i,t=2}(c_{i,t=2})]$. If the investor neither buys or sells at $t = 1$, $w_{ik}$ takes a value between 0 and 1 such that Equation (3) holds.

Rearranging Equation (3) to solve for optimal shareholdings $S^*_i$, dividing both sides by $(R - T_i)$, rescaling and defining $\delta_i = \delta_{ik}/(R - T_i)$, and $t_i = (1 - T_i)/(R - T_i)$, leads to:

$$ S^*_i = \theta_i (\delta_i + E[D] t_i - P) \Omega^{-1} $$

(6)

Summing Equation (6) over all investors, imposing the market clearing condition, dividing both sides by $\theta_m = \sum \theta_i$ and rearranging, leads to:

$$ S^*_i = (\theta_i/\theta_m)1 + \theta_i((\delta_i - \delta) + E[D](t_i - t_m))\Omega^{-1} $$

(7)

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Note: $w_{ik} = -\partial \alpha_{ik}/\partial S_{ik}$. 

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where \( t_m = \sum \theta_i t_i \), \( \hat{\delta} = (\sum \theta \delta_i) / \theta_m \) and \( \mathbf{1} \) denotes a vector of ones representing the single share outstanding in each security \( k = 1, \ldots, K \).

Equation (7) indicates that an investor’s optimal holdings of shares depend on more than the ratio of individual to market risk tolerance, \( \theta_i / \theta_m \). The vector of investor deferral terms, \( \delta_i \), measures the extent to which an investor is locked-in due to accrued capital gains. The vector of average deferral terms, \( \bar{\delta} \), indicates that the extent to which other investors are locked-in due to accrued capital gains also affects the optimal portfolio composition of a given investor \( i \). Since the sign on \( \bar{\delta} \) is negative, the lock-in of other investors acts to offset the effect of lock-in due to an investor’s own accrued capital gains, \( \delta_i \).

It should be noted that \( S^* \), in Equation (7) depends on the vector of equilibrium prices, \( \mathbf{P} \), through the individual and average deferral terms. These equilibrium prices, \( \mathbf{P} \), depend in turn on optimal portfolio weights. In presence of this simultaneity problem, we revert to Sharpe (1991) and Kwan (1997), among others, and sum Equation (6) across all investors and rearrange as follows to get the equilibrium price for stock \( k \):

\[
P_k = R^{-1}(\hat{\delta}_k + P_k + (E[D_k] - P_k)(1 - T) - \theta_m^{-1} \mathbf{1} \mathbf{\Omega} \mathbf{t}_i) \tag{8}
\]

where \( T = (\sum \theta_i t_i) / \theta_m \) and \( \hat{\delta}_k = (\sum \theta \delta_{ik}) / \theta_m \) which is element \( k \) in \( \hat{\delta} \) defined above.

These Equations (7) and (8) are similar to the results in Klein (1998) with the simplified assumptions of this paper. In the presence of capital gains taxes, these
expressions should determine the pre-SEO prices and optimal holdings after all secondary market trading is over.
3. Effect of an SEO

We now use the model developed in the previous section to show the equilibrium price reaction to an increase in the number of shares outstanding because of an SEO. We start by re-writing Equation (8) to focus on a single stock $k = SEO$ which is assumed to be the company that is conducting the SEO:

$$P_{SEO} = R^{-1}(\hat{\delta}_{SEO} + P_{SEO} + (E[D_{SEO}] - P_{SEO})(1 - T) - \theta_m^{-1}1_{SEO}\Omega_{SEO})$$  \hspace{1cm} (9)

where $1_{SEO}$ is the vector of the number of shares outstanding in stocks $k = 1, \ldots, K$, i.e., a matrix of ones with $N_{SEO}$ in position $k = SEO$ where $N_{SEO}$ is the number of shares of the firm that is doing the SEO; and $t_{SEO} = t_k$ where $k = SEO$.

We want to show that the equilibrium price ($P_{SEO}$) is strictly lower when $N_{SEO}$ increases. In order to do so we need to consider each investor’s response to the increase in $N_{SEO}$. Equation (7) can be written for stock $k = SEO$ as:

$$S_{t_{SEO}} - (\theta_i/\theta_m)N_{SEO} - \theta_i((\hat{\delta}_i - \hat{\theta} + E[D]) (t_i - t_m))\Omega^{-1}t_{SEO} = 0$$  \hspace{1cm} (10)

**Proposition 1.** When the number of shares of a given stock increases because of an SEO, any investor who was locked-in will be less so. Any investor who purchased at the equilibrium price will still not be locked-in and will purchase more; and any investor who sold or did not trade will be less locked-in, i.e., will either now purchase or not trade.
To prove this proposition, we implicitly differentiate Equation (10) with respect to \( N_{SEO} \):

\[
\frac{\partial S_{i,SEO}^*}{\partial N_{SEO}} = \frac{\theta_i}{\theta_m} + \theta_i \left[ \partial_i (\delta_i - \bar{\delta}) \partial N_{SEO} \right] \Omega^{-1} \iota_{SEO} \tag{11}
\]

Equation (11) simplifies\(^7\) to:

\[
\frac{\partial S_{i,SEO}^*}{\partial N_{SEO}} = \frac{\theta_i}{\theta_m} + \psi_i \frac{\partial w_{i,SEO}}{\partial N_{SEO}} \tag{12}
\]

where \( \psi_i = (\theta_i (1 - 1/\theta_m) \Omega^{-1} \iota_{SEO}) (T_i (P_{SEO} - B_{i,SEO}) (R - 1)) \) which is a positive scalar.

We want to show the lefthand side of Equation (12) is greater than or equal to zero for all investors. We also want to analyze the value of \( \frac{\partial w_{i,SEO}}{\partial N_{SEO}} \) because it will help determine the effect of an SEO on \( \delta_{SEO} \) in Equation (9).

For ease of exposition, we re-write Equation (12) as:

\[
CS = RTS + CW \tag{13}
\]

where \( CS \) refers to “change in shareholding” due to the increase in \( N_{SEO} \) because of the SEO (i.e., \( CS = \frac{\partial S_{i,SEO}^*}{\partial N_{SEO}} \)); \( RTS \) refers to “risk tolerance share” of the shares issued under

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\(^7\) This simplification results because changes in all deferral terms except for \( k = SEO \) can be ignored and because the inverse of the variance-covariance matrix is positive semi-definite.
the SEO (i.e., $RTS = \frac{\theta_i}{\theta_m}$); and $CW$ refers to the change in investor $i$’s deferral term due to the change in $w_{i,SEO}$ (i.e., $CW = \psi_i \frac{\partial w_{i,SEO}}{\partial N_{SEO}}$).

Before the increase in the number of shares because of the SEO, $i$ either buys, sells, or does not trade at the equilibrium price. As the number of shares for $k = SEO$ increases because of the SEO, investor $i$ will also either buy, sell, or not trade. Table 1 below outlines these 9 possible cases of investor activity before and after an SEO.

Table 1: Changes in $w_{i,SEO}$ (CW) and changes in $\frac{\partial S^*_{i,SEO}}{\partial N_{SEO}}$ (CS) from pre-SEO to post-SEO

<table>
<thead>
<tr>
<th>Case #</th>
<th>Behavior at Equilibrium (pre-SEO)</th>
<th>Behavior post-SEO</th>
<th>CS$^1$</th>
<th>CW$^1$</th>
<th>Feasibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Buyer</td>
<td>Buyer</td>
<td>&gt; 0</td>
<td>0</td>
<td>Feasible</td>
</tr>
<tr>
<td>2</td>
<td>Buyer</td>
<td>No Trade</td>
<td>0</td>
<td>&gt; 0</td>
<td>Infeasible$^2$</td>
</tr>
<tr>
<td>3</td>
<td>Buyer</td>
<td>Seller</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>Infeasible$^3$</td>
</tr>
<tr>
<td>4</td>
<td>No Trade</td>
<td>Buyer</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>Feasible</td>
</tr>
<tr>
<td>5</td>
<td>No Trade</td>
<td>No Trade</td>
<td>0</td>
<td>&lt; 0$^2$</td>
<td>Feasible</td>
</tr>
<tr>
<td>6</td>
<td>No Trade</td>
<td>Seller</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td>Infeasible$^3$</td>
</tr>
<tr>
<td>7</td>
<td>Seller</td>
<td>Buyer</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>Feasible</td>
</tr>
<tr>
<td>8</td>
<td>Seller</td>
<td>No Trade</td>
<td>0</td>
<td>&lt; 0$^2$</td>
<td>Feasible</td>
</tr>
<tr>
<td>9</td>
<td>Seller</td>
<td>Seller</td>
<td>&lt; 0</td>
<td>0</td>
<td>Infeasible$^3$</td>
</tr>
</tbody>
</table>

Notes:
1. CS = “Change in shareholding” = $\frac{\partial S^*_{i,SEO}}{\partial N_{SEO}}$; CW = “Change in $w_{i,SEO}$ x $\psi_i$” = $\psi_i \frac{\partial w_{i,SEO}}{\partial N_{SEO}}$; RTS = “Risk tolerance share” = $\theta_i/\theta_m$. Thus, CS = RTS + CW from eq. (13)
2. CS = 0 ⇒ CS = RTS + CW = 0 ⇒ CW < 0 since RTS > 0 using eq. (13). This makes case 2 infeasible. In case 5 and case 8, this prevents CW = 0
3. CS < 0 ⇒ CS = RTS + CW < 0 ⇒ CW < 0 since RTS > 0 using eq. (13). This makes cases 3, 6 and 9 infeasible.

Table 1 considers the value for the change in shareholding (CS) because of an increase in the number of shares for $k = SEO$. For example, if $i$ buys at the pre-SEO equilibrium price (cases 1, 2 and 3) there are three possible values for CS. CS is positive if
$i$ would buy more due to an SEO; CS is zero if $i$ would not trade and CS is negative if $i$ would sell because of the SEO. The values of CS are similar for cases 4, 5 and 6 if investor $i$ did not trade at the equilibrium price prior to the SEO. They are also similar for cases 7, 8 and 9 if $i$ sold at the pre-SEO equilibrium price.

Table 1 also considers the change in investor $i$’s $w$ term (CW) when the number of shares of $k = SEO$ increases because of an SEO. For example, if $i$ buys at equilibrium (cases 1, 2 and 3), CW = 0 if $i$ still buys as the number of shares of $k = SEO$ increases (case 1); this is because $w_{i,seo} = 0$ when $i$ buys. CW > 0 if $i$ does not trade (case 2); this is because $w_{i,seo}$ changes from zero to strictly positive (but also < 1). CW > 0 if $i$ sells (case 3); this is because $w_{i,seo}$ would change from 0 to 1. The CW terms for cases 4 to 9 can be determined by similar reasoning. Note in cases 5 and 8, the possibility that CW = 0 is ruled out by Equation (13).\(^8\)

The last column in Table 1 indicates which cases are feasible. Due to Equation (13), cases 2, 3, 6 and 9 are infeasible.\(^9\) Considering only the remaining cases means CW must be zero for an investor who was buying at the equilibrium price and can only be strictly negative for all other investors (i.e., those investors who sold and those investors who did not trade). Thus, proposition 1 has been shown.

We now consider the effect of an SEO on the equilibrium price. In order to do so

\(^8\) See note 2 in Table 1.
\(^9\) See note 3 in Table 1.
we need to consider the aggregate effect for all investors based on the effect on each individual investor as described in Proposition 1.

**Proposition 2.** \( P_{\text{SEO}} \) is less than it was before SEO; the drop is larger if \( k = \text{SEO} \) has larger accrued gains.

Proposition 2 follows directly from differentiating \( P_{\text{SEO}} \) in Equation (9) with respect to \( N_{\text{SEO}} \):

\[
\frac{\partial P_{\text{SEO}}}{\partial N_{\text{SEO}}} = R^{-1}(\frac{\partial \delta_{\text{SEO}}}{\partial N_{\text{SEO}}} - \partial \Omega^{-1} \Omega_{\text{SEO}} \Omega^{-1} \Omega_{\text{SEO}}) \tag{14}
\]

The second term on the right hand side represents the decrease in \( P_{\text{SEO}} \) because investors need to be compensated for accepting the additional risk because of the SEO. The term itself is strictly positive which means, given it has a negative sign, the effect on the equilibrium price is strictly negative. The first term on the right hand side is also strictly negative from proposition 10. Since, \( \delta_{\text{SEO}} \) is increasing in investors’ accrued capital gains, it implies \( P_{\text{SEO}} \) drops more for stocks that have created large capital gains for their investors. Thus, Proposition 2 has been shown.

These two propositions allow us to comment on how the company doing the SEO should set the price at which it should offer its shares (the “SEO offer price”) and how that price compares with the post-SEO equilibrium price that is provided by Equation (9).

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10 Unless no investors are locked-in, in which case this term equals zero.
First, we note from Table 1 that none of the investors will sell shares of the company that is doing the SEO; all investors either buy more or do not trade.\textsuperscript{11} Thus the sole seller of shares is the company doing the SEO. If the SEO offer price is above the post-SEO equilibrium price, there will be insufficient interest from investors and the SEO will be unsuccessful. If the SEO price is below the post-SEO equilibrium price, investors will eagerly purchase these new shares from the company. The equilibrium price after the SEO will be established as per Equation (9) which is lower than it would have been if there had been no SEO.

We note that our model does not formally distinguish between the time the SEO is announced and the time the new shares are issued. Our model only shows that the equilibrium price drops when the number of shares increases due to an SEO. Our results are consistent, however, with an announcement day effect since no investor would want to buy at the higher pre-SEO equilibrium price knowing the number of shares is going to increase when the SEO is completed. If the actual number of shares issued differs from what was announced, the post-SEO equilibrium price would adjust accordingly.

\textsuperscript{11} We recognize in reality that expectations of some investors may change, because of signalling, for example, when an SEO is announced. Thus, the company may not be the only seller of shares. In order to focus on the effect of lock-in, our model excludes this possibility.
4. Numerical Examples

Table 2 presents numerical examples to provide further insight to the model. The table assumes there are two securities \((k = 1 \text{ and } 2)\) and each security initially has three shares outstanding. The securities pay no dividends except for a terminal dividend \((E[D_k] = 10)\). There are three investors \((i = A, B \text{ and } C)\) who have homogenous expectations about the mean and variance of the terminal dividends of the two securities. We assume the SEO does not change these expectations, i.e., we assume no signaling so that our examples can focus on the effect of lock-in on equilibrium prices after an SEO. The three investors have the same risk tolerance, and risk tolerance is assumed to be constant. The only difference among the investors is their initial endowments and accrued gains. Parameter values are arbitrarily selected, but these values are consistent with a one-year investment horizon as in Klein (1998).

<table>
<thead>
<tr>
<th>Stock</th>
<th>(H_a)</th>
<th>(H_B)</th>
<th>(d_a)</th>
<th>(d_B)</th>
<th>(S^a)</th>
<th>(S^B)</th>
<th>(P_a)</th>
<th>Change in (P_a)</th>
</tr>
</thead>
</table>
| I. Pre-SEO for stocks with "no lock-in"
1 | 1.300 | 1.300 | 0.400 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 8.670 |
| 2 | 1.000 | 1.000 | 1.000 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 8.670 |
| II. Post-SEO for stocks with "no lock-in"
1 | 1.000 | 1.000 | 1.000 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 1.100 | 1.100 | 1.100 | 8.600 | -0.070 |
| 2 | 1.000 | 1.000 | 1.000 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 8.647 | -0.023 |
| III. Pre-SEO for stocks with "high lock-in"
1 | 1.300 | 1.300 | 0.400 | 0.700 | 7.700 | 8.670 | 0.064 | 0.015 | 0.000 | 0.026 | 1.150 | 0.955 | 0.895 | 8.702 |
| 2 | 1.000 | 1.000 | 1.000 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 0.925 | 1.022 | 1.052 | 8.677 |
| IV. Post-SEO for stocks with "high lock-in"
1 | 1.150 | 0.955 | 0.895 | 0.700 | 7.700 | 8.670 | 0.019 | 0.000 | 0.000 | 0.000 | 1.150 | 1.075 | 1.075 | 8.609 | -0.093 |
| 2 | 0.925 | 1.022 | 1.052 | 8.670 | 8.670 | 8.670 | 0.000 | 0.000 | 0.000 | 0.000 | 0.975 | 1.012 | 1.012 | 8.650 | -0.027 |

Calculations for investors \(i = A, B, C\) and securities \(k = 1, 2\) are based on eqs. 3, 7 and 8 in the text. Unless otherwise noted, parameter values are as follows: \(\sigma^2 = \sigma^2 = 1\); \(\sigma_{12} = 0.5\); \(E[D_k] = 10\); \(\theta_i = 3\) (constant risk tolerance); \(K = 1.05\); \(T_i = 0.3\); for \(i = A, B, C\).
The equilibrium prices are determined through a Walrasian auction as in Klein (1998). At first, a trial set of equilibrium prices for the two securities is assumed. Based on the endowed shareholdings \( (H_{ik}) \) and the endowed cost basis \( (B_{ik}) \), Equation (3) is used to determine whether an investor buys, sells or does not trade at the trial set of prices and to determine each investor's deferral term \( (\delta_{ik}) \). Equation (6) is used to determine the optimal shareholdings of each investor in the two securities; if the sum of these optimal shareholdings does not equal the number of securities outstanding, the process is repeated until it does. The next iteration starts using the optimal shareholdings from Equation (6) and the set of prices from Equation (8) from the previous iteration.

Panel I in Table 2 shows a base case in which none of the investors has an accrued gain on either security. Investors A and B are endowed with more than the optimal number of shares and investor C starts with less than the optimal number of shares. Investors A and B sell to investor C and the equilibrium price of each security is $8.67.

Panel II in Table 2 illustrates the effect of an SEO under the same assumptions as in Panel I except that the number of shares of security 1 is increased by 10% to 3.3 shares outstanding. We have assumed that the starting shareholdings are equal to what was optimal in the equilibrium described in Panel I, i.e., before the SEO. The post-SEO equilibrium price of $8.60 for security 1 is lower than the pre-SEO equilibrium price of $8.67.

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12 In models with frictions such as transactions costs or taxes, it is usually assumed that trading occurs only once the equilibrium price has been determined and posted by the auctioneer.

13 We set \( w_{ik} = 0, 1 \) or a value between 0 and 1 depending on whether the investor buys/sells/does not trade as in Klein (1998).

14 We believe this is the appropriate assumption when analyzing the effect of an SEO on the price of a security that has already been trading in the market.
$8.67 because of the greater supply of shares. The equilibrium price of security 2 also declines but by a smaller amount than for security 1; this decline is because the investors now require a greater compensation for bearing any type of security market risk because of the higher number of shares outstanding in security 1. Note that in Panel II all three investors will purchase more shares because of the SEO. This result is consistent with Equation (13); since no investors are locked-in, the CW term disappears and only the RTS term applies, which is strictly positive.

Panels III and IV repeat the examples provided in the first two panels but when there is lock-in. Panel III determine the pre-SEO price of security 1 which is used as a base case for comparison with what happens in Panel IV. Panel IV illustrates the effect of an SEO under the same assumptions as in Panel III except that the number of shares of security 1 is increased by 10% to 3.3 shares outstanding. Note we once again assume in Panel IV that starting endowments are equal to what was optimal before the SEO as shown in Panel III. The post-SEO equilibrium price of $8.61 for security 1 is lower than the pre-SEO equilibrium price of $8.70 because of the greater supply of shares. The equilibrium price of security 2 also declines because of the SEO in security 1, but by a smaller amount.

Panels III and IV also demonstrate how investors’ deferral terms $(\delta_{ik})$ change with the SEO in security 1. In Panel III, both investors A and B are locked-in to their starting

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15 This is the second term on the right hand side of Equation (14). We note the model in this paper does not allow for other reasons for this price decline, such as signalling.

16 We note the pre-SEO equilibrium price for security 1 is higher in Panel III when there is lock-in than what it is in Panel I when there is no lock-in. This difference is consistent with the results in Klein (1998).

17 This is the second term on the right hand side of Equation (14). We note the model in this paper does not allow for other reasons for this price decline, such as signalling.
position in security 1. Despite being locked-in, investor B is willing to sell some of its shares to investor C because its cost basis is not much below the equilibrium price and its deferral term is small. In contrast, investor A is not willing to trade because it is more locked-in than B and has a larger deferral term for security 1. This is also because of its larger starting position and lower cost basis. After the SEO as shown in Panel IV, investor A is still locked-in and continues to hold a greater number of shares in security 1 than would be optimal in the absence of lock-in as shown in Panel II. Investor A’s deferral term decreases, however, because it is less locked-in than it was before. This is because the additional 0.3 shares offered through the SEO has increased the number of shares investor A would optimally hold if it were not locked-in, as shown in Panel II. These results are consistent with Proposition 1.

In contrast with the results in Panel II, it is no longer the case that all three investors purchase more shares because of the SEO. The two terms on the right hand side of Equation (13) and the 9 cases in Table 1 provide guidance on this issue. Since investor C is not locked-in, the CW term in Equation (13) equals zero and only the RTS term applies; this case corresponds to Case 1 in Table 1. Investor B sells at the pre-SEO equilibrium price but purchases at the post-SEO equilibrium price; this case corresponds to Case 7 in Table 1. Investor A does not trade at the pre-SEO equilibrium price and also does not trade at the post-SEO equilibrium price; this corresponds to Case 5 in Table 1.

The remaining two feasible cases in Table 1 do not apply to any of the investors’ situations described in the numerical examples provided in Table 2. If we changed the
example so that the number of shares issued in the SEO were twice as large, i.e., 0.6 new shares of security 1, investor B would also purchase shares at the post-SEO equilibrium price and Case 4 would apply instead. Case 8 of Table 1 would apply if investor B has a lower cost basis and only sold 0.01 share in Panel III and the SEO in Panel IV is one quarter as large, i.e., only 0.075 new shares of security 1. The other cases in Table 1 (cases 2, 3, 6 and 9) are infeasible as discussed earlier.

Comparing the results in Panels I and II with the results in Panels III and IV provides an example of Proposition 2. The SEO-induced drop in equilibrium price for stock 1 in Panels I and II when there is no lock-in is $0.07 which is less than the $0.09 drop when there is lock-in as shown in Panels III and IV. This difference arises because of the first term on the right hand side of Equation (14) which is more negative for stocks with larger accrued capital gains. We note these price drops are based on one-year investment horizon and likely understate the magnitude of this lock-in effect in a more realistic setting of a longer investment horizon. We call for empirical research to determine if this effect is found in the cross-section of SEO-induced stock price movements.

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18 These numerical examples treat the risk-free rate and investors’ risk tolerance as constants. We can see that as investors become more locked-in, they sell less. So, current consumption would decrease as compared to future consumption, which will influence both the risk-free rate and the degree of risk tolerance. These changes would affect the pricing of all securities and may mitigate the extent of the lock-in effect on the market portfolio. These changes would not, however, eliminate the cross-sectional differences among equilibrium stock returns due to the lock-in effect, as outlined in Klein (1998).

19 See the last panel in the numerical examples of Klein (1998) which provides an example of how much larger the lock-in effect can be with a longer and more realistic investment horizon. Also see Klein (1999, 2001) which provide multi-period models.
5. Conclusion

In this paper, we show that investors are less locked-in when a company issues more shares. As a result, the equilibrium price of those shares should decline, and that decline is larger for stocks that have created large accrued capital gains for its investors.

This finding is consistent with the empirical literature which reports a decline in a company’s stock price when it conducts an SEO. Our results also provide a rational explanation on how the cross-section of these price responses should depend on investors’ accrued capital gains. We call for empirical research on this issue.
References


