Experimental Economics and the New Commodities Problem

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Abstract

Brynjolfsson, Collis, Diewert, Eggers and Fox (2018) have used experimental economics to measure the welfare benefits of free commodities. In this paper, their methodological approach is adapted to measuring the benefits of new commodities which may or may not be free. Their approach leads to a new method for estimating Hicksian reservation prices. The new methodology in the present paper requires experimental estimates for household willingness to pay for new commodities or estimates for the compensation required for households to give up their use of a new commodity.

JEL Classification Numbers

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1. Introduction

A major problem facing statistical agencies is how to adjust household price and quantity indexes for increases in the choice of commodities. The main concept for dealing with this problem is to use a framework suggested by Hicks (1940), where it is assumed that households have (latent) preferences defined over products before they actually appear in the marketplace. If reservation prices for these unavailable products can be estimated for the period prior to their introduction to the market place, then normal index number theory, based on the economic approach to index numbers, can be applied. The practical problem is: how exactly can these reservation prices be determined?

Hausman (1996) (1999) and Diewert and Feenstra (2017) adapted household demand theory to estimate these unobserved reservation prices while de Haan and Krsinich (2014) used hedonic regression techniques. In the present paper, following the example of Brynjolfsson, Eggers and Gannamaneni (2018) and Brynjolfsson, Collis, Diewert, Eggers and Fox (BCDEF) (2018), we suggest a third method for determining reservation prices. This rests on laboratory or online experiments that elicit what compensation is required in order for a household to give up its consumption of a new product.[2]

The idea of compensating households for price changes, in such a way that their utility would be held constant, is due to Hicks (1939; 40-41) (1946; 331-332).[3]

2. The Case of N Continuing Commodities and One New Commodity

We assume that we have price and quantity data for a household (or a homogeneous group of households) for two periods. In period 0, the observed price and quantity vectors are $p^0 \equiv [p_1^0,...,p_N^0]$ and $q^0 \equiv [q_1^0,...,q_N^0]$. In period 1, we have the new price and quantity vectors, $p^1 \equiv [p_1^1,...,p_N^1]$ and $q^1 \equiv [q_1^1,...,q_N^1]$ and in addition, the household is consuming $z^1 > 0$ units of a new commodity that is sold at the price $w^1 > 0$. The household maximizes a linearly homogeneous, increasing, continuous and concave utility function, $f(q,z)$, subject to a budget constraint in each period. However, in period 0, we constrain z to equal 0. The utility of consuming q in period 0 is given by $f(q,0)$ so we are making the

[2] This paper is inspired by the work of Brynjolfsson, Eggers and Gannamaneni (2018) and Brynjolfsson, Collis, Diewert, Eggers and Fox (2018). Our methodology uses somewhat different assumptions.

[3] Suppose a utility maximizing household has the utility function $f(q)$ where q is a consumption vector. Let $u = f(q)$ and let p be a positive vector of prices that the household faces. The household's cost or expenditure function is defined as $C(u,p) \equiv \min_q \{p\cdot q : f(q) \geq u\}$. Diewert and Mizobuchi (2009; 344) used the cost function to define the family of *Hicksian price variation functions* as $P_H(p^0,p^1,q) \equiv C[f(q),p^1] - C[f(q),p^0]$. These functions are difference counterparts to the family of Konüs (1939) true cost of living indexes, $C[f(q),p^1]/C[f(q),p^0]$. Hicks (1945; 68-69) called $P_H(p^0,p^1,q^0)$ *the price compensating variation* and called $P_H(p^0,p^1,q^1)$ the *price equivalent variation*. This latter price variation will play an important role in what follows. Samuelson (1974) defined the family of *money metric utility changes* as follows: $Q_S(q^0,q^1,p) \equiv C[f(q^1),p] - C[f(q^0),p]$. These functions are difference counterparts to the family of Allen (1949) price indexes, $C[f(q^1),p]/C[f(q^0),p]$. Henderson (1941; 118) defined the (quantity) *compensating variation* as $Q_S(q^0,q^1,p^1)$ for the case of two commodities and Hicks (1942; 128) defined it for the case of N commodities. Hicks (1942; 127) also defined the (quantity) *equivalent variation* for a general N as $Q_S(q^0,q^1,p^0)$.

assumption that Hicks (1940; 114) made many years ago; i.e., that the household has the *same tastes* in each period, including the period when the new commodity was not available.

Our aim is to obtain a Hicksian reservation price for the new commodity in period 0 using (experimental) information on how much *compensation* must be paid to households in period 1 for *not* consuming the new commodity. Once an appropriate reservation price for the new commodity is obtained for period 0, normal index number theory can be used to measure welfare change and changes in the Konüs (1939) true cost of living index.[4]

Define the utility level in period 1 as follows:

(1) $u^1 \equiv f(q^1,z^1)$.

Define the *household's conditional cost function*, $c(u,p,z)$, as follows:[5]

(2) $c(u,p,z) \equiv \min_q \{p\cdot q : f(q,z) \geq u\}$.

This cost function minimizes the costs of consuming a bundle of commodities q, conditional on having z units of the new commodity, that will achieve the target level of utility u. The household's *regular cost function*, $C(u,p,w)$ is defined as follows:

$$
\begin{aligned}
(3)\ C(u,p,w) &\equiv \min_{q,z} \{p\cdot q + wz : f(q,z) \geq u\} \\
&= \min_z \{\min_q \{p\cdot q : f(q,z) \geq u\} + wz\} \\
&= \min_z \{c(u,p,z) + wz\}.
\end{aligned}
$$

We assume that (q^1, z^1) is a solution to the cost minimization problem defined by $C(u^1,p^1,w^1)$ and q^1 is a solution to the conditional cost minimization problem defined by $c(u^1,p^1,z^1)$. Thus using (3) for (u^1, p^1, w^1), we have the following equalities: [6]

$$
\begin{aligned}
(4)\ p^1\cdot q^1 + w^1z^1 &= C(u^1,p^1,w^1) \\
&= \min_z \{c(u^1,p^1,z) + w^1z\} \\
&= c(u^1,p^1,z^1) + w^1z^1.
\end{aligned}
$$

We assume that $c(u^1,p^1,z)$ is differentiable with respect to z at $z = z^1 > 0$. Thus the first order necessary condition for the minimization problem in (4) implies the following equality:

(5) $\partial c(u^1,p^1,z^1)/\partial z = -w^1$.

Note that (4) also implies the following equation:

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[4] See for example de Haan and Krsinich (2014) and Diewert, Fox and Schreyer (2018).

[5] Notation: $p\cdot q \equiv \Sigma_{n=1}^N p_nq_n$ where $p \equiv [p_1,...,p_N]$ and $q \equiv [q_1,...,q_N]$.

[6] From (1), $f(q^1,z^1) = u^1$. Thus the cost minimization problems in (4) will hold if we replace the utility constraints in definitions (2) and (3) with equalities.

(6) $c(u^1,p^1,z^1) = p^1 \cdot q^1$.

Experimental economics comes into play at this point by asking households in period 1: how much money will it take for the household to give up its use of the new commodity? Put another way: what is the income required for the household to achieve the utility level u^1 using commodities that are available in both periods (and excluding the use of the new commodity)? The answer to this question is the following conditional cost:

$$
\begin{aligned}
(7)\ c(u^1,p^1,0) &\equiv \min{}_q \{p^1 \cdot q : f(q,0) = u^1\} \\
&> c(u^1,p^1,z^1)
\end{aligned}
$$

where the inequality follows from the assumptions that f is increasing in its arguments and that $z^1 > 0$. Define the monetary compensation m^1 that is additional to $p^1 \cdot q^1$ that is required to keep the household at the utility level u^1 without using z^1 as follows:[7]

$$(8)\ m^1 \equiv c(u^1,p^1,0) - p^1 \cdot q^1 = c(u^1,p^1,0) - c(u^1,p^1,z^1)$$

where we have used (6) to derive the second equality. Note that utility and the prices of continuing commodities are held constant on the right hand side of (8).[8] Assuming that m^1 can be estimated through controlled experiments, it can be seen that $c(u^1,p^1,0) = p^1 \cdot q^1 + m^1$ can be determined.[9] We convert m^1 into a period 1 *average compensation price per unit of z foregone* by setting m^1 equal to $w^{C1}z^1$:

$$(9)\ w^{C1} \equiv m^1/z^1.$$

Using (8) and (9), we can write the cost difference, $c(u^1,p^1,0) - c(u^1,p^1,z^1)$, as follows:

$$(10)\ c(u^1,p^1,0) - c(u^1,p^1,z^1) = w^{C1}z^1.$$

At this point, we assume that $c(u^1,p^1,z)$ is also differentiable with respect to z at z = 0 (a one sided derivative exists at this point). Thus we can form the following two first-order Taylor series approximations:

$$
\begin{aligned}
(11)\ c(u^1,p^1,0) &\approx c(u^1,p^1,z^1) + [\partial c(u^1,p^1,z^1)/\partial z][0 - z^1] \\
&= c(u^1,p^1,z^1) - w^1[0 - z^1] && \text{using (5)} \\
&= c(u^1,p^1,z^1) + w^1z^1.
\end{aligned}
$$

$$
\begin{aligned}
(12)\ c(u^1,p^1,z^1) &\approx c(u^1,p^1,0) + [\partial c(u^1,p^1,0)/\partial z][z^1 - 0] \\
&= c(u^1,p^1,0) - w^{R1}[z^1 - 0] \\
&= c(u^1,p^1,0) - w^{R1}z^1,
\end{aligned}
$$

[7] This is equation (27) of BCDEF (2018), which they describe as a global willingness to accept function.
[8] Thus the right hand side of (8) does not equal either a Hicksian price or quantity variation; it is a Hicksian like *mixed variation*.
[9] In the context of free digital commodities and services, this is what BCDEF (2018) called "total income": actual income ($p^1 \cdot q^1$) plus the additional income (m^1) required to achieve the same level of utility as with a positive amount of the free commodity z.

where w^{R1} is the *Hicksian reservation price* $-\partial c(u^1,p^1,0)/\partial z$. This reservation price is not directly observable but we will be able to solve for it shortly. The approximate equality (12) can be rewritten as:

$$(13)\ c(u^1,p^1,0) \approx c(u^1,p^1,z^1) + w^{R1}z^1.$$

A more accurate approximation to the difference $c(u^1,p^1,0) - c(u^1,p^1,z^1)$ can be obtained if we take the following arithmetic average of the two first order approximations (11) and (13):

$$(14)\ c(u^1,p^1,0) - c(u^1,p^1,z^1) \approx \tfrac{1}{2}(w^1 + w^{R1})z^1.$$

The approximation given by (14) will be an exact one if $c(u^1,p^1,z)$ is a quadratic function of z between 0 and z^1; see the quadratic approximation lemma in Diewert (1976).

Note that the left hand side of (10) is equal to the left hand side of (14). Thus the right hand sides are approximately equal to each other and we obtain the following approximate equality:

$$(15)\ w^{C1}z^1 \approx \tfrac{1}{2}(w^1 + w^{R1})z^1.$$

Recall that $z^1 > 0$ and w^{C1} and w^1 are observable.[10] Thus we can use (15) (as an equality) to solve for the unknown reservation price w^{R1}. The solution is:

$$(16)\ w^{R1} \approx 2w^{C1} - w^1.$$

If households are reluctant to surrender their units of z, so that the average compensation price w^{C1} is greater than the market price w^1, then from (16) the period 1 reservation price w^{R1} will be greater than the observed period 1 price for a unit of z, w^1. Note that if the z commodity is free, then $w^1 = 0$ and an approximate to the reservation price is then twice the compensation price, $w^{R1} \approx 2w^{C1}$.[11]

3. The Case where N = 1

We have found a reservation price, w^{R1}, for period 1 indifference curve but what we want is a reservation price for period 0. In order to obtain this reservation price, we temporarily restrict ourselves to the case where N = 1, so that $q = q_1$ is now a scalar.

Consider Figure 1. The observed (optimal) period 0 consumption bundle is $(q_1^{\,0},0)$, is represented by point A, where the household consumes 0 units of z and $q_1^{\,0}$ units of the

[10] Recall that w^1 is the observed market price for z^1 and w^{C1} is the period 1 compensation price per unit of z foregone, as elicited from experimental evidence; see equation (9).

[11] It is unclear how good this approximation would be for truly novel products. BCDEF (2018) argue that a reservation price of twice the compensation price is too low, at least for innovative digital products with few substitutes.

always available commodity. The observed (optimal) period 1 consumption bundle is (q_1^1,z^1), represented by point B, where the household consumes z^1 units of the new commodity and q_1^1 units of the continuing commodity.

The period 1 indifference curve is the set of q_1 and z combinations that are on the indifference curve indexed by $u^1 \equiv f(q_1,z)$.[12] Point B is on this indifference curve as is the bundle $(q_1^{1*},0)$, where q_1^{1*} is the solution to the conditional cost minimization problem defined by $c(u^1,p_1^1,0)$. Thus $f(q_1^1,z^1) = f(q_1^{1*},0) = u^1$ where $q_1^{1*} > q_1^1 > 0$ and $z^1 > 0$.

The period 1 observed price for a unit of q_1 is $p_1^1 > 0$ and the period 1 observed price for a unit of z is $w^1 > 0$. The slope of the period 1 budget line is $-w^1/p_1^1$ and this budget line is tangent to the period 1 indifference curve at point B.

The slope of the period 1 indifference curve at the point $(q_1^{1*},0)$ is $-w^{R1}/p_1^1$ where w^{R1} is the period 1 reservation price for the new commodity. Finally, the slope of the straight line joining $(q_1^{1*},0)$ to (q_1^1,z^1) is $-w^{C1}/p_1^1$, where w^{C1} is the average compensation price for forgoing the consumption of z.

Let p_1^0 be the observed price of q_1 in period 0 and let $w^{R0} \equiv -\partial c(u^0,p_1^0,0)/\partial z$ be the period 0 Hicksian reservation price for the new commodity in period 0. The slope of the period 0 indifference curve at point A is $-w^{R0}/p_1^0$. Because $f(q_1,z)$ is homogeneous of degree 1, the first order partial derivatives of this function will be homogeneous of degree 0. This means that every indifference curve (in both periods) will have the same slope at its intersection point with the q_1 axis. Hence we have $-w^{R1}/p_1^1 = -w^{R0}/p_1^0$ and we can solve for the new commodity's reservation price in period 0:

(17) $w^{R0} = w^{R1}/[p_1^1/p_1^0]$;

i.e., the period 0 reservation price is the *inflation adjusted carry backward period 1 reservation price*; that is, the period 1 reservation price w^{R1} for the new commodity deflated by inflation of the continuing commodity q_1 between periods 0 and 1, p_1^1/p_1^0.[13]

Thus, using (17) together with the approximation in (16), we can get an estimate of the period 0 reservation price for the new commodity using observable information, so long as we can have estimates of the average compensation price, w^{C1}.

While in principle (17) can be applied to periods that are several years apart, the quality of the estimate hinges on the plausibility of the assumption of unchanged preferences. This underlines the importance of early introduction of new products into price indexes:

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[12] The period 1 indifference curve is the function $q_1 = g(z,u^1)$ where $g(z,u)$ is implicitly defined by the equation $u = f(g(z,u),z)$. The indifference curve function $g(z,u)$ will be decreasing in z, increasing in u and linearly homogeneous in z,u together since we have assumed that $f(q_1,z)$ is linearly homogeneous. Thus $\partial g(z,u)/\partial z$ will be homogeneous of degree 0 in z,u. The conditional cost function, $c(u,p_1,z)$ is equal to $p_1 g(z,u)$.

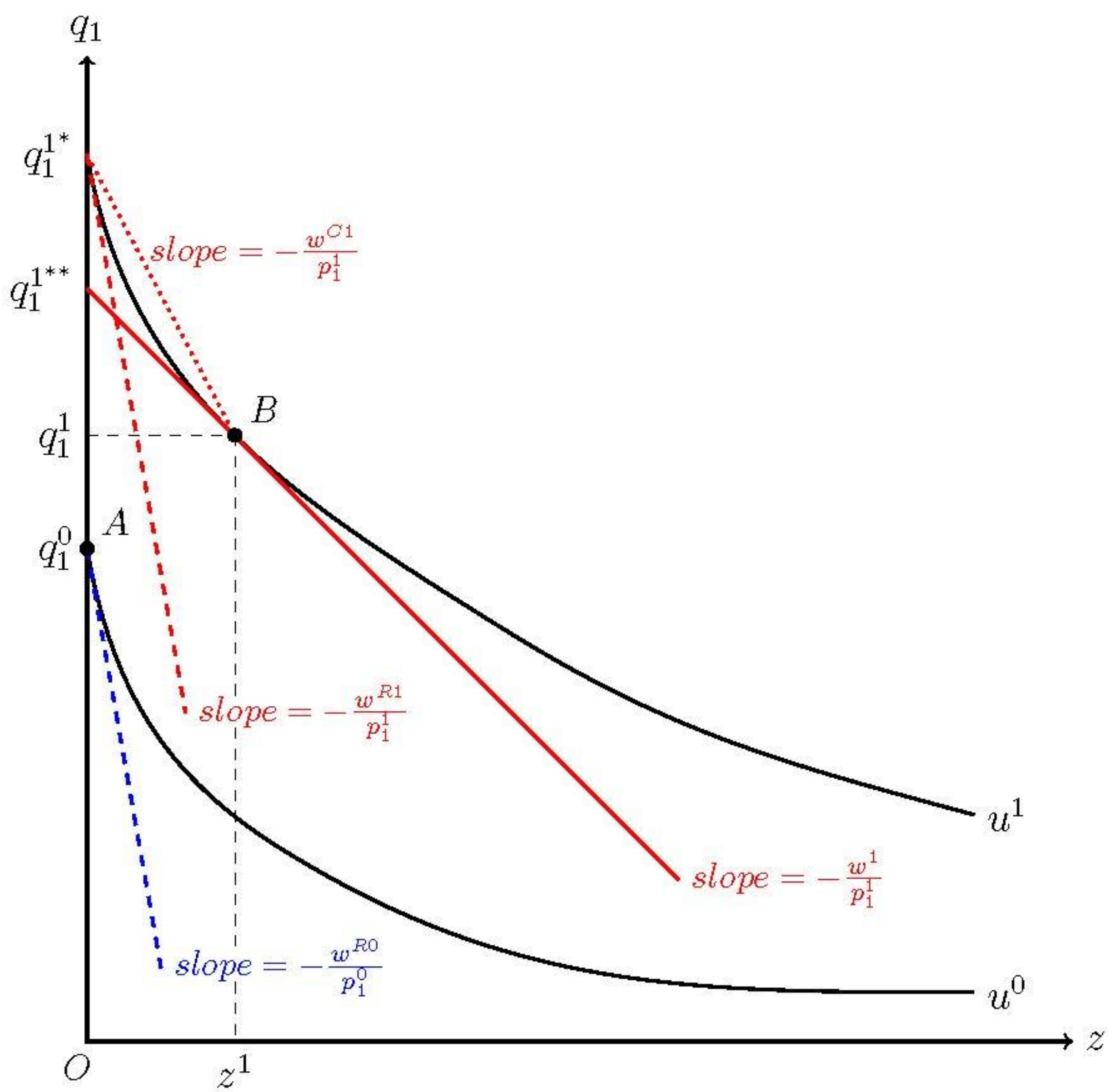
[13] See Diewert, Schreyer and Fox (2018) for more on carry backward prices.

the earlier w^1 is actually measured, the more plausible the assumption of an unchanged utility function.

From the figure, it can be seen that the average of the prices w^1 and w^{R1} is reasonably close to w^{C1}, meaning that in this case the approximation in (15), and hence in (16), is quite good.

Note that if the u^1 indifference curve is linear, so that the commodities are perfect substitutes, then the approximations given by (11) and (13) are exact. In this case, the reservation price w^{R1}, the observed price w^1 and the average compensation price w^{C1} are all equal (and the points q_1^{1*} and q_1^{1**} will coincide).

Figure 1: The Two Commodity Case



The above methodology can be adapted to the case where the new commodity is provided at a price of zero in period 1, as is the case with many free digital commodities. In this case $w^1 = 0$ but the above algebra is still valid. In terms of Figure 1, the u^1 indifference curve becomes parallel to the z axis at the point z^1. Thus the slope of the indifference curve at this point of satiation becomes 0.

The next step in our analysis is to determine how important are the estimated reservation prices to the more accurate measurement of household consumption.[14] Typically, statistical agencies cannot estimate reservation prices and so they use *maximum overlap* price indexes to deflate nominal household expenditures to form real consumption estimates; a maximum overlap index only includes products which are present in both periods. In our present two commodity situation, the maximum overlap price index is the price ratio for the continuing commodity, p_1^1/p_1^0. Thus the statistical agency maximum overlap quantity index between the two periods is the following one:

$$\begin{aligned}(18)\ Q_{MO} &\equiv \{[p_1^1q_1^1+w^1z^1]/[p_1^0q_1^0]\}/[p_1^1/p_1^0] \\ &= \{[p_1^1q_1^1+w^1z^1]/\ p_1^1\}/\{[p_1^0q_1^0]/p_1^0]\} \\ &= [q_1^1 + (w^1/p_1^1)z^1]/q_1^0.\end{aligned}$$

Note that $[p_1^1q_1^1+w^1z^1]/[p_1^0q_1^0]$ is the ratio of nominal consumption for the two periods and p_1^1/p_1^0 is the maximum overlap consumption price deflator.

To form the "true" index of real consumption, we will construct the Fisher quantity index using the reservation price for z in period 0. We use the following Laspeyres and Paasche "true" real consumption indexes, Q_L and Q_p respectively:

$$(19)\ Q_L \equiv [p_1^0q_1^1 + w^{R0}z^1]/[p_1^0q_1^0 + w^{R0}0] = [q_1^1 + (w^{R0}/p_1^0)z^1]/q_1^0\ ;$$
$$(20)\ Q_P \equiv [p_1^1q_1^1 + w^1z^1]/[p_1^1q_1^0 + w^10] \quad = [q_1^1 + (w^1/p_1^1)z^1]/q_1^0\ .$$

Note that from (18) and (20), $Q_{MO} = Q_P$. The "true" Fisher quantity index is the geometric mean of Q_L and Q_P defined by (19) and (20). We will approximate this Fisher index as the arithmetic mean of Q_L and Q_P. Thus we have:

$$\begin{aligned}(21)\ Q_F &\approx ½Q_L + ½Q_P \\ &= ½[q_1^1 + (w^{R0}/p_1^0)z^1]/q_1^0 + ½[q_1^1 + (w^1/p_1^1)z^1]/q_1^0 \\ &= ½[q_1^1 + (w^{R1}/p_1^1)z^1]/q_1^0 + ½[q_1^1 + (w^1/p_1^1)z^1]/q_1^0 \qquad \text{since } w^{R0}/p_1^0 = (w^{R1}/p_1^1) \\ &= \{q_1^1 + ½(w^{R1}/p_1^1)z^1 + ½(w^1/p_1^1)z^1\}/q_1^0.\end{aligned}$$

The amount by which the approximate Q_F will exceed the statistical agency Q_{MO} is as follows:

$$\begin{aligned}(22)\ Q_F - Q_{MO} &\approx \{½(w^{R1}/p_1^1)z^1 - ½(w^1/p_1^1)z^1\}/q_1^0 \\ &= ½(w^{R1} - w^1)z^1/p_1^1q_1^0 \\ &\approx (w^{C1} - w^1)z^1/p_1^1q_1^0 \qquad \text{using (16)}\end{aligned}$$

[14] The analysis here is closely related to that of BCDEF (2018).

$$= [(w^{C1} - w^1)z^1/(p_1{}^1/p_1{}^0)]/p_1{}^0q_1{}^0.$$

Typically, the reservation price for z in period 1, w^{R1}, will be greater than the corresponding market price for z in period 1, w^1. Deflating nominal consumption growth by the maximum overlap index will then lead to an underestimate of real consumption in period 1. The real amount of this understatement is approximately equal to $\frac{1}{2}(w^{R1} - w^1)z^1$ deflated by the period 1 price level, $p_1{}^1$.

With reference to Figure 1, note that we can write $Q_F \approx q_1{}^{1*}/q_1{}^0$ and $Q_{MO} = q_1{}^{1**}/q_1{}^0$, with the distance between points $q_1{}^{1*}$ and $q_1{}^{1**}$ representing the amount of underestimation from using Q_{MO}.[15] There will be no understatement if $w^{R1} = w^1$ or if $q_1{}^{1*} = q_1{}^{1**}$ in Figure 1.

If $w^1 = 0$ so that the new commodity is a free good in period 1, then

$$
\begin{aligned}
(23)\; Q_F - Q_{MO} &\approx \tfrac{1}{2}w^{R1}\, z^1/p_1{}^1q_1{}^0 && \\
&\approx w^{C1}\, z^1/p_1{}^1q_1{}^0 && \text{using (16)} \\
&= (m^1/p_1{}^1)/q_1{}^0 && \text{using (9)} \\
&= [m^1/(p_1{}^1/p_1{}^0)]/p_1{}^0q_1{}^0. &&
\end{aligned}
$$

That is, the difference between the quantity indexes is approximately equal to the income, $m^1 = w^{C1}\, z^1$, needed to compensate for giving up the new commodity z^1, deflated back to period 0 by the price inflation of continuing commodities, $(p_1{}^1/p_1{}^0)$, divided by the period 0 income $p_1{}^0q_1{}^0$. The right hand side of (23) is then exactly equal to the adjustment to GDP growth from the Total Income approach of BCDEF (2018).

4. The Case of N Continuing Commodities

To generalize the above analysis to the case of N continuing commodities, assume that the utility function has the following *separable functional form*:

$$(24)\; f(q,z) = h(F(q),z)$$

where both F(q) and h(Q,z) are linearly homogeneous, increasing and concave in their arguments. Since F(q) is linearly homogeneous, it has a dual unit cost function, $c^*(p)$ where $c^*(p) \equiv \min_q \{p\cdot q : F(q) = 1\}$. We assume that q^t solves the cost minimization problem $\min_q \{p^t\cdot q : F(q) = F(q^t)\}$ for t = 0,1. It can be shown that these assumptions imply the following equalities:[16]

$$(25)\; p^t\cdot q^t = c^*(p^t)F(q^t) \equiv P^tQ^t \qquad t = 0,1.$$

where the period t *aggregate price and aggregate quantity for the continuing commodities* are defined by $P^t \equiv c^*(p^t)$ and $Q^t \equiv F(q^t)$ for t = 0,1. Now pick a functional

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[15] See Diewert and Fox (2001; 180-181) for a similar diagram, essentially based on Romer (1994; 12-14).
[16] See Konüs and Byushgens (1926), Shephard (1953), Samuelson and Swamy (1974) and Diewert (1976).

form for F(q) (or for the dual $c^*(p)$) that has an exact index number formula associated with it and replace the p_1^t and q_1^t in the previous section by the appropriate aggregate P^t and Q^t, for t = 0,1.[17]

Then (17) and (22) become the following equations:

$$(26)\ w^{R0} = w^{R1}/[c^*(p^1)/c^*(p^0)] = w^{R1}/ P_{MO}$$

$$(27)\ Q_F - Q_{MO} \approx [(w^{C1} - w^1)z^1/(c^*(p^1)/c^*(p^0))]/p^0\cdot q^0 = [(w^{C1} - w^1)z^1/P_{MO}]/p^0\cdot q^0$$

where $c^*(p)$ is the unit cost function that is dual to F(q) and $c^*(p^1)/c^*(p^0) = P_{MO}$, an exact price index defined over the continuing commodities.

For free commodities, the right hand side of (27) is exactly equal to the percentage point adjustment to GDP from the Total Income approach of BCDEF (2018). BCDEF proposed the following index of real growth for free commodities and services: $Q_T = [(p^1\cdot q^1+w^{c1}z^1)/P_{MO}]/p^0\cdot q^0$, where P_{MO} is a maximum overlap price index (i.e. a price index that a national statistical office would use) and Q_T is the "total income" quantity index, i.e. the numerator in the square brackets is the income required to achieve the same utility through consuming only continuing commodities as would be achieved consuming both continuing and new commodities. Comparing Q_T with Q_{MO}, we get: $Q_T - Q_{MO} = (w^{c1}z^1)/(P_{MO}p^0\cdot q^0) = [w^{c1}z^1/P_{MO}]/p^0\cdot q^0$, which is exactly equal to the right hand side of equation (27) above for $w^1 = 0$.

Thus, (27) generalizes the BCDEF Total Income approach to the case where the z commodity has a non-zero price in period 1. It says that if the approximation in equation (16), $w^{R1} \approx 2w^{C1} - w^1$, is a good one then the difference between the Total Income quantity index and the maximum overlap quantity index can be interpreted as the amount by which a maximum overlap index understates an approximate "true" Fisher index.

5. Conclusion

We have shown how experimental estimates of willingness to forgo consumption can be used to get otherwise unobservable reservation prices for new commodities; that is, prices for the commodities in the period before they exist. Having such prices allows standard index number theory to be applied. We provide an approximation to the percentage point discrepancy between an approximate "true" Fisher quantity index (calculated using reservation prices for new commodities) and a maximum overlap index (as typically used by national statistical offices).

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[17] Diewert (1976) gives many examples of suitable exact index number formula that can approximate a linearly homogeneous F(q) or $c^*(p)$ to the second order. The Fisher (1922) index is included in this class of superlative index number formulae.

We believe that these results advance understanding of mismeasurement from not appropriately accounting for new commodities, and provide a simple method for assessing the effects on real consumption if valuations of new commodities become available that reflect the willingness to forego consumption. In addition, the geometric explanation of the relationship between reservation prices and experimental prices should prove to be helpful in other contexts.

However, our results rested on various approximations and assumptions which may prove to be restrictive in practice. In particular, our assumption of homothetic preferences and the separability assumption made in section 4 may prove to be problematic in some situations.

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