The Digital Economy, New Products and Consumer Welfare

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Erwin Diewert, Kevin J. Fox and Paul Schreyer,[1]
Discussion Paper 17-09,
Vancouver School of Economics,
University of British Columbia,
Vancouver, B.C., Canada, V6T 1L4.

Abstract

Benefits of the Digital Economy are evident in everyday life, but there are significant concerns that these benefits may not be appropriately reflected in official statistics. Statistical agencies are typically unable to measure the benefits that result from the introduction of such new goods and services. The measurement of the net benefits of new and disappearing products depends on what type of index the statistical agency is using to deflate final demand aggregates. We examine this measurement problem when the agency uses any standard price index formula for its deflation of the value aggregate, such as GDP. An Appendix applies the methodology to the problem of measuring the effects of product substitutions for disappearing items.

Keywords

Maximum overlap indexes, Hicksian reservation prices, Laspeyres, Paasche, Fisher and Törnqvist price indexes, new goods, free goods, welfare measurement, willingness to pay measures, willingness to sell measures, quality adjustment, replacement sampling.

JEL Classification Numbers

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[1] W. Erwin Diewert: Vancouver School of Economics, University of British Columbia, Vancouver B.C., Canada, V6T 1Z1 and the School of Economics, UNSW Sydney, NSW 2052, Australia (erwin.diewert@ubc.ca); Kevin J. Fox: CAER & School of Economics, UNSW Sydney, NSW 2052, Australia (K.Fox@unsw.edu.au) and Paul Schreyer, OECD Statistics Directorate, Paris (paul.schreyer@oecd.org). We thank Jan de Haan and Michael Webster for helpful comments. The first author gratefully acknowledges the financial support of the SSHRC of Canada, and the first two authors gratefully acknowledge the financial support of the Australian Research Council (DP150100830). The views expressed in this paper reflect those of the authors and not necessarily those of the OECD or its Member countries.

1. Introduction

The debate regarding the benefits of the Digital Economy regularly features the suggestion that mismeasurement by national statistical offices is playing a major role in obscuring productivity, economic growth and welfare gains.[2] If measurement is lacking, through methodological challenges, statistical agency budgets or data availability, then we are severely hampered in our ability to understand the impact of new technologies, goods and services. In this paper, we build on a earlier attempt by Diewert and Fox (2017) and develop new frameworks for measuring welfare change and real consumption growth in the presence of new and disappearing goods (and services); such goods are frequently synonymous with the Digital Economy.

New, often very specialized digital goods are now part of daily consumption for many, accompanied by the disappearance of previously consumed commodities. We provide a framework for quantifying the welfare benefits and costs of new and disappearing products.[3] The basic idea is the following. Statistical agencies typically use a "matched model" approach when they construct price indexes, and these are used to deflate a final demand value aggregate; i.e., when constructing a particular price index that compares the prices of a group of products over two periods, the scope of the index is usually restricted to the set of commodities or products that are present in both periods. The resulting index is called a *maximum overlap index*.[4] However, if one is using the economic approach to index number theory that was originally developed by Konüs (1924), then *reservation prices* for the missing products should be matched up with the zero quantities for the missing products in each period; the reservation price for a missing product is the price which would induce a utility maximizing potential purchaser of product to demand zero units of it. Normal index number theory can then be applied to the resulting augmented data set for the two periods under consideration.[5]

This reservation price approach for the treatment of new goods is due to Hicks (1940; 114). Hofsten (1952; 95-97) extended his approach to cover the case of disappearing goods as well. If reservation prices are estimated, elicited from surveys,[6] or guessed, then the "true" price index can be calculated and compared to its maximum overlap

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[2] See, for example, Feldstein (2017), Reinsdorf and Schreyer (2017), Syverson (2017), Groshen, Moyer, Aizcorbe, Bradley and Friedman (2017), Ahmad, Ribarsky and Reinsdorf (2017), Hulten and Nakamura (2017), Ahmad and Schreyer (2016), Byrne, Fernald and Reinsdorf (2016).

[3] Diewert and Fox (2017) did not consider the case of disappearing goods, but considered the case of free goods. While free goods often have an implicit price, this price is usually unobserved. In this case, a price of zero is applied by national statistical offices, resulting in the positive quantities of these goods having no measured value. Hence their benefits to consumers go unmeasured, and they do not appear in nominal and real output. See Brynjolfsson and Oh (2012) on measuring the value of free digital services, and Nakamura, Samuels and Soloveichik (2016) for examples of how to think about the valuation of free media.

[4] This type of index dates back to Marshall (1887). Keynes (1930; 94) called it the highest common factor method while Triplett (2004; 18) called it the overlapping link method. See Diewert (1993; 52-56) for additional material on the early history of the new goods problem.

[5] See Diewert (1976) for practical applications of the economic approach to index number theory.

[6] See Brynjolfsson, Eggers, and Gannamaneni (2017) on the use of online choice experiments to elicit valuations of goods and services.

counterpart.[7] Thus an estimate of the bias in the deflator can be formed. This bias in the deflator translates into a corresponding bias in the real output aggregate. We will evaluate this bias in the context of a statistical agency that uses maximum overlap Törnqvist indexes in section 3. The context we consider is one in which transaction level data are available so that indexes can be calculated from the elementary level. In a similar manner, we will evaluate the bias in the Laspeyres, Paasche and Fisher (1922) maximum overlap indexes in section 4. Section 2 develops some general relationships in the expenditure shares of a true index relative to its maximum overlap counterpart. These relationships will be used in sections 3 and 4.

Finally, an Appendix applies the algebra developed in sections 3 and 4 to to the problem of measuring the effects of product substitutions for disappearing items.

2. The Relationships between True Shares and Maximum Overlap Shares

Consider two periods, 0 and 1. There are three classes of commodities. Class 1 products are present in both periods with positive prices and quantities for all N products in this group. Denote the period t price and quantity vectors for this group of products as $p_1^t \equiv [p_{11}^t,...,p_{1N}^t] >> 0_N$ and $q_1^t \equiv [q_{11}^t,...,q_{1N}^t] > 0_N$ for t = 0,1.

Class 2 products are the *new* goods and services that are not available in period 0 but are available in period 1. Denote the period 0 price and quantity vectors for this group of K products as $p_2^{0*} \equiv [p_{21}^{0*},...,p_{2K}^{0*}] >> 0_N$ and $q_2^0 \equiv [q_{11}^0,...,q_{1K}^0] = 0_N$. The prices in the vector p_2^{0*} are the positive reservation prices that make the demand for these products in period 0 equal to zero. These reservation prices have to be estimated somehow. The period 1 price and quantity vectors for these K products are $p_2^1 \equiv [p_{21}^1,...,p_{2K}^1] >> 0_N$ and $q_2^1 \equiv [q_{21}^1,...,q_{2K}^1] > 0_N$ and these vectors are observable.

Class 3 products are the *disappearing* goods and services that were available in period 0 but are not available in period 1.[8] Denote the period 0 price and quantity vectors for this group of M products as $p_3^0 \equiv [p_{31}^0,...,p_{3M}^0] >> 0_N$ and $q_3^0 \equiv [q_{31}^0,...,q_{3M}^0] > 0_N$. The period 1 price and quantity vectors for these M products are $p_3^{1*} \equiv [p_{31}^{1*},...,p_{3M}^{1*}] >> 0_N$ and $q_3^1 \equiv [q_{31}^1,...,q_{3M}^1] = 0_N$. The prices in the vector p_3^{1*} are the positive reservation prices that make the demand for these products in period 1 equal to zero. Again, these reservation prices have to be estimated somehow.

[7] Feenstra (1994) uses our suggested general methodological approach in the context of purchasers who have CES preferences and he uses the Sato (1976) Vartia (1976) maximum overlap index number formula which is exact for CES preferences. An advantage of his methodology is that the Hicksian reservation prices in the CES context are equal to $+\infty$ and thus there is no need to estimate these reservation prices using his approach. However, it is likely that his approach overestimates the benefits of new products and moreover, the CES functional form is not fully flexible as are the preferences that are exact for the Törnqvist and Fisher indexes; see Diewert (1976). Hausman (1996) (1999) (2003) and Hausman and Leonard (2002) develop an expenditure function approach that uses a flexible functional form to estimate reservation prices.

[8] Diewert and Fox (2017) did not consider the case of disappearing goods. Another difference between Diewert and Fox (2017) and the current paper is that here we use a traditional index number approach, whereas the previous paper used an indicator approach, i.e. a difference framework.

Define the *true expenditure shares* for product n in Group 1 for periods 0 and 1, s_{1n}^0 and s_{1n}^1, in the usual way:

$$(1)\ s_{1n}^0 \equiv p_{1n}^0q_{1n}^0/[p_1^0\cdot q_1^0 + p_2^{0*}\cdot q_2^0 + p_3^0\cdot q_3^0]\ ; \qquad n = 1,\ldots,N;$$
$$= p_{1n}^0q_{1n}^0/[p_1^0\cdot q_1^0 + p_3^0\cdot q_3^0] \qquad \text{since } q_2^0 = 0_N;$$
$$(2)\ s_{1n}^1 \equiv p_{1n}^1q_{1n}^1/[p_1^1\cdot q_1^0 + p_2^1\cdot q_2^1 + p_3^{1*}\cdot q_3^1]\ ; \qquad n = 1,\ldots,N;$$
$$= p_{1n}^1q_{1n}^1/[p_1^1\cdot q_1^1 + p_2^1\cdot q_2^1]^9 \qquad \text{since } q_3^1 = 0_N.$$

Note that these shares can be calculated using observable data; i.e., these shares do not depend on the imputed prices p_2^{0*} and p_3^{1*}.

Define the *true expenditure shares* for product k in Group 2 for periods 0 and 1, s_{2k}^0 and s_{2k}^1, as follows:

$$(3)\ s_{2k}^0 \equiv p_{2k}^0q_{2k}^0/[p_1^0\cdot q_1^0 + p_2^{0*}\cdot q_2^0 + p_3^0\cdot q_3^0]\ ; \qquad k = 1,\ldots,K;$$
$$= p_{2k}^0q_{2k}^0/[p_1^0\cdot q_1^0 + p_3^0\cdot q_3^0] \qquad \text{since } q_2^0 = 0_N;$$
$$= 0\ ; \qquad \text{since } q_{2k}^0 = 0;$$
$$(4)\ s_{2k}^1 \equiv p_{2k}^1q_{2k}^1/[p_1^1\cdot q_1^0 + p_2^1\cdot q_2^1 + p_3^{1*}\cdot q_3^1]\ ; \qquad k = 1,\ldots,K;$$
$$= p_{2k}^1q_{2k}^1/[p_1^1\cdot q_1^1 + p_2^1\cdot q_2^1] \qquad \text{since } q_3^1 = 0_N.$$

Note that these shares can also be calculated using observable data.

Define the *true expenditure shares* for product m in Group 3 for periods 0 and 1, s_{3m}^0 and s_{3m}^t, as follows:

$$(5)\ s_{3m}^0 \equiv p_{3m}^0q_{3m}^0/[p_1^0\cdot q_1^0 + p_2^{0*}\cdot q_2^0 + p_3^0\cdot q_3^0]\ ; \qquad m = 1,\ldots,M;$$
$$= p_{3m}^0q_{3m}^0/[p_1^0\cdot q_1^0 + p_3^0\cdot q_3^0] \qquad \text{since } q_2^0 = 0_N;$$
$$(6)\ s_{3m}^1 \equiv p_{3m}^1q_{3m}^1/[p_1^1\cdot q_1^0 + p_2^1\cdot q_2^1 + p_3^{1*}\cdot q_3^1]\ ; \qquad m = 1,\ldots,M;$$
$$= p_{3m}^1q_{3m}^1/[p_1^1\cdot q_1^1 + p_2^1\cdot q_2^1] \qquad \text{since } q_3^1 = 0_N;$$
$$= 0 \qquad \text{since } q_{3m}^1 = 0.$$

Note that these shares can also be calculated using observable data.

Now define the expenditure shares for product Group 1 using just the products that are in Group 1. These are the shares that are relevant for the maximum overlap indexes which will be defined shortly. The *maximum overlap share* for product n in period t, s_{1n}^{tO}, is defined as follows:

$$(7)\ s_{1nO}^t \equiv p_{1n}^tq_{1n}^t/p_1^t\cdot q_1^t\ ; \qquad t = 0,1;\ n = 1,\ldots,N.$$

These maximum overlap shares are also observable. It can be seen that the following relationships hold between the true Group 1 shares and the maximum overlap Group 1 shares:[10]

[9] The inner product of the vectors p_1^t and q_1^t is denoted as $p_1^t\cdot q_1^t \equiv \Sigma_{n=1}^N\, p_{1n}^tq_{1n}^t$, etc.

$$(8)\ s_{1n}^{0} = s_{1nO}^{0}p_{1}^{0}\cdot q_{1}^{0}/[p_{1}^{0}\cdot q_{1}^{0} + p_{3}^{0}\cdot q_{3}^{0}]\ ; \qquad n = 1,...,N;$$
$$= s_{1nO}^{0}[1 - \Sigma_{m=1}^{M} s_{3m}^{0}]\ ;$$
$$(9)\ s_{1n}^{1} = s_{1nO}^{1}p_{1}^{1}\cdot q_{1}^{1}/[p_{1}^{1}\cdot q_{1}^{1} + p_{2}^{0}\cdot q_{2}^{0}]\ ; \qquad n = 1,...,N;$$
$$= s_{1nO}^{1}[1 - \Sigma_{k=1}^{K} s_{2k}^{1}]\ .$$

3. The Törnqvist Price Index Decomposition

Let P_{TO} denote the *Törnqvist maximum overlap index*. The logarithm of this index is defined as follows:

$$(10)\ \ln P_{TO} \equiv \Sigma_{n=1}^{N}\ (1/2)(s_{1nO}^{0} + s_{1nO}^{1})\ln(p_{1n}^{1}/p_{1n}^{0}).$$

The logarithm of the *true Törnqvist index*, P_T, is defined as follows:

$$\begin{aligned}(11)\ \ln P_{T} &\equiv \Sigma_{n=1}^{N}\ (1/2)(s_{1n}^{0} + s_{1n}^{1})\ln(p_{1n}^{1}/p_{1n}^{0}) + \Sigma_{k=1}^{k}\ (1/2)(s_{2k}^{0} + s_{2k}^{1})\ln(p_{2k}^{1}/p_{2k}^{0*}) \\ &\quad + \Sigma_{m=1}^{M}\ (1/2)(s_{3m}^{0} + s_{3m}^{1})\ln(p_{3m}^{1*}/p_{3m}^{0}) \\ &= \Sigma_{n=1}^{N}\ (1/2)(s_{1n}^{0} + s_{1n}^{1})\ln(p_{1n}^{1}/p_{1n}^{0}) + \Sigma_{k=1}^{k}\ (1/2)(0 + s_{2k}^{1})\ln(p_{2k}^{1}/p_{2k}^{0*}) \\ &\quad + \Sigma_{m=1}^{M}\ (1/2)(s_{3m}^{0} + 0)\ln(p_{3m}^{1*}/p_{3m}^{0}) \qquad \text{using (3) and (6)} \\ &= \Sigma_{n=1}^{N}\ (1/2)\{s_{1nO}^{0}[1-\Sigma_{m=1}^{M}\ s_{3m}^{0}]+s_{1nO}^{1}[1 - \Sigma_{k=1}^{K}\ s_{2k}^{1}]\}\ln(p_{1n}^{1}/p_{1n}^{0}) \\ &\quad + \Sigma_{k=1}^{k}\ (1/2)(s_{2k}^{1})\ln(p_{2k}^{1}/p_{2k}^{0*})\ + \Sigma_{m=1}^{M}\ (1/2)(s_{3m}^{0})\ln(p_{3m}^{1*}/p_{3m}^{0}) \\ &\qquad \text{using (8) and (9)} \\ &= \ln P_{TO} + (1/2)\Sigma_{k=1}^{K}\ s_{2k}^{1}[\ln(p_{2k}^{1}/p_{2k}^{0*}) - \Sigma_{n=1}^{N}\ s_{1nO}^{1}\ \ln(p_{1n}^{1}/p_{1n}^{0})] \\ &\quad + (1/2)\Sigma_{m=1}^{M}\ s_{3m}^{0}[\ln(p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{n=1}^{N}\ s_{1nO}^{0}\ \ln(p_{1n}^{1}/p_{1n}^{0})] \qquad \text{using (10)} \\ &= \ln P_{TO} + \ln\kappa + \ln\mu\end{aligned}$$

where the logarithms of the terms κ and μ are defined as:

$$\begin{aligned}(12)\ \ln\kappa &\equiv (1/2)\Sigma_{k=1}^{K}\ s_{2k}^{1}[\ln(p_{2k}^{1}/p_{2k}^{0*}) - \Sigma_{n=1}^{N}\ s_{1nO}^{1}\ \ln(p_{1n}^{1}/p_{1n}^{0})] \\ &= (1/2)\Sigma_{k=1}^{K}\ s_{2k}^{1}[\ln(p_{2k}^{1}/p_{2k}^{0*}) - \ln P_{JO}^{1}];\end{aligned}$$
$$\begin{aligned}(13)\ \ln\mu &\equiv (1/2)\Sigma_{m=1}^{M}\ s_{3m}^{0}[\ln(p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{n=1}^{N}\ s_{1nO}^{0}\ \ln(p_{1n}^{1}/p_{1n}^{0})] \\ &= (1/2)\Sigma_{m=1}^{M}\ s_{3m}^{0}[\ln(p_{3m}^{1*}/p_{3m}^{0}) - \ln P_{JO}^{0}]\end{aligned}$$

where the (weighted) *Jevons index* using the maximum overlap share weights of period 1 is P_{JO}^{1} and the (weighted) *Jevons index* using the maximum overlap share weights of period 0 is P_{JO}^{0}; i.e., the logarithm of these two indexes are defined as follows:[11]

$$(14)\ \ln P_{JO}^{1} \equiv \Sigma_{n=1}^{N}\ s_{1nO}^{1}\ \ln(p_{1n}^{1}/p_{1n}^{0});$$
$$(15)\ \ln P_{JO}^{0} \equiv \Sigma_{n=1}^{N}\ s_{1nO}^{0}\ \ln(p_{1n}^{1}/p_{1n}^{0}).$$

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[10] These relationships are due to de Haan and Krsinich (2012; 31-32).

[11] These could also be described as Cobb Douglas indexes, and (14) has been called a geometric Paasche index and (15) has been called a geometric Laspeyres index.

Exponentiating both sides of (11) leads to the following relationship between the "true" cost of living index P_T and the price index P_{TO} that is defined over products that are available in both periods:[12]

(16) $P_T = P_{TO} \times \kappa \times \mu$.

The above κ and μ terms are counterparts to Feenstra's (1994; 159) λ_{t-1} and λ_t terms that he derived for bias due to changes in the availability of commodities in the context of CES preferences.

The term κ defined by (12) can be regarded as a measure of the *reduction* in the true cost of living due to the introduction of new products. The period 0 imputed price for new product k, $p_{2k}{}^{0*}$, is likely to be higher than the actual price for new product k in period 1 adjusted for general inflation, $p_{2k}{}^1/P_{JO}{}^1$, and thus κ is likely to be less than 1. The bigger is the share of new products in period 1, $\Sigma_{k=1}{}^K s_{2k}{}^1$, the more κ will be less than 1. Note that the logarithmic contribution of each new product to the reduction in the true cost of living can be measured using the additive decomposition that definition (12) provides.

The inflation adjustment term μ defined by (13) can be regarded as a measure of the *increase* in the true cost of living due to the disappearance of existing products. The period 1 imputed price for disappearing product m, $p_{3m}{}^{1*}$, is likely to be higher than the actual price for product m in period 0 adjusted for general inflation, $p_{3m}{}^0P_{JO}{}^0$, and thus μ is likely to be greater than 1. The bigger is the share of disappearing products in period 0, $\Sigma_{m=1}{}^M s_{3m}{}^0$, the more μ will be greater than 1.

The decomposition defined by (11) is also useful in the context of defining *imputed carry backward or carry forward prices* for products that may be new or unavailable. Recall that the imputed reservation prices in period 0 are the prices $p_{2k}{}^{0*}$ and the imputed reservation prices in period 1 are the prices $p_{3m}{}^{1*}$. Rough estimates or more precise econometric estimates have to be made for these reservation or virtual prices.[13] However, it is possible to use available information on prices and quantities for periods 0 and 1 in order to define the following *carry backward prices* $p_{2kb}{}^0$ for the missing products in period 0 and the following *carry forward prices* $p_{3mf}{}^1$ for the missing products in period 1:

(17) $p_{2kb}{}^0 \equiv p_{2k}{}^1/P_{JO}{}^1$; $\qquad$ k = 1,...,K;
(18) $p_{3mf}{}^1 \equiv p_{3m}{}^0P_{JO}{}^0$; $\qquad$ m = 1,...,M.

Thus the inflation adjusted carry forward price defined by (18) for the missing product m in period 1 takes the observed price for product m in period 0, $p_{3m}{}^0$ and adjusts it for

[12] This formula was first derived by de Haan and Krsinich (2012; 31-32) (2014; 344). Their imputed prices for the missing products were obtained by using hedonic regressions whereas our imputed prices are interpreted as Hicksian reservation prices but the algebra is the same in both contexts. For additional discussion on this formula and its variants, see de Haan (2017).
[13] Rothbarth (1941) introduced this term for the Hicksian (1940) reservation prices. Hicks did not give a name to his pricing concept.

general inflation for the group of products that are present in both periods 0 and 1 using the (weighted) maximum overlap Jevons index P_{JO}^0. Similarly, the inflation adjusted carry backward price defined by (17) for the missing product k in period 0 takes the observed price for product k in period 1, p_{2k}^1 and deflates it by the (weighted) Jevons maximum overlap price index, P_{JO}^1. The above inflation adjusted imputed prices are more reasonable than the often used *constant carry forward prices*, p_{3m}^0, or *constant carry backward prices*, p_{2k}^1. From (12) and (13) and (11), it can be seen that if the reservation prices are equal to their inflation adjusted carry forward prices (so that $p_{3m}^{1*} = p_{3mf}^1$ for m = 1,...,M) and inflation adjusted carry backward prices (so that $p_{2k}^{0*} = p_{2kb}^0$ for k = 1,...,K), then the true Törnqvist index P_T will equal its maximum overlap counterpart, P_{TO}.

However, in general, economic theory suggests that the reservation prices will be greater than their inflation adjusted carry forward or backward prices. Thus we define the following *margin terms*, κ_k and μ_m, which express how much higher each reservation price is from its inflation adjusted carry forward or backward price counterpart:

$$(19)\ 1 + \kappa_k \equiv p_{2k}^{0*}/p_{2kb}^0\ ; \qquad k = 1,\ldots,K;$$
$$(20)\ 1 + \mu_m \equiv p_{3m}^{1*}/p_{3mf}^0\ ; \qquad m = 1,\ldots,M.$$

Now substitute definitions (17)-(20) into (11) and we obtain the following *exact relationship* between the true Törnqvist index P_T and its maximum overlap counterpart P_{TO}:

$$(21)\ \ln(P_T/P_{TO}) = -\Sigma_{k=1}^{K} (1/2)s_{2k}^1 \ln(1 + \kappa_k) + \Sigma_{m=1}^{M} (1/2)s_{3m}^0 \ln(1 + \mu_m).$$

Exponentiate both sides of (21) and subtract 1 from both sides of the resulting expression. Define the right hand side of the resulting expression as the function $g(\kappa_1,\ldots,\kappa_K, \mu_1,\ldots,\mu_M)$ and approximate g by taking the first order Taylor series approximation to g evaluated at $0 = \kappa_1 = \ldots = \kappa_K = \mu_1 = \ldots = \mu_M$. The resulting approximation to $(P_T/P_{TO}) - 1$ is the following one:[14]

$$(22)\ (P_T/P_{TO}) - 1 \approx \Sigma_{m=1}^{M} (1/2)s_{3m}^0\, \mu_m - \Sigma_{k=1}^{K} (1/2)s_{2k}^1 \kappa_k.$$

From equations (1) and (2), the period 0 and 1 value aggregates for the goods and services in the group of N + K + M commodities under consideration, v^0 and v^1, are defined as follows:

$$(23)\ v^0 \equiv p_1^0 \cdot q_1^0 + p_3^0 \cdot q_3^0;\ v^1 \equiv p_1^1 \cdot q_1^1 + p_2^1 \cdot q_2^1.$$

The "true" *implicit Törnqvist quantity index* Q_T is defined as the value ratio, v^1/v^0, deflated by the "true" Törnqvist price index, P_T ; i.e., we have:

[14] This formula is similar in spirit to the highly simplified approximate formulae obtained by Diewert (1987; 779) (1998; 51-54).

(24) $Q_T \equiv [v^1/v^0]/P_T$.

Statistical agencies can use maximum overlap Törnqvist price indexes to deflate final demand aggregates in order to construct aggregate quantity or volume indexes.[15] Thus in our context, the *maximum overlap Törnqvist quantity index*, Q_{TO}, is defined as follows:

(25) $Q_{TO} \equiv [v^1/v^0]/P_{TO}$.

The bias in Q_{TO} relative to its true counterpart Q_T can be measured by the ratio Q_T/Q_{TO}:

(26) $Q_T/Q_{TO} = P_{TO}/P_T$

where we have used definitions (24) and (25) to derive (26). An exact expression for the logarithm of P_{TO}/P_T can be obtained from (21):

(27) $\ln(P_{TO}/P_T) = \Sigma_{k=1}^{K} (1/2)s_{2k}^{1} \ln(1 + \kappa_k) - \Sigma_{m=1}^{M} (1/2)s_{3m}^{0} \ln(1 + \mu_m)$.

Exponentiate both sides of (26) and subtract 1 from both sides of the resulting expression. Define the right hand side of the resulting expression as the function $h(\kappa_1,...,\kappa_K, \mu_1,...,\mu_M)$ and approximate h by taking the first order Taylor series approximation to h evaluated at $0 = \kappa_1 = ... = \kappa_K = \mu_1 =... = \mu_M$. The resulting approximation to $(Q_T/Q_{TO}) - 1$ is the following one:

(28) $(Q_T/Q_{TO}) - 1 \approx \Sigma_{k=1}^{K} (1/2)s_{2k}^{1}\kappa_k - \Sigma_{m=1}^{M} (1/2)s_{3m}^{0} \mu_m$.

Thus if there are no disappearing goods, the right hand side of (28) becomes $\Sigma_{k=1}^{K} (1/2)s_{2k}^{1}\kappa_k$ and this number is a measure of the downward bias in the maximum overlap Törnqvist quantity index for the value aggregate in percentage points. That is, (28) gives the downward bias in welfare from ignoring new goods and services.

In the following section, we develop analogous bias formulae for price and quantity aggregates that are constructed using maximum overlap Laspeyres, Paasche or Fisher indexes.

4. The Laspeyres, Paasche and Fisher Decompositions

Using the notation that was defined in section 1 above, define the *true Laspeyres price index*, P_L, as follows:[16]

(29) $P_L \equiv [p_1^{1}\cdot q_1^{0} + p_2^{1}\cdot q_2^{0} + p_3^{1*}\cdot q_3^{0}]/[p_1^{0}\cdot q_1^{0} + p_2^{0*}\cdot q_2^{0} + p_3^{0}\cdot q_3^{0}]$

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[15] The US Bureau of Labor Statistics uses the Törnqvist price index as its target index for its chained CPI. Typically, there are no adjustments for new and disappearing products so these Törnqvist price indexes are essentially maximum overlap price indexes.

[16] The Lowe index, which uses a fixed base that may not be either of the periods under consideration, is used in constructing the CPI in many countries. We do not explicitly consider this index here, but similar results as for the Laspeyres index can of course be derived.

$$= [p_1^1 \cdot q_1^0 + p_3^{1*} \cdot q_3^0]/[p_1^0 \cdot q_1^0 + p_3^0 \cdot q_3^0] \qquad \text{using } q_2^0 = 0_K$$
$$= \Sigma_{n=1}^N s_{1n}^0 (p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^{1*}/p_{3m}^0).$$

Define the *maximum overlap Laspeyres price index* P_{LO} that is defined only over products that are present in both periods as follows:

$$(30)\ P_{LO} \equiv p_1^1 \cdot q_1^0/p_1^0 \cdot q_1^0$$
$$= \Sigma_{n=1}^N s_{1nO}^0 (p_{1n}^1/p_{1n}^0)$$

where the maximum overlap shares s_{1nO}^0 are defined above by definitions (7).

Recall equations (8) which exhibited the relationship between the true share weights for continuing products, s_{1n}^0, and the share weights for the commodities present in each period, s_{1nO}^0. Using definitions (29) and (30) and equations (8), we can derive the following relationship between P_L and P_{LO}:

$$(31)\ P_L \equiv \Sigma_{n=1}^N s_{1n}^0 (p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^{1*}/p_{3m}^0)$$
$$= \Sigma_{n=1}^N s_{1nO}^0[1 - \Sigma_{m=1}^M s_{3m}^0](p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^{1*}/p_{3m}^0)$$
$$= P_{LO} - \Sigma_{n=1}^N s_{1nO}^0(\Sigma_{m=1}^M s_{3m}^0)(p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^{1*}/p_{3m}^0)$$
$$= P_{LO} + \Sigma_{m=1}^M s_{3m}^0 [(p_{3m}^{1*}/p_{3m}^0) - \Sigma_{n=1}^N s_{1nO}^0(p_{1n}^1/p_{1n}^0)]$$
$$= P_{LO} + \Sigma_{m=1}^M s_{3m}^0 [(p_{3m}^{1*}/p_{3m}^0) - P_{LO}]$$
$$= P_{LO}(1 + \Sigma_{m=1}^M s_{3m}^0 [(p_{3m}^{1*}/p_{3m}^0 P_{LO}) - 1])$$
$$= P_{LO}(1 + \alpha)$$

where $\alpha \equiv \Sigma_{m=1}^M s_{3m}^0 [(p_{3m}^{1*}/p_{3m}^0 P_{LO}) - 1]$ is a bias term which when multiplied by P_{LO} and added to the maximum overlap Laspeyres price index, P_{LO}, enables us to obtain the true Laspeyres price index, P_L. There is a presumption that this error term will be *positive* in which case P_{LO} has a downward bias and must be adjusted upward by this term in order to account for the effective increase in the price level which is due to the disappearance of the products in Group 3 during period 1.

The decomposition defined by (31) is also useful in the context of defining *imputed carry forward prices* for products that become unavailable in period 1. Thus define the carry forward prices for the disappearing products as follows:

$$(32)\ p_{3mf}^1 \equiv p_{3m}^0 P_{LO}\ ; \qquad m = 1,\ldots,M.$$

Thus the period 0 price for disappearing product m, p_{3m}^0, is adjusted for general group inflation using the maximum overlap Laspeyres price index P_{LO} and is carried forward to period 1. These carry forward prices are different from the carry backward prices in the previous section which used the Jevons index P_{JO}^0 as the adjusting index of general inflation.

Define the following *margin terms*, α_m, which express how much higher each reservation price is from its Laspeyres inflation adjusted carry forward price counterpart:

$$(33)\ 1 + \alpha_m \equiv p_{3m}^{1*}/p_{3mf}^{0}\ ; \qquad m = 1,...,M.$$

Now substitute definitions (33) into (31) and we obtain the following *exact relationship* between the true Laspeyres price index P_L and its maximum overlap counterpart P_{LO}:

$$\begin{aligned}(34)\ P_L &\equiv P_{LO} + \Sigma_{m=1}^{M} s_{3m}^{0}\,[(p_{3m}^{1*}/p_{3m}^{0}) - P_{LO}] \\ &= P_{LO}(1 + \Sigma_{m=1}^{M} s_{3m}^{0}\,[(1 + \alpha_m) - 1]) \\ &= P_{LO}(1 + \Sigma_{m=1}^{M} s_{3m}^{0}\,\alpha_m).\end{aligned}$$

Thus we have the following exact formula for $(P_L/P_{LO}) - 1$:

$$(35)\ (P_L/P_{LO}) - 1 = \Sigma_{m=1}^{M} s_{3m}^{0}\,\alpha_m.$$

We expect the margins α_m to be positive in general. In this case, it can be seen that the maximum overlap Laspeyres price index will understate inflation as measured by the true Laspeyres price index, provided that there are disappearing products in period 1.

The “true” implicit quantity index that matches up with the true Laspeyres price index is the *Paasche quantity index* Q_P defined as follows:

$$(36)\ Q_P \equiv [v^1/v^0]/P_L.$$

The *maximum overlap Paasche quantity index*, Q_{PO}, is the implicit quantity index that deflates the value ratio by the maximum overlap Laspeyres price index:

$$(37)\ Q_{PO} \equiv [v^1/v^0]/P_{LO}.$$

The bias in Q_{PO} relative to its true counterpart Q_P can be measured by the ratio Q_P/Q_{PO}:

$$\begin{aligned}(38)\ Q_P/Q_{PO} &= P_{LO}/P_L && \text{using (36) and (37)} \\ &= 1/(1 + \Sigma_{m=1}^{M} s_{3m}^{0}\,\alpha_m) && \text{using (34).}\end{aligned}$$

where we have used definitions (24) and (25) to derive (26).

Define the right hand side of (38) as the function $f(\alpha_1,...,\alpha_M)$ and approximate f by taking the first order Taylor series approximation to f evaluated at $0 = \alpha_1 =... = \alpha_M$. The resulting approximation to $(Q_P/Q_{PO}) - 1$ is the following one:

$$(39)\ (Q_P/Q_{PO}) - 1 \approx - \Sigma_{m=1}^{M} (1/2)s_{3m}^{0}\,\alpha_m.$$

Thus the use of a maximum overlap Laspeyres price index leads to a resulting maximum overlap quantity index which will tend to *overstate volume growth* if there are disappearing products in period 1.

A similar analysis can be carried out for the Paasche price index. Define the *true Paasche price index*, P_P, as follows:

$$(40)\ P_P \equiv [p_1{}^1\cdot q_1{}^1 + p_2{}^1\cdot q_2{}^1 + p_3{}^{1*}\cdot q_3{}^1]/[p_1{}^0\cdot q_1{}^1 + p_2{}^{0*}\cdot q_2{}^1 + p_3{}^0\cdot q_3{}^1]$$
$$= [p_1{}^1\cdot q_1{}^1 + p_2{}^1\cdot q_2{}^2]/[p_1{}^0\cdot q_1{}^2 + p_2{}^{0*}\cdot q_2{}^2] \qquad \text{using } q_3{}^1 = 0_M$$
$$= [\Sigma_{n=1}{}^N s_{1n}{}^1 (p_{1n}{}^1/p_{1n}{}^0)^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1 (p_{2k}{}^1/p_{2k}{}^{0*})^{-1}]^{-1} .$$

Define the *maximum overlap Paasche price index* P_{PO} that is defined only over products that are present in both periods as follows:

$$(41)\ P_{PO} \equiv p_1{}^1\cdot q_1{}^1/p_1{}^0\cdot q_1{}^1$$
$$= [\Sigma_{n=1}{}^N s_{1nO}{}^1 (p_{1n}{}^1/p_{1n}{}^0)^{-1}]^{-1}$$

where the maximum overlap shares $s_{1nO}{}^1$ were defined above by definitions (7).

Recall equations (9) which exhibited the relationship between the period 1 true share weights for continuing products, $s_{1n}{}^1$, and the corresponding share weights for the commodities present in each period, $s_{1n}{}^{1*}$. Using definitions (40) and (41) and equations (9), we can derive the following relationship between P_P and P_{PO}:

$$(42)\ (P_P)^{-1} \equiv \Sigma_{n=1}{}^N s_{1n}{}^1 (p_{1n}{}^1/p_{1n}{}^0)^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1 (p_{2k}{}^1/p_{2k}{}^{0*})^{-1}$$
$$= \Sigma_{n=1}{}^N s_{1nO}{}^1[1 - \Sigma_{k=1}{}^K s_{2k}{}^1](p_{1n}{}^1/p_{1n}{}^0)^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1(p_{2k}{}^1/p_{2k}{}^{0*})^{-1}$$
$$= (P_{PO})^{-1} - \Sigma_{n=1}{}^N s_{1nO}{}^{1*}\Sigma_{k=1}{}^K s_{2k}{}^1(p_{1n}{}^1/p_{1n}{}^0)^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1 (p_{2k}{}^1/p_{2k}{}^{0*})^{-1}$$
$$= (P_{PO})^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1[(p_{2k}{}^1/p_{2k}{}^{0*})^{-1} - \Sigma_{n=1}{}^N s_{1nO}{}^1(p_{1n}{}^1/p_{1n}{}^0)^{-1}]$$
$$= (P_{PO})^{-1} + \Sigma_{k=1}{}^K s_{2k}{}^1[(p_{2k}{}^1/p_{2k}{}^{0*})^{-1} - (P_{PO})^{-1}]$$
$$= (P_{PO})^{-1}\{1+ \Sigma_{k=1}{}^K s_{2k}{}^1[(p_{2k}{}^1/p_{2k}{}^{0*} P_{PO})^{-1} - 1]\}$$
$$= (P_{PO})^{-1}\{1+ \Sigma_{k=1}{}^K s_{2k}{}^1[(p_{2k}{}^{0*} P_{PO}/p_{2k}{}^1) - 1]\}$$
$$= (P_{PO})^{-1}(1 + \beta)$$

where $\beta \equiv \Sigma_{k=1}{}^K s_{2k}{}^1[(p_{2k}{}^{0*} P_{PO}/p_{2k}{}^1) - 1]$ is a bias term which allows us to adjust the reciprocal of the Paasche price index defined over continuing products, $1/P_{PO}$, so that the adjusted index is equal to the reciprocal of the true Paasche price index, $1/P_P$. The relationship (42) can be rewritten as follows:

$$(43)\ P_P = P_{PO}(1 + \beta)^{-1}.$$

There is a presumption that the error term β will be *positive* in which case P_{PO} has an upward bias relative to P_P and hence must be adjusted downward by dividing P_{PO} by $(1+\beta)$. This adjustment term $1/(1+\beta)$ accounts for the effective decrease in the true index P_P which is due to the appearance of the new products in Group 2 during period 1.

The decomposition defined by (42) is also useful in the context of defining *imputed carry backward prices* for products that are available in period 1 but not period 0. Thus define the carry backward prices for the new products as follows:[17]

[17] These new carry backward prices use the maximum overlap deflator P_{PO} instead of the maximum overlap Jevons deflator $P_{JO}{}^1$ which was used in our earlier definitions of carry backward prices, (17).

$$(44)\ p_{2kb}{}^{0} \equiv p_{2k}{}^{1}/P_{PO}\ ; \qquad k = 1,...,K.$$

Thus the period 1 price for new product k, $p_{2k}{}^{1}$, is adjusted for general group inflation using the maximum overlap Paasche price index P_{PO} and is carried backward to period 0.

Define the following *margin terms*, β_k, which express how much higher each reservation price is from its Paasche inflation adjusted carry backward price counterpart:

$$(45)\ 1 + \beta_k \equiv p_{2k}{}^{0*}/p_{2kb}{}^{0}\ ; \qquad k = 1,...,K.$$

Now substitute definitions (45) into (42) and we obtain the following *exact relationship* between the true Paasche price index P_P and its maximum overlap counterpart P_{PO}:

$$\begin{aligned}(46)\ (P_P)^{-1} &= (P_{PO})^{-1}\{1+ \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}[(p_{2k}{}^{1}/p_{2kb}{}^{0}\, P_{PO})^{-1} - 1]\} \\ &= (P_{PO})^{-1}\{1+ \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}[(p_{2k}{}^{0*}\, P_{PO}/p_{2k}{}^{1}) - 1]\} \\ &= (P_{PO})^{-1}\{1+ \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}[(1 + \beta_k) - 1]\} \\ &= (P_{PO})^{-1}(1 + \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}\, \beta_k).\end{aligned}$$

Thus we have the following exact formula for $(P_{PO}/P_P) - 1$:

$$(47)\ (P_{PO}/P_P) - 1 = \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}\, \beta_k.$$

We expect the margins β_k to be positive in general. In this case, it can be seen that the maximum overlap Paasche price index P_{PO} will overstate inflation as measured by the true Paasche price index P_P, provided that there are new products in period 1.

Formula (35) gave an expression for (P_L/P_{LO}). It is convenient to have a companion formula for (P_P/P_{PO}) to match up with the formula for (P_L/P_{LO}) given by (35). Rearranging (46) we have the following exact formula:

$$\begin{aligned}(48)\ (P_P/P_{PO}) - 1 &= (1 + \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}\, \beta_k)^{-1} - 1 \\ &\approx - \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}\, \beta_k\end{aligned}$$

where the approximation in (48) follows by taking a first order Taylor series approximation to the function $(1 + \Sigma_{k=1}{}^{K}\, s_{2k}{}^{1}\, \beta_k)^{-1} - 1$ around the point $\beta_1 = \beta_2 = ... = \beta_K = 0$.

The "true" implicit quantity index that matches up with the true Paasche price index is the *Laspeyres quantity index* Q_L defined as follows:

$$(49)\ Q_L \equiv [v^1/v^0]/P_P.$$

The *maximum overlap Laspeyres quantity index*, Q_{LO}, is the implicit quantity index that deflates the value ratio by the maximum overlap Paasche price index:

(50) $Q_{LO} \equiv [v^1/v^0]/P_{PO}$.

The bias in Q_{LO} relative to its true counterpart Q_L can be measured by the ratio Q_L/Q_{LO}:

(51) $(Q_L/Q_{LO}) - 1 = (P_{PO}/P_P) - 1$ using (49) and (50)
$= \Sigma_{k=1}^{K} s_{2k}^{1} \beta_k$ using (47).

Thus the upward bias in the maximum overlap Paasche price index P_{PO} translates into a downward bias in the companion maximum overlap Laspeyres quantity index, Q_{LO}.

Finally, we look at the bias in the maximum overlap Fisher indexes relative to their true counterparts.

Define the true Fisher index, P_F, as the geometric mean of the true Laspeyres and Paasche indexes and define the maximum overlap Fisher index over commodities present in both periods, P_{FO}, as the geometric mean of the maximum overlap Laspeyres and Paasche indexes, P_{LO} and P_{PO}:

(52) $P_F \equiv (P_L P_P)^{1/2}$;
(53) $P_{FO} \equiv (P_{LO} P_{PO})^{1/2}$.

The exact relationship between P_F and P_{FO} can be determined by substituting the exact decompositions for P_L and P_P given by (31) and (43) into definition (52):

(54) $P_F \equiv (P_L P_P)^{1/2}$
$= [P_{LO}(1 + \alpha)P_{PO}(1 + \beta)^{-1}]^{1/2}$
$= P_{FO}[(1 + \alpha)(1 + \beta)^{-1}]^{1/2}$ using definition (53)
$\approx P_{FO}[(1 + \alpha)(1 - \beta)]^{1/2}$ using the approximation $(1+\beta)^{-1} \approx 1 - \beta$
$\approx P_{FO}[(1/2)(1 + \alpha) + (1/2)(1 - \beta)]$
approximating the geometric mean by the arithmetic mean
$= P_{FO}[1 + (1/2)\alpha - (1/2)\beta]$.

Rearranging (54) and using $\alpha = \Sigma_{m=1}^{M} s_{3m}^{0} \alpha_m$ and $\beta = \Sigma_{k=1}^{K} s_{2k}^{1} \beta_k$, we obtain the following approximate relationship:

(55) $(P_F/P_{FO}) - 1 \approx (1/2) \Sigma_{m=1}^{M} s_{3m}^{0} \alpha_m - (1/2) \Sigma_{k=1}^{K} s_{2k}^{1} \beta_k$.

The two approximations used in deriving the final approximation in (55) will be quite accurate if α and β are close to 1. Typically, this will be the case. The Fisher index approximate decomposition defined by (55) is a counterpart to the Törnqvist approximate decomposition defined earlier by (22). Typically, they will give much the same answer.

The "true" *implicit Fisher quantity index* Q_F is defined as the value ratio, v^1/v^0, deflated by the "true" Fisher price index P_F ; i.e., we have:

(56) $Q_F \equiv [v^1/v^0]/P_F$.

Some statistical agencies use maximum overlap Fisher price indexes to deflate final demand aggregates in order to construct aggregate quantity or volume indexes. Thus in our context, the *maximum overlap Fisher quantity index*, Q_{FO}, is defined as follows:

(57) $Q_{FO} \equiv [v^1/v^0]/P_{FO}$.

The bias in Q_{FO} relative to its true counterpart Q_F can be measured by the ratio Q_F/Q_{FO}:

(58) $Q_F/Q_{FO} = P_{FO}/P_F$

where we have used definitions (56) and (57) to derive (58). An exact expression for P_{FO}/P_F can be obtained from (54):

$$(59)\ P_{FO}/P_F = [(1+\alpha)^{-1}(1+\beta)]^{1/2}$$
$$= [(1+\Sigma_{m=1}^{M} s_{3m}^{0}\,\alpha_m)^{-1}(1+\Sigma_{k=1}^{K} s_{2k}^{1}\,\beta_k)]^{1/2}.$$

Regard the right hand side of (59) as a function of $\alpha_1,...,\alpha_M$ and $\beta_1,...,\beta_K$ and form the first order Taylor series approximation to this function around the point $0 = \alpha_1 = ... = \alpha_M = \beta_1 = ... = \beta_K$. This approximation leads to the following approximation for Q_F/Q_{FO}:

$$(59)\ (Q_F/Q_{FO}) - 1 \approx (1/2)\,\Sigma_{k=1}^{K} s_{2k}^{1}\,\beta_k - (1/2)\,\Sigma_{m=1}^{M} s_{3m}^{0}\,\alpha_m.$$

This approximation is very similar to our earlier approximation for $(Q_T/Q_{TO}) - 1$; see (28) above.

In the Appendix, we will adapt the algebra developed in this section to the problems associated with replacing disappearing products with closely related substitute products.

5. Conclusion

Statistical agencies almost always construct estimates of real consumption by deflating the value aggregate by using a particular index number formula. However, in the context of new and disappearing products instead of using the "true" index number formula for the price index, statistical agencies often use a maximum overlap index that is restricted to the products present in both periods being compared. We evaluated the bias in the use of maximum overlap indexes for the Törnqvist, Laspeyres, Paasche and Fisher price and quantity indexes in sections 3 and 4. The resulting bias formulae are very simple but unfortunately, they require estimates of Hicksian reservation prices for the missing products in both periods. These reservation prices are not easy to compute, and in some cases they may be out of scope for official price indices.[18]

[18] See Reinsdorf and Schreyer (2017).

Appendix: Bias Formulae for Replacement Samples

Triplett (2004; 12-40) provides an excellent discussion of existing statistical agency practices to deal with quality change in the context of *replacement sampling*. In most elementary strata, products disappear from a sample of similar products while new products appear. It is common for statistical agencies to refresh their sample of products by substituting replacement products for the disappearing products. However, in order to make the replacement products comparable to the disappearing products, the statistical agency may make some *quality adjustments* to the new products. Triplett systematically describes the main methods of quality adjustment used by statistical agencies in his Handbook. We will discuss most of methods he reviewed in this Appendix.[19]

We will study the possible bias in sample replacement methods in the transaction data (e.g. scanner data) context; i.e., we will assume that price and quantity data are available to the statistical agency for the set of products under consideration.[20] We will also adapt our Hicksian reservation price methodology that was described in section 4 to the product replacement context; i.e., we will continue to assume that M products disappear in period 1 but in this Appendix, we will assume that the K new products introduced in period 1 are replacement products for the M disappearing products (so that now K = M). We will adapt the algebra in section 4 first to the case where the Laspeyres index is the target index. Subsequently, we will consider the companion case where the Paasche index is the target index. The case of the Fisher index as the target is left to the reader.

The Laspeyres Case

We now make the assumption that the new products in Group 2 that appear in period 1 are *replacement products* for the M disappearing products in period 1. Thus we set K = M and the p_{2k}^1 which appear in sections 2-4 above now become the M observable replacement prices in Group 2, which we label as p_{2m}^1 for m = 1,...,M.

However, the statistical agency may quality adjust these replacement prices to make them more comparable to the period 0 product prices for Group 3, the p_{3m}^0. The quality adjustment factor for product m is A_m^1 for m = 1,...,M. Thus the statistical agency *quality adjusted replacement price* for disappearing product m in period 1 is p_{3mr}^1 defined as the

[19] Triplett's (2004) Handbook is mostly about the use of hedonic regression models to perform quality adjustment on new products to make them comparable to disappearing products. However, he realized that there was considerable opposition to the use of hedonic regression models by official statistical agencies due to their "subjective" nature. Triplett probably wrote his Chapter II as a response to these criticisms; i.e., he showed that existing statistical agency practices in dealing with disappearing products also had large subjective aspects!

[20] It can be argued that with transaction level data, replacement strategies are unnecessary: "Given some index number formula, quality adjustment is a matter of imputation or prediction of 'missing' prices or price relatives. What is needed is an estimate of what the price or price relative of a disappearing (new) item would have been, had it been sold during the current (base) period. With prediction comes statistical modelling, which is usually hedonic modelling in this context." Jan de Haan (2007; 1). However, many statistical agencies still use a traditional sample replacement approach.

period 1 observable Group 2 (replacement) product price p_{2m}^1 times the quality adjustment factor A_m^1:

$$(A1)\ p_{3mr}^1 \equiv A_m^1 p_{2m}^1 ; \qquad m = 1,...,M.$$

We define the *replacement Laspeyres index*, P_{LR}, in the same way as we defined the true Laspeyres index P_L as in (29) except that the true reservation prices, p_{3m}^{1*}, are replaced by the quality adjusted replacement prices p_{3mr}^1 defined by (A1). Thus we have the following definition:

$$\begin{aligned}(A2)\ P_{LR} &\equiv \Sigma_{n=1}^N s_{1n}^0 (p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (p_{3mr}^1/p_{3m}^0) \\ &= \Sigma_{n=1}^N s_{1n}^0 (p_{1n}^1/p_{1n}^0) + \Sigma_{m=1}^M s_{3m}^0 (A_m^1 p_{2m}^1/p_{3m}^0).\end{aligned}$$

We can form an estimate for the *bias* in P_{LR} by taking the difference between the true Laspeyres index P_L defined by (29) and (A2):

$$\begin{aligned}(A3)\ P_L - P_{LR} &= \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^{1*}/p_{3m}^0) - \Sigma_{m=1}^M s_{3m}^0 (p_{3mr}^1/p_{3m}^0) \\ &= \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^0)^{-1} (p_{3m}^{1*} - A_m^1 p_{2m}^1).\end{aligned}$$

Thus if $A_m^1 \geq p_{3m}^{1*}/p_{2m}^1$ for m = 1,..,M with at least one strict inequality (so that the quality adjustment factors A_m^1 are *too large*), then P_{LR} will be less than the true Laspeyres index and hence will have a *downward bias*. If $A_m^1 \leq p_{3m}^{1*}/p_{2m}^1$ for m = 1,..,M with at least one strict inequality (so that the quality adjustment factors A_m^1 are *too small*), then P_{LR} will be greater than the true Laspeyres index and hence will have an *upward bias*.

Now we can analyze some of Triplett's (2004; 21-29) special cases.

Special Case 1: Triplett's Direct Comparison Method

This is the special case of (A3) where the quality adjustment factors are all chosen to equal 1; i.e., we have $A_m^1 = 1$ for m = 1,...,M. Thus the period 1 replacement product prices p_{2m}^1 are regarded as exact substitute prices for the corresponding disappearing product prices p_{3m}^1 (which are missing). Substitute the equations $A_m^1 = 1$ into (A3) above in order to obtain the following bias formula for the *replacement Laspeyres index that uses the direct comparison method*:

$$(A4)\ P_L - P_{LR} = \Sigma_{m=1}^M s_{3m}^0 (p_{3m}^0)^{-1}[p_{3m}^{1*} - p_{2m}^1].$$

If product 2m is a perfect substitute for product 3m for m = 1,...,M, then p_{3m}^{1*} will equal p_{2m}^1 for all m and P_{LR} will equal the true Laspeyres price index, P_L. If product 2m has approximately the same quality as product 3m for each m but the product pairs are not perfect substitutes, then the reservation prices p_{3m}^{1*} will tend to be higher than the corresponding prices p_{2m}^1 (these products are actually available in period 1 and hence should be less than their reservation prices) and thus the right hand side of (A4) will tend to be positive. Hence P_{LR} will tend to have a downward bias relative to the true Laspeyres index P_L.[21]

Special Case 2: Triplett's Link to Show No Change Case (Carry Forward Method)

Perhaps a better description of this method for quality adjustment would be to call this method *the price carry forward with no inflation adjustment method*; i.e., we simply assume that the period 1 replacement price for product m is the corresponding base period price p_{3m}^{0}. Thus in this case, we have $p_{3mr}^{1} \equiv p_{3m}^{0} = A_m^{1}p_{2m}^{1}$ for m = 1,...,M. Hence in this case, the quality adjustment factors are $A_m^{1} = p_{3m}^{0}/p_{2m}^{1}$. Using (A3), the resulting bias formula is now the following one:

$$
\begin{aligned}
\text{(A5)}\quad P_L - P_{LR} &= \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3mr}^{1}/p_{3m}^{0}) \\
&= \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3m}^{0}/p_{3m}^{0}) \qquad \text{using } p_{3mr}^{1} = p_{3m}^{0} \\
&= \Sigma_{m=1}^{M} s_{3m}^{0} [(p_{3m}^{1*}/p_{3m}^{0}) - 1].
\end{aligned}
$$

If there is general inflation for the commodity group under consideration going from period 0 to 1, then P_{LR} will tend to have a downward bias due to the fact that the period 0 prices p_{3m}^{0} are not adjusted upwards for inflation. There may be another dose of downward bias in P_{LR} due to the fact that the reservation prices p_{3m}^{1*} will tend to be higher than the prices of substitute products that are available in period 1. If there is general deflation for the commodity group under consideration going from period 0 to 1, then P_{LR} will tend to have an upward bias due to the fact that the period 0 prices p_{3m}^{0} are not adjusted downwards for this deflation. Some of this downward bias could be offset by the fact that the reservation prices p_{3m}^{1*} will tend to be higher than the prices of substitute products that are available in period 1. In general, this method is not recommended.

Special Case 3: Triplett's Deletion Method (Maximum Overlap Method)

Perhaps a better description of this method for quality adjustment would be to call this method *the price carry forward with a maximum overlap inflation adjustment method*. In this case, we assume that the period 1 replacement price for missing product m is the corresponding base period price p_{3m}^{0} times an index of general inflation. The index of general inflation that we use is the maximum overlap Laspeyres price index, P_{LO}, defined by (30) in in section 4. Thus the replacement prices for the missing M products in period 1 are defined as follows for this method:[22]

$$
\begin{aligned}
\text{(A6)}\quad p_{3mr}^{1} &\equiv p_{3m}^{0}P_{LO}\,; \qquad m = 1,...,M \\
&= p_{3mf}^{1}
\end{aligned}
$$

where the second set of equalities in (A6) follows from definitions (32) in the main text; i.e., the p_{3mf}^{1} are the *Laspeyres maximum overlap carry forward prices* for the disappearing products that were defined in section 4.

[21] Thus this downward replacement bias will tend to offset some of the upward substitution bias in the true Laspeyres index.

[22] In this case, the Triplett quality adjustment factors are defined as $A_m^{1} \equiv P_{LO}\, p_{3m}^{0}/p_{2m}^{1}$.

Substitute equations (A6) into equation (A3) and we obtain the following equations:[23]

$$
\begin{aligned}
\text{(A7) } P_L - P_{LR} &= \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3mr}^{1}/p_{3m}^{0}) \\
&= \Sigma_{m=1}^{M} s_{3m}^{0} (p_{3m}^{1*}/p_{3m}^{0}) - \Sigma_{m=1}^{M} s_{3m}^{0} (P_{LO}) && \text{using (A6)} \\
&= P_{LO}\, \Sigma_{m=1}^{M} s_{3m}^{0} [(p_{3m}^{1*}/P_{LO}\, p_{3m}^{0}) - 1] \\
&= P_{LO}\, \Sigma_{m=1}^{M} s_{3m}^{0} [(p_{3m}^{1*}/p_{3mf}^{1}) - 1] && \text{using (A6).}
\end{aligned}
$$

Thus the difference between the true Laspeyres index P_L and the Laspeyres index using inflation adjusted carry forward prices p_{3mf}^{1} as approximations to the Hicksian reservation prices p_{3m}^{1*} hinges on the differences between these two sets of prices. In general, we would expect the reservation prices p_{3m}^{1*} to be greater than their inflation adjusted carry forward price counterparts p_{3mf}^{1}. Thus in general, we expect P_{LR} to have a downward bias using this method.[24] However, if general inflation is positive, then the bias using this method should be considerably less than the bias using Method 2 above.

The Paasche Case

We now consider how the replacement methodology used above works in the context of evaluating a true Paasche index and various approximations to it. The true Paasche index, P_P, was defined by (40) in the main text. However, in the present context, there are M new products instead of K new products. These M new products are Group 2 products that replace the Group 3 products that disappeared in period 1. For convenience, we write the reciprocal of the true Paasche index in the present context as follows:

$$\text{(A8) } (P_P)^{-1} = \Sigma_{n=1}^{N} s_{1n}^{1} (p_{1n}^{1}/p_{1n}^{0})^{-1} + \Sigma_{m=1}^{M} s_{2m}^{1} (p_{2m}^{1}/p_{2m}^{0*})^{-1}.$$

The prices for the new products which have appeared in period 1 as replacement products are now the prices p_{2m}^{1} for m = 1,..., M and we require *reservation prices* for these products in period 0 which we now denote as p_{2m}^{0*} for m = 1,...,M. If these replacement products were not available in period 0, then the p_{2m}^{0*} are true Hicksian reservation prices. However, it may be the case that products m existed in period 0 in which case, the period 0 observed price for this product, p_{2m}^{0}, could be observed. In this case, the reservation price should be taken to be the observed price; i.e., in this case, we have:

$$\text{(A9) } p_{2m}^{0*} \equiv p_{2m}^{0} \text{ if product m exists in period 0.}$$

In the general case where either the M replacement products did not exist in period 0 or where the statistical agency is unable to collect observed period 0 prices for these products, then in order to construct an approximation to the true Paasche price index, the

[23] In this case, P_{LR} coincides with P_{LO}. The bias formula (A7) is a generalization of a bias formula due to Triplett (2004; 25).

[24] This general verdict on the method agrees with Triplett's (2004; 29) general assessment of the bias in this method. However, Triplett goes on to state that in the case of computers or other products that have rapid downward price change, the bias may go in the other direction. Thus the direction of bias will depend on the context.

statistical agency may insert *estimated prices* or *replacement prices* p_{2mr}^0 in place of the "true" period 0 reservation prices p_{2m}^{0*} for m = 1,...,M. In order to make the analysis of the Paasche replacement index symmetric to our treatment of the Laspeyres replacement index, we introduce a new set of *product specific inflation adjustment factors*, A_m^0 that convert the observed period 1 prices p_{2m}^1 into their estimated counterpart prices that approximate the period 0 reservation prices p_{2m}^{0*}:[25]

$$\text{(A10)}\quad p_{2mr}^0 \equiv p_{2m}^1/A_m^0\,; \qquad m = 1,...,M.$$

Now define *the replacement Paasche index*, P_{PR}, in the same way as one would define the true Paasche index P_P as in (A8) except that the true period 0 reservation prices, p_{2m}^{0*}, are replaced by the inflation adjusted replacement prices p_{2mr}^0 defined by (A10). Thus we have the following definition for the reciprocal of P_{PR}:

$$\begin{aligned}\text{(A11)}\quad (P_{PR})^{-1} &= \Sigma_{n=1}^{N} s_{1n}^1 (p_{1n}^1/p_{1n}^0)^{-1} + \Sigma_{m=1}^{M} s_{2m}^1 (p_{2m}^1/p_{2mr}^0)^{-1} \\ &= \Sigma_{n=1}^{N} s_{1n}^1 (p_{1n}^1/p_{1n}^0)^{-1} + \Sigma_{m=1}^{M} s_{2m}^1 (A_m^0)^{-1} \qquad \text{using (A10).}\end{aligned}$$

We can form an estimate for the *bias* in P_{LR} by taking the difference between (A8) and (A11):

$$\begin{aligned}\text{(A12)}\quad (P_P)^{-1} - (P_{PR})^{-1} &= \Sigma_{m=1}^{M} s_{2m}^1 (p_{2m}^1/p_{2m}^{0*})^{-1} - \Sigma_{m=1}^{M} s_{2m}^1 (A_m^0)^{-1} \\ &= \Sigma_{m=1}^{M} s_{2m}^1 [(p_{2m}^1/p_{2m}^{0*})^{-1} - (A_m^0)^{-1}].\end{aligned}$$

Thus if $A_m^0 \geq p_{2m}^1/p_{2m}^{0*}$ for m = 1,..,M with at least one strict inequality (so that the product specific inflation adjustment factors A_m^0 are *too large*), then $(P_{PR})^{-1}$ will be less than the reciprocal of the true Paasche index and hence will have a *downward bias*. Thus P_{PR} will be greater than the true Paasche index P_P. If $A_m^0 \leq p_{2m}^1/p_{2m}^{0*}$ for m = 1,..,M with at least one strict inequality (so that the product specific inflation adjustment factors A_m^0 are *too small*), then P_{PR} will be less than the true Paasche index P_P.

We will consider three special cases of the above general methodology.

Special Case 1: The Use of Inflation Adjusted Paasche Carry Backward Prices

In this case, the replacement prices for period 0 are set equal to the inflation adjusted carry backward prices defined by equations (44) in the main text. Thus using our present notation, we have the following replacement prices:

$$\text{(A13)}\quad p_{2mr}^0 \equiv p_{2m}^1/A_m^0 = p_{2mb}^0 \equiv p_{2m}^1/P_{PO}\,; \qquad m = 1,...,M$$

where P_{PO} is the Paasche Maximum Overlap price index defined by (41) in the main text, which in this case is equal to the Paasche Replacement price index defined by (A11)

[25] Quality adjustment is not an issue in defining the "quality" adjustment factors A_m^0 if all of the replacement products actually existed in period 0. Thus we will refer to the A_m^0 as inflation adjustment factors.

where the replacement prices p_{2mr}^0 are defined by equations (A13). Note that in this case, all of the product specific inflation factors A_m^0 are equal to the maximum overlap Paasche index, P_{PO}, which in turn is also equal to P_{PR} in this case. Thus the bias in this case can be determined by evaluating the following version of equation (A12) where the A_m^0 are all equal to P_{PO}:

$$\text{(A14)}\ (P_P)^{-1} - (P_{PR})^{-1} = \Sigma_{m=1}^M s_{2m}^1 [(p_{2m}^1/p_{2m}^{0*})^{-1} - (A_m^0)^{-1}]$$
$$= \Sigma_{m=1}^M s_{2m}^1 [(p_{2m}^1/p_{2m}^{0*})^{-1} - (P_{PO})^{-1}].$$

If the Group 2 products were missing in period 0, then the reservation prices p_{2m}^{0*} are likely to be relatively high and the price ratios p_{2m}^1/p_{2m}^{0*} are likely to be less than the maximum overlap Paasche index P_{PO}, so that we are likely to have $(p_{2m}^1/p_{2m}^{0*})^{-1} > (P_{PO})^{-1}$ so the right hand side of (A14) is likely to be positive. Thus $(P_P)^{-1} > (P_{PR})^{-1}$ which in turn implies $P_P < P_{PR} = P_{PO}$. Thus in this case, P_{PR} is likely to have an upward bias.

Special Case 2: The Case of Carry Backward Prices with No Inflation Adjustment

This is the Paasche counterpart to Special Case 2 for the Laspeyres index. In this case, the replacement prices for period 0 are set equal to the period 1 product prices for the replacement products. Thus we have the following replacement prices:

$$\text{(A15)}\ p_{2mr}^0 \equiv p_{2m}^1/A_m^0 \equiv p_{2m}^1\ ; \qquad m = 1,\ldots,M.$$

In this case, $A_m^0 = 1$ for all m. The bias in this case can be determined by evaluating the following version of equation (A12) where the A_m^0 are all equal to 1:

$$\text{(A16)}\ (P_P)^{-1} - (P_{PR})^{-1} = \Sigma_{m=1}^M s_{2m}^1 [(p_{2m}^1/p_{2m}^{0*})^{-1} - (A_m^0)^{-1}]$$
$$= \Sigma_{m=1}^M s_{2m}^1 [(p_{2m}^1/p_{2m}^{0*})^{-1} - (1)^{-1}].$$

It is difficult to evaluate the bias of P_{PR} in this case. If there is a large amount of general inflation or deflation between periods 0 and 1 for the product category under consideration, then we are probably safe in asserting that the bias in using this method will be greater than the bias using the method described in the first special case (because the present method does not account for general inflation between periods 0 and 1).

Special Case 3: Prices for the Replacement Products are Available in Period 0

In this case, we assume that the statistical agency can observe these period 0 prices, p_{2m}^0 for m = 1,...,M. Thus in this case, the reservation prices p_{2m}^{0*} are equal to the observed prices, p_{2m}^0, for m = 1,...,M. The product specific inflation adjustment factors A_m^0 in this case are the observed price ratios for the products:

$$\text{(A17)}\ A_m^0 \equiv p_{2m}^1/p_{2m}^0\ ; \qquad m = 1,\ldots,M.$$

In this case, the reciprocal of the replacement Paasche index, P_{PR}, is defined as follows:

$$(A18)\ (P_{PR})^{-1} = \Sigma_{n=1}^{N} s_{1n}^{1} (p_{1n}^{1}/p_{1n}^{0})^{-1} + \Sigma_{m=1}^{M} s_{2m}^{1} (A_m^{0})^{-1}$$
$$= \Sigma_{n=1}^{N} s_{1n}^{1} (p_{1n}^{1}/p_{1n}^{0})^{-1} + \Sigma_{m=1}^{M} s_{2m}^{1} (p_{2m}^{1}/p_{2m}^{0})^{-1}$$
$$\equiv P_P^{-1}.$$

Thus under these conditions, the replacement Paasche index is equal to the true Paasche index. Thus if possible, the statistical agency should attempt to get transactions prices in the previous period for the replacement products it uses in its index computations for the current period. Of course, this will usually not be possible.

It can be seen that Triplett was quite right in flagging the problem of sample attrition and replacement methodologies as a serious problem with traditional statistical agency procedures.

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