INDEX NUMBER THEORY AND MEASUREMENT ECONOMICS

W. Erwin Diewert[1] March 16, 2015.
University of British Columbia and the University of New South Wales
Email: erwin.diewert@ubc.ca
Website: http://www.economics.ubc.ca/faculty-and-staff/w-erwin-diewert/

Chapter 15: Lowe Indexes and the Practical Construction of a CPI

1. Properties and Alternative Representations for the Lowe Index

Practical index number construction does not proceed exactly along the lines outlined in the previous Chapters. Even at higher levels of aggregation where expenditure information by commodity class is available, usually this expenditure information comes from household expenditure surveys[2] and there are delays in processing this information. Usually, this aggregate expenditure information will be one to five years out of date. This lack of up to date expenditure information means that all of the approaches to index number theory which were carefully explained in previous Chapters that relied on the availability of current expenditure information cannot be applied.[3]

Thus practical Consumer Price Indexes use *current price information* at a monthly frequency[4] but they use *quantity or expenditure weights* (at the higher levels of aggregation) that pertain to a past base year. Thus there are *two base periods or reference periods* in a practical CPI: the base year for the reference quantities or expenditures and the base month for prices. This type of index is known in the literature as a *Lowe* (1823) *index*; recall its introduction in Chapter 1 above.[5]

---

[1] This Chapter (and the next three chapters) were prepared for the Preconference Training Sessions on Price Measurement held on August 23-24, 2014, sponsored by the 33rd General Conference of the International Association for Research in Income and Wealth held in Rotterdam, Netherlands, August 24-30, 2014. These chapters draw heavily on Diewert (2012).

[2] In the UK, the household expenditure survey is called the Living Costs and Food Survey (LCF). This survey relates to private households only, excluding highest income and pensioner households; see the ONS (2012; 36). For the Retail Prices Index (RPI), the LCF survey expenditure information is used to obtain weights at higher levels of aggregation. However, for the Consumer Price Index (or CPI which is also the Eurostat mandated HICP), the higher level weights are largely calculated using Household Final Monetary Consumption Expenditure (HFMCE); see the ONS (2012; 35-36).

[3] Obviously, the superlative indexes described in previous chapters could be calculated on a delayed basis (at least at higher levels of aggregation) provided that household expenditure information is collected on a continuous basis and if we were willing to wait until expenditure information on the current period became available. If scanner data is used in the construction of a CPI, then the situation changes since up to date information on prices and quantities may be available on a current basis at the detailed item level. Problems associated with consumer price index construction when scanner data are available will be discussed in Chapter 17.

[4] New Zealand and Australia use current price information at a quarterly frequency.

[5] We also discussed the use of the Lowe index as a CPI in more detail in Chapter 7 of these notes. In Chapter 7, we assumed that month to month price relatives were used to update the previous month's Lowe index. This is what is done in many countries. However, in Europe instead of using month to month price relatives to update the Lowe index of the previous month, long term price relatives are used to construct the

It seems a bit odd to use annual quantity or expenditure weights with monthly price information but statistical agencies give two reasons for the use of annual weights:[6]

- Expenditure information collected from household surveys is often unreliable when collected for short periods of time and this variability can be reduced by using annual information;
- Some expenditures are seasonal in nature and thus the pattern of expenditure for any given month will not be representative for the annual average expenditures by commodity class.[7]

We will discuss the above explanations in more detail in section 3 below but for now, we will accept the above arguments and proceed to explain alternative ways for representing the Lowe index.

Historically, the Lowe index $P_{Lo}(p^0,p^t,q^b)$ was defined in terms of a base period quantity vector, $q^b \equiv [q_1^b,...,q_N^b]$ (which we will take to be the base year quantity vector of household purchases), a vector of base month prices p^0 where period 0 represents the base month for pricing purposes and a sequence of 12 consecutive monthly household price vectors $p^t \equiv [p_1^t,...,p_N^t]$ for m =1,2,...,12 which follow month 0:

$$(1)\ P_{Lo}(p^0,p^t,q^b) \equiv\ p^t\cdot q^b/p^0\cdot q^b \equiv \textstyle\sum_{n=1}^N p_n^tq_n^b / \sum_{n=1}^N p_n^0q_n^b\ ; \qquad t = 0,1,...,12.$$

Thus the level of prices in month t of the current (augmented) year[8] relative to month 0 is simply the cost of purchasing the commodity basket q^b at the prices p^t of month t, $p^t\cdot q^b$, divided by the cost of purchasing the same annual commodity basket q^b at the prices p^0 of the base month 0, $p^0\cdot q^b$. This is an index number concept that is relatively easy to explain to the public.

At this point, it will be useful to introduce the notation that corresponds to the type of Lowe index that is used by many statistical agencies including Eurostat and the Office for National Statistics (ONS) in the UK. For the sake of definiteness, we consider the Harmonized Index of Consumer Prices (HICP) that Eurostat constructs for countries belonging to the European Union (with the cooperation of the various national statistical agencies). The HICP uses January of each year as the base month.[9] Thus if in January of year y, the historical level of the RPI were $P_{RPI}(y:1)$, then the February level in year y

Lowe index for a run of 12 consecutive months. This alternative methodological approach will be explained in more detail below.

[6] See the ONS (2012; 12) for these reasons.

[7] In fact, the use of the Lowe index $P_{Lo}(p^0,p^t,q^b)$ in the context of seasonal commodities corresponds to Bean and Stine's (1924; 31) Type A index number formula. Bean and Stine made 3 additional suggestions for the construction of price indexes in the context of seasonal commodities.

[8] An augmented year is a string of 13 consecutive months.

[9] An alternative CPI index is also constructed by the ONS: the Retail Prices Index (RPI). The RPI uses December as the base month instead of January but the index calculations for the RPI are otherwise similar to the HICP index calculations. There are other differences between national CPIs and the HICP; for a discussion of these differences, see Diewert (2002).

would be $P_{RPI}(y{:}1)\times P_{Lo}(p^{y,1},p^{y,2},q^b)$, the March level would be $P_{RPI}(y{:}1)\times P_{Lo}(p^{y,1},p^{y,3},q^b)$,..., the December year y level would be $P_{RPI}(y{:}1)\times P_{Lo}(p^{y,1},p^{y,12},q^b)$ and the January year y+1 level would be $P_{RPI}(y+1{:}1) \equiv P_{RPI}(y{:}1)\times P_{Lo}(p^{y,1},p^{y,13},q^b)$. Note that the index comparisons within year y are of the *fixed base type*; i.e., the prices of month m, $p^{y,m}$, are compared directly with the base month prices in January, $p^{y,1}$.[10] After the calculation of the index for January of year y+1, at this point, a new vector of quantity weights would be introduced, say q^{b*} and there would be a new sequence of 13 consecutive monthly price vectors $p^{y+1,m}$ say[11] that started in January of year t+1 and finished in January of year t+2. Thus the February year t+1 level would be $P_{RPI}(y+1{:}1)\times P_{Lo}(p^{y+1,1},p^{y+1,2},q^{b*})$, the March year y+1 level would be $P_{RPI}(y+1{:}1)\times P_{Lo}(p^{0*},p^{2*},q^{b*})$ and so on.

We will rewrite the Lowe index defined by (1) using the notation introduced in the previous paragraph so that p^0 is now replaced by the year y, month 1 vector of prices, $p^{y,1} \equiv [p_1^{y,1},p_2^{y,1},...,p_N^{y,1}]$, and p^t is replaced by the (augmented) year, month m vector of prices, $p^{y,m} \equiv [p_1^{y,m},p_2^{y,m},...,p_N^{y,m}]$ for m = 1,2,...,13.[12] In what follows, the Lowe index defined above will be written in alternative forms.

The first alternative way of rewriting the sequence of Lowe indexes for year y is in the following *hybrid share form*:

$$
\begin{aligned}
(2)\ P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b}) &\equiv \textstyle\sum_{n=1}^N p_n^{y,m} q_n^{\ b} / \sum_{n=1}^N p_n^{y,1} q_n^{\ b}\ ; \qquad m = 1,2,...,13;\\
&= \textstyle\sum_{n=1}^N (p_n^{y,m}/p_n^{y,1}) p_n^{y,1} q_n^{\ b} / \sum_{n=1}^N p_n^{y,1} q_n^{\ b}\\
&= \textstyle\sum_{n=1}^N (p_n^{y,m}/p_n^{y,1})\, s_n^{y,1,b}
\end{aligned}
$$

where the *hybrid expenditure shares* $s_n^{y,1,b}$ corresponding to the (annual) quantity weights vector q^b for the base year b and to the (monthly) prices $p^{y,1}$ for the prices base month (which is January of year y) are defined by:[13]

$$
(3)\ s_n^{y,1,b} \equiv p_n^{y,1} q_n^{\ b} / \textstyle\sum_{i=1}^N p_i^{y,1} q_i^{\ b} \qquad \text{for } n = 1,\ldots,N.
$$

Before proceeding to other representations of the Lowe index, we note that it has very good axiomatic properties *for the comparison of prices within a given augmented year*;

[10] Not all statistical agencies use a within year fixed base price comparison where the prices in the current month are compared with the corresponding prices in the previous December or January; i.e., some countries *chain* their elementary price relatives within the year. This alternative methodology is explained in the *Consumer Price Index Manual*; see the ILO (2004; 271-272). We will not explore the properties of the chaining method in this Chapter since this method is discussed in the *Manual* whereas the use of fixed base price relatives was not discussed in detail in the *Manual*.

[11] If there were no changes in the commodity classification, $p^{y+1,1}$ should equal $p^{y,13}$; i.e., the old January year y+1 vector of prices $p^{y,13}$ should coincide with the first of the new string of 13 consecutive monthly prices $p^{y+1,m}$ for the calendar year y+1 plus January of year y+2.

[12] The price vector $p^{y,13}$ is the January of year y+1 price vector where the commodity classification that is used in year y is used to calculate the prices in $p^{y,13}$.

[13] Fisher (1922; 53) used the terminology "weighted by a hybrid value" while Walsh (1932; 657) used the term "hybrid weights". The two representations of the Lowe index given by the first and last lines of equations (2) are given in the ONS (2012; 12)

i.e., it is straightforward to show that the Lowe index satisfies all of the axioms T1-T12 listed in section 4 of Chapter 4 on elementary indexes except for the following two tests: test T8 (the symmetric treatment of outlets test) and test T9 (the price bouncing test).[14] However, when comparing price vectors across two different augmented years, more tests fail: T2 (the identity test), T7 (the mean value test), T10 (the time reversal test) and T11 (the circularity test).

The failure of the identity test if we compare prices across two different years is particularly troublesome but it is straightforward to find some sufficient conditions that will ensure that this test holds. In order to establish these conditions, it will be necessary to be more explicit on how to convert the present Lowe index methodology (with a fixed January base for an augmented year but then chaining the indexes across years) into indexes that can be put into the format of an elementary index.

Suppose that we start the sequence of index levels at January of year y (and the price level is set equal to unity for this month). Let the sequence of monthly price vectors starting in January of year y and ending in January of year y+1 be denoted by $p^{y,1}$, $p^{y,2}$,..., $p^{y,13}$ and denote the annual base year reference vector of quantities by q^b. Then the sequence of *Lowe index price levels*, $P_{LoL}(y;m)$, *for month m in the augmented year y* will be the following sequence of fixed base indexes:

$$(4)\ P_{LoL}(y;m) \equiv P_{Lo}(p^{y,1},p^{y,m},q^b) = p^{y,m}\cdot q^b/p^{y,1}\cdot q^b\ ; \qquad m = 1,2,...,13.$$

The elementary index tests in section 4 of Chapter 4 involved two price vectors: a base period price vector p^0 and a current period price vector p^1 and a function, $P(p^0,p^1)$. In the current situation where we are discussing the properties of Lowe indexes, as long as the base year quantity vector q^b remains fixed, we can use the tests in Chapter 4 to evaluate the properties of these Lowe indexes (with respect to prices) and this is what will be done below. Thus redefine the base period price vector p^0 as $p^{y,r}$ and the current period price vector p^1 as $p^{y,s}$ where r and s refer to two months in the augmented year y. As usual, q^b is the annual basket vector that is used to compute Lowe indexes for year y. The "elementary" price index $P(p^{y,r},p^{y,s})$ that corresponds to the comparison of the Lowe index price level in month s relative to month r in the augmented year y is defined as follows:

$$\begin{aligned}(5)\ P(p^{y,r},p^{y,s}) &\equiv P_{Lo}(p^{y,1},p^{y,s},q^b)/P_{Lo}(p^{y,1},p^{y,r},q^b)\\ &= [p^{y,s}\cdot q^b/p^{y,1}\cdot q^b]/[p^{y,r}\cdot q^b/p^{y,1}\cdot q^b]\\ &= p^{y,s}\cdot q^b/p^{y,r}\cdot q^b.\end{aligned}$$

It can be verified that $P(p^{y,s},p^{y,r}) \equiv p^{y,s}\cdot q^b/p^{y,r}\cdot q^b$ passes the tests T1-T7 and T10-T12 listed in section 4 of Chapter 4. So far, all is well.

[14] The test T8 and T9 are not really relevant in the present context. The representation of the Lowe index given by (2) is useful in establishing that the Lowe index satisfies test T7, the mean value test in section 4 of Chapter 4.

Now suppose that we wish to compare the prices in month s of year y+1 with the prices of month r in year y. We first construct the sequence of *price levels* for months m in augmented year y+1, $P_{LoL}(y+1;m)$. Let the sequence of monthly price vectors starting in January of year y+1 and ending in January of year y+2 be denoted by $p^{y+1,1}$, $p^{y+1,2}$,..., $p^{y+1,13}$ and denote the new annual base year reference vector of quantities for year y+1 by q^{b+1}.[15] The January level of the price index for year y+1 has already been determined as $P_{LoL}(y;12) \equiv p^{12}\cdot q^{b}/p^{0}\cdot q^{b}$; see (4) with m = 12. The sequence of *Lowe index price levels*, $P_L(y+1;m)$, *for month m in the augmented year y+1* will be the product of the already determined January value of the fixed base Lowe index for month 12 in year t, $P_{Lo}(p^{y,1},p^{y,13},q^{b}) = P_{LoL}(y,13)$ times the new fixed base Lowe index for month m in augmented year y+1, $P_{Lo}(p^{y+1,1},p^{y+1,m},q^{b+1})$; i.e., we have the following sequence of *price levels for augmented year y+1*:

$$(6)\ P_{LoL}(y+1;m) \equiv P_{LoL}(y,13)P_{Lo}(p^{y+1,1},p^{y+1,m},q^{b+1}) \qquad m = 1,2,...,13$$
$$= [p^{y,13}\cdot q^{b}/p^{y,1}\cdot q^{b}][p^{y+1,m}\cdot q^{b+1}/p^{y+1,1}\cdot q^{b+1}]\ ;$$

Now let $p^{y,r}$ and $p^{y+1,s}$ be the price vectors for month r in augmented year y and for month s in augmented year y+1. The "elementary" price index $P(p^{y,r},p^{y+1,s})$ that compares the month s prices in the vector $p^{y+1,s}$ for year y+1 relative to the month r prices in the vector $p^{y,r}$ for year is defined as the ratio of the price level in month s of year y+1, $P_{LoL}(y+1;s)$, relative to the price level in month r of year y, $P_{LoL}(y;r)$:

$$(7)\ P^{*}(p^{y,r},p^{y+1,s}) \equiv P_{LoL}(y+1;s)/P_{LoL}(y;r)$$
$$= [p^{y,13}\cdot q^{b}/p^{y,1}\cdot q^{b}][p^{y+1,s}\cdot q^{b+1}/p^{y+1,1}\cdot q^{b+1}]/[p^{y,r}\cdot q^{b}/p^{y,1}\cdot q^{b}] \text{ using (90) and (92)}$$
$$= [p^{y+1,s}\cdot q^{b+1}/p^{y+1,1}\cdot q^{b+1}]/[p^{y,r}\cdot q^{b}/p^{y,13}\cdot q^{b}].$$

Thus $P^{*}(p^{y,r},p^{y+1,s})$, which compares the level of prices in month s of year y+1 to the level of prices in month r of year y, is equal to the year y+1 Lowe index $p^{y+1,s}\cdot q^{b+1}/p^{y+1,1}\cdot q^{b+1}$ (which compares the prices in month s to month 0 (January) in year y+1) divided by the year y Lowe index $p^{y,r}\cdot q^{b}/p^{y,13}\cdot q^{b}$ (which compares the prices $p^{y,r}$ in month r in year y to the prices $p^{y,13}$ in January of year y+1, using the year y classification scheme).

It can be verified that the "elementary" index P^{*} defined by (7) satisfies the following tests listed in section 4 of Chapter 4: T1 (continuity), T3 (monotonicity in the components of $p^{y+1,s}$), T4 (monotonicity in the components of $p^{y,r}$), T5 (homogeneity of degree 1 in the components of $p^{y+1,s}$), T6 (homogeneity of degree −1 in the components of $p^{y,r}$), and T12 (commensurability). The following tests are not in general satisfied by P^{*}: T2 (identity), T7 (mean value test), T8 (symmetric treatment of outlets test), T9 (price bouncing test), T10 (time reversal test) and T11 (circularity). The failures of the identity test and the time reversal test are fairly serious.

[15] If the commodity classification remains the same in years y and y+1, then $p^{y,13}$ will equal $p^{y+1,1}$. However, typically, there will be small changes in the commodity classifications going from one year to the next and these changes may make $p^{y,13} \neq p^{y+1,1}$.

However, if we make some extra assumptions, the "elementary" index defined by (7) simplifies and satisfies more tests. The extra assumptions are listed in (8):

(8) $p^{y,13} = p^{y+1,1}$ and $q^b = q^{b+1.}$

If there are no changes in the commodity classification, then $p^{y,13}$ (the vector of January prices for year y+1 using the classification scheme of year y) will indeed be equal to $p^{y+1,1}$ (the vector of January prices for year t+1 using the classification scheme of year y+1) so the first equality in (8) will hold. If the reference annual quantity vector for year b, q^b, happens to remain unchanged for the following year b+1, then $q^b = q^{b+1}$ and the second equality in (8) will hold. Thus if conditions (8) hold, then (7) simplifies as follows:

(9) $P^*(p^{y,r},p^{y+1,s}) = p^{y+1,s}\cdot q^b/p^{y,r}\cdot q^b$.

The "elementary" index P^* defined by (9) will satisfy all of the tests T1-T12 with the exception of tests T8 and T9.

Generally speaking, $p^{y,13}$ will be close to $p^{y+1,1}$ so the first equality in (8) will be approximately true. The second equality in (8) will also be approximately true (household purchases of commodities do not change all that much going from one year to the next) but the degree of approximation will not be as close as the closeness of the two price vectors. In particular, if there are long run trends in prices, then there will be substitution effects that will cause q^{b+1} to systematically diverge from q^b.

From the economic perspective, the *Consumer Price Index Manual* showed that if there are long term trends in prices, then the Lowe index was likely to have some substitution bias;[16] however, for statistical agencies that update their weights every year, this substitution bias is likely to be small.

In order to provide the additional alternative characterizations of the Lowe index (within an augmented year), it is necessary to introduce *base year expenditures by commodity*, say e_n^b for n = 1,...,N, and *base year average prices by commodity*, say p_n^b for n = 1,...,N. Of course, average annual prices, annual quantities and annual expenditures should satisfy the following equations:

(10) $e_n^b = p_n^b q_n^b$; n = 1,...,N.

Now let $p^b \equiv [p_1^b,...,p_N^b]$ and rewrite the sequence of within the (augmented) year Lowe indexes defined by (2) as follows:

(11) $P_{Lo}(p^{y,1},p^{y,m},q^b) = p^{y,m}\cdot q^b/p^{y,1}\cdot q^b$; m = 1,2,...,13;

$$= [p^{y,m}\cdot q^b/p^b\cdot q^b]/[p^{y,1}\cdot q^b/p^b\cdot q^b]$$

$$= \sum_{n=1}^N s_n^b(p_n^{y,m}/p_n^b)/\sum_{n=1}^N s_n^b(p_n^{y,1}/p_n^b)$$

[16] See ILO (2004; 273).

where the *base year expenditure shares* s_n^b are defined as

$$(12)\ s_n^b \equiv p_n^b q_n^b / \sum_{i=1}^N p_i^b q_i^b = e_n^b / \sum_{i=1}^N e_i^b\ ; \qquad n = 1,...,N.$$

The Laspeyres index between two price vectors p^0 and p^1 can be defined as $P_L(p^0,p^1,q^0) \equiv p^1 \cdot q^0 / p^0 \cdot q^0$. Thus using (11), it can be seen that the Lowe index can be written as the ratio of the Laspeyres index $P_L(p^b,p^{y,m},q^b)$ that compares the prices of month m in year y, $p^{y,m}$, to the base year prices p^b and the Laspeyres index $P_L(p^b,p^{y,1},q^b)$ that compares the prices of month 1 in year y, $p^{y,1}$, to the base year prices p^b:[17]

$$\begin{aligned}(13)\ P_{Lo}(p^{y,1},p^{y,m},q^b) &= [p^t \cdot q^b / p^b \cdot q^b] / [p^0 \cdot q^b / p^b \cdot q^b] \qquad m = 1,2,...,13; \\ &= P_L(p^b,p^{y,m},q^b) / P_L(p^b,p^{y,1},q^b).\end{aligned}$$

It is useful to explain how the annual price and quantity vectors, p^b and q^b, can be obtained from monthly price and expenditure data on each commodity during the chosen base year b. Let $p_n^{b,m}$ be the *monthly (unit value) price for commodity n in month m of the base year b* and let $e_n^{b,m}$ be the corresponding *monthly expenditure for the reference population for commodity n in month m of the base year b* for n = 1,...,N and m = 1,2,...,12. The *annual total consumption for commodity n for base year b for the reference population*, q_n^b, can be obtained by deflating monthly values and summing over months in the base year b as follows:

$$(14)\ q_n^b \equiv \sum_{m=1}^{12} e_n^{b,m} / p_n^{b,m} = \sum_{m=1}^{12} q_n^{b,m}\ ; \qquad n = 1,\ldots,N$$

where $q_n^{b,m} \equiv e_n^{b,m} / p_n^{b,m}$ for n = 1,...,N and m = 1,...,12. In practice, the above equations will be evaluated using aggregate expenditures over closely related commodities and the price $p_n^{b,m}$ will be the month m price index for this elementary commodity group n in year b relative to the first month of year b.

Following national income accounting conventions, a reasonable[18] *annual price for commodity n for the base year b*, p_n^b, which matches up with the annual quantity q_n^b defined by (14) is the *value* of total consumption of commodity n in year b divided by q_n^b. Thus we have:

$$\begin{aligned}(15)\ p_n^b &\equiv \left(\sum_{m=1}^{12} e_n^{b,m}\right) / q_n^b && n = 1,\ldots,N \\ &= \sum_{m=1}^{12} e_n^{b,m} / \left[\sum_{m=1}^{12} e_n^{b,m} / p_n^{b,m}\right] && \text{using (14)} \\ &= \left[\sum_{m=1}^{12} s_n^{b,m} (p_n^{b,m})^{-1}\right]^{-1}\end{aligned}$$

[17] This formula for the Lowe index can be found in ILO (2004; 271).
[18] Hence these annual commodity prices are essentially unit value prices. Under conditions of high inflation, the annual prices defined by (15) may no longer be "reasonable" or representative of prices during the entire base year because the expenditures in the final months of the high inflation year will be somewhat artificially blown up by general inflation. Under these conditions, the annual prices and annual commodity expenditure shares should be interpreted with caution. For more on dealing with situations when there is high inflation within a year, see Hill (1996).

where the *share of annual expenditure on commodity n in month m of the base year* is

(16) $s_n^{b,m} \equiv e_n^{b,m} / \sum_{k=1}^{12} e_n^{b,k}$; $\quad$ n = 1,…,N ; m = 1,…,12.

Thus the annual base year price for commodity n, p_n^b, turns out to be a monthly expenditure weighted *harmonic mean* of the monthly prices for commodity n in the base year, $p_n^{b,1}$, $p_n^{b,2}$,…, $p_n^{b,12}$.

Once the base year prices p_n^b have been calculated, the hybrid shares s_n^{0b} defined by (3) can be calculated by multiplying the base year expenditures e_n^b by $(p_n^{y,1}/p_n^b)$, the ratio of the January price for commodity n in year y, $p_n^{y,1}$ to the base year price for commodity n, p_n^b. Thus the nth hybrid share can be written as follows:

(17) $s_n^{y,1,b} \equiv p_n^{y,1}q_n^b / \sum_{i=1}^N p_i^{y,1}q_i^b = (p_n^{y,1}/p_n^b)e_n^b/\sum_{k=1}^N (p_k^{y,1}/p_k^b)e_k^b$.

The operation of multiplying the base year expenditure weight for commodity n, e_n^b, by the corresponding nth price ratio, $p_n^{y,1}/p_n^b$, is known as *price updating* the base year expenditure weights. Substitution of (17) into the last line of (2) leads to our *third formula* [19] for the Lowe index:

(18) $P_{Lo}(p^{y,1},p^{y,m},p^b,e^b) = \sum_{n=1}^N (p_n^{y,m}/p_n^{y,1})(p_n^{y,1}/p_n^b)e_n^b/\sum_{k=1}^N (p_k^{y,1}/p_k^b)e_k^b$; m =1,2,...,13

where $e^b \equiv [e_1^b,...,e_N^b]$ is the vector of base year expenditure shares on the N commodities. The above formula shows how the Lowe indexes are functions of four sets of variables: $p^{y,1}$ (the month 1 price vector for year y),$p^{y,m}$ (the month m price vector for year y), p^b (the vector of annual commodity prices for the base year b) and e^b (the vector of annual household expenditures for the reference population in year b).[20]

Formula (18) for the Lowe index leads directly to our final formula for this index. Thus divide both numerator and denominator on the right hand side of (18) by total annual expenditures by the reference population in the base year b, $\sum_{n=1}^N e_n^b$, and using definitions (16) which define the annual expenditure shares s_n^b for the base year, it can be seen that the Lowe index can be written as the *ratio of two Young* (1812) *indexes*:

(19) $P_{Lo}(p^{y,1},p^{y,m},p^b,s^b) = \sum_{n=1}^N (p_n^{y,m}/p_n^b)s_n^b/\sum_{k=1}^N (p_k^{y,1}/p_k^b)s_k^b$; $\quad$ m =1,2,...,13
$\qquad = P_Y(p^b,p^{y,m},s^b)/P_Y(p^b,p^{y,1},s^b)$

where the *Young index* $P_Y(p^0,p^1,s)$ which compares the prices p^1 to the prices p^0 using the share weights s is defined as

[19] The first two formulae are (1) and (2).

[20] Actually, it can be seen that the Lowe index depends only on three vectors: the *within year y vector of relative prices* $[p_1^{y,m}/p_1^{y,1},p_2^{y,m}/p_2^{y,1},...,p_N^{y,m}/p_N^{y,1}]$ that compares the prices in month m of year y with the corresponding commodity prices of month 1 of year y; the *vector of relative prices* $[p_1^{y,1}/p_1^b,...,p_N^{y,1}/p_N^b]$ that compares the prices in month 1 of year y with the corresponding commodity prices of the base year and the *annual expenditures vector for the base year* b, $e^b \equiv [e_1^b,...,e_N^b]$.

$$(20)\ P_Y(p^0,p^1,s) \equiv \sum_{n=1}^N (p_n^{\,1}/p_n^{\,0})s_n.$$

Comparing the Young index defined by (20) with the Carli index defined in Chapter 4, it can be seen that the Carli index is a special case of the Young index when the weights s_n are all equal to 1/N. Unfortunately, the Young index has the same upward bias problem that made the Carli index an unattractive choice of elementary index; i.e., it can be shown that the Young index fails the time reversal test with the likelihood of an upward bias. In particular, the following inequality holds which is the counterpart to the same inequality for the Carli index:[21]

$$(21)\ P_Y(p^0,p^1,s)P_Y(p^1,p^0,s) \geq 1$$

where the strict inequality in (21) holds unless p^1 is proportional to p^0.

However, since the Lowe index is a ratio of Young indexes, there is no obvious bias in an index that is equal to a ratio of Young indexes, as in (21).

Having introduced the concept of a Young index, it is useful to contrast the following Young index, $P_Y(p^{y,1},p^{y,m},s^b)$, which compares the prices in month m of year y, $p^{y,m}$, to the prices of month 1 in year m, $p^{y,1}$, using the base year expenditure shares s^b as weights:

$$(22)\ P_Y(p^{y,1},p^{y,m},s^b) \equiv \sum_{n=1}^N (p_n^{\,y,m}/p_n^{\,y,1})\, s_n^{\,b}\ ; \qquad m = 1,2,...,13.$$

The above Young index can be compared to the representation of the corresponding Lowe index given by (2), $P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b})$, which compared the same monthly price vectors, $p^{y,1}$ and $p^{y,m}$ in year m, but used the hybrid expenditure shares $s^{y,1,b}$ defined by (3) as weights in place of the base year expenditure shares s^b:

$$(23)\ P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b}) \equiv \sum_{n=1}^N (p_n^{\,y,m}/p_n^{\,y,1})\, s_n^{\,y,1,b}\ ; \qquad m = 1,2,...,13.$$

The hybrid expenditure shares, $s_n^{\,y,1,b}$, can be regarded as price updated versions of the base year expenditure shares; i.e., starting with definitions (3), we have the following representation for the hybrid shares:

$$
\begin{aligned}
(24)\ s_n^{\,y,1,b} &\equiv p_n^{\,y,1}q_n^{\,b} / \textstyle\sum_{i=1}^N p_i^{\,y,1}q_i^{\,b} \qquad \text{for } n = 1,\ldots,N \\
&= (p_n^{\,y,1}/p_n^{\,b})p_n^{\,b}q_n^{\,b} / \textstyle\sum_{i=1}^N (p_i^{\,y,1}/p_i^{\,b})p_i^{\,b}q_i^{\,b} \\
&= (p_n^{\,y,1}/p_n^{\,b})e_n^{\,b} / \textstyle\sum_{i=1}^N (p_i^{\,y,1}/p_i^{\,b})e_i^{\,b} \\
&= (p_n^{\,y,1}/p_n^{\,b})s_n^{\,b} / \textstyle\sum_{i=1}^N (p_i^{\,y,1}/p_i^{\,b})s_i^{\,b}
\end{aligned}
$$

where the last equality follows by dividing numerator and denominator by the base year annual expenditures, $\sum_{n=1}^N e_n^{\,b}$, and using definitions (12) which define the base year expenditure shares, $s_n^{\,b}$.

[21] See the ILO (2004; 277).

At first glance, it would appear that the indexes defined by (22) and (23) should be numerically close. Obviously, if inflation is uniform across all commodity classes going from the base year to the base month 1 in year y, so that $p_n^{y,1} = \lambda p_n^b$ for n = 1,...,N for some scalar $\lambda > 0$, then $s_n^{y,1,b}$ will equal s_n^b for all n with the consequence that the Young index $P_Y(p^{y,1},p^{y,m},s^b)$ defined by (22) will equal the Lowe index defined by (23). However, this price proportionality assumption is unlikely to hold so we will develop a more general necessary and sufficient condition for the equality of the Lowe and Young indexes defined by (22) and (23).

In order to simplify the notation, define the *relative price* r_n *between months m and 1 for commodity n in year y* and the *relative price* t_n *between month 1 in year y for commodity n relative to the average price of commodity n in the base year b* as follows:

$$(25)\ r_n \equiv p_n^{y,m}/p_n^{y,1}\ ;\ t_n \equiv (p_n^{y,1}/p_n^b)\ ; \qquad n = 1,...,N.$$

Define r^* as the *share weighted average of the* r_n and t^* as the *share weighted average of the* t_n, where the base year expenditure shares, s_n^b, are used as weights as follows:

$$(26)\ r^* \equiv \sum_{n=1}^N s_n^b r_n = P_Y(p^{y,1},p^{y,m},s^b)\ ;\ t^* \equiv \sum_{n=1}^N s_n^b t_n = P_Y(p^b,p^{y,1},s^b).$$

Note that r^* is equal to the Young index $P_Y(p^{y,1},p^{y,m},s^b)$ defined by (22). It will also be useful to define the *weighted covariance between the relative price vectors* $r \equiv [r_1,...,r_N]$ and $t \equiv [t_1,...,t_N]$ using the base year shares s_n^b as weight as follows:

$$(27)\ \mathrm{Cov}(r,t,s^b) \equiv \sum_{n=1}^N (r_n - r^*)(t_n - t^*)s_n^b = \sum_{n=1}^N r_n t_n s_n^b - r^* t^*.$$

Substituting definitions (25) into (23) leads to the following expression for the Lowe index $P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b})$:

$$\begin{aligned}(28)\ P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b}) &\equiv \sum_{n=1}^N r_n s_n^{y,1,b}\ ; && m = 1,2,...,13\\ &= \sum_{n=1}^N r_n t_n s_n^b / \sum_{n=1}^N t_n s_n^b && \text{using (23) and (24)}\\ &= [\mathrm{Cov}(r,t,s^b) + r^* t^*]/t^* && \text{using (26) and (27)}\\ &= [\mathrm{Cov}(r,t,s^b)/t^*] + P_Y(p^{y,1},p^{y,m},s^b) && \text{using (26).}\end{aligned}$$

Thus we obtain the following simple relationship between the Young index $P_Y(p^{y,1},p^{y,m},s^b)$ which uses the base year shares s_n^b as weights for the relative prices $p_n^{y,m}/p_n^{y,1}$ and the corresponding Lowe index $P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b})$ which uses the hybrid share vector $s_n^{y,1,b}$ as weights:

$$(29)\ P_{Lo}(p^{y,1},p^{y,m},s^{y,1,b}) - P_Y(p^{y,1},p^{y,m},s^b) = \mathrm{Cov}(r,t,s^b)/t^* = \mathrm{Cov}(r,t,s^b)/P_Y(p^b,p^{y,1},s^b)$$

where the last equality follows using (26). Thus the difference between the Lowe index for month m in year y (that uses the price updated hybrid shares $s_n^{y,1,b}$ as weights for the price relatives $p_n^{y,m}/p_n^{y,1}$) and the corresponding Young index for month m in year y (that uses the base year expenditure shares s_n^b as weights for the price relatives $p_n^{y,m}/p_n^{y,1}$) is equal to the weighted covariance between the within year y price relatives $r_n \equiv p_n^{y,m}/p_n^{y,1}$

and the price relatives $t_n \equiv p_n^{y,1}/p_n^b$ between the base year and month 1 of year y, $Cov(r,t,s^b)$, divided by the Young index $P_Y(p^b,p^{y,1},s^b)$, which measures price inflation going from the base year b to month 1 of the current year y.

Since $P_Y(p^b,p^{y,1},s^b)$ will generally be a number which is slightly larger than 1, the key term which will explain the difference between the Lowe and Young indexes is the covariance, $Cov(r,t,s^b)$. If price change over all commodity groups proceeds smoothly with long run trends in most strata, then this covariance will be positive and the Lowe index will exceed the corresponding Young index. However, if there is mean reversion of prices (so that a relatively high average price p_n^b for commodity n in the base year is followed by relatively low monthly prices $p_n^{y,m}$ for this commodity in the current year y), then the covariance will be negative.[22] The situation is also complicated by the existence of seasonality in the monthly prices for year y; this seasonality could cause the covariance $Cov(r,t,s^b)$ to be either positive or negative.[23]

Our conclusion at this point is that the Lowe index is a reasonably satisfactory index concept for the construction of a practical consumer price index.[24] In particular, its axiomatic properties are satisfactory. However, these axioms do not deal adequately with seasonal baskets and so later in the following Chapter, we will suggest indexes that deal more adequately with seasonal commodities. If the statistical agency using the Lowe index methodology updates its annual expenditure weights on a continuous basis, the substitution bias that is inherent in a fixed basket index will be relatively low under normal conditions. However as noted above, there are some problems with the use of annual weights that are used in conjunction with monthly prices and these problems will be discussed in the following sections.

2. Problems with the Estimation of Annual Prices and Quantities for the Base Year

The algebra in the previous section implicitly assumed that the statistical agency collected price and expenditure data for every distinct product that is sold to households in the country over the course of a year. This is an oversimplification: expenditures are split up into strata and within each stratum, specific products within the strata are chosen to be priced. The underlying assumption is that the sampled specific product prices capture the trend for all products in the strata. Most statistical agencies collect specific item prices for say 300-1000 products but these collected prices are generally stratified

[22] This appears to be the case for computations based on some Israeli data on fresh fruits. Diewert, Finkel and Artsev (2009) computed Lowe and Young indexes for fresh fruits for the 72 months starting in January 1997 and extending through December 2002 and the sample means of the Lowe and Young indexes were 1.1220 and 1.1586 respectively.

[23] Our discussion of the use of Young indexes in the place of Lowe indexes is not irrelevant for some statistical agencies. At lower levels of aggregation, the some agencies use replication weights that are not price updated and so their Lowe type index is not a "pure" Lowe index but rather has some elements of Young indexes in their procedures; for example, see the ONS (2012; 38-41). The above algebra suggests that if $Cov(r,t,s^b)$ is small in magnitude for the stratum under consideration, then price updating the weights at lower levels of aggregation will not materially affect the overall index.

[24] It is reasonably satisfactory from the viewpoint of the test approach to index number theory but it is not satisfactory from the viewpoint of the economic approach; i.e., the Lowe index will be subject to a certain amount of substitution bias.

by location (say 5 to 50 locations) and by shop type (say 2-5 types). In the end, expenditure weights for 5000-10000 strata may be constructed.

At higher levels of aggregation (i.e., at the level of the 5000 or so strata for which there is expenditure weight information), the statistical agency may use the Lowe index as its target index concept. However, note that within each stratum for which expenditure weights are available, an elementary price index is used in place of true micro prices for each product in each stratum. The problems associated with the choice of elementary aggregate formula were reviewed in more detail in Chapter 4. In this section, we will examine more closely how the true micro prices are aggregated into elementary indexes that can be matched with expenditure information.

Thus a practical problem which we have not considered up to now is the fact that the Lowe indexes that statistical agencies calculate do not use the single stage of aggregation methodology which was used in the previous section. The prices p_n^b and $p_n^{y,m}$ which appear in the various formulae for the Lowe index in section 1 are actually elementary indexes for the N strata under consideration. Under what conditions will these various formulae for the Lowe index be equal to a true Lowe index? This is the question which we will now address.

The problems associated with reconciling two stage aggregation with single stage aggregation can be illustrated if the overall index consists of only two strata. Some new notation needs to be introduced. Let p_{nk}^b, q_{nk}^b and e_{nk}^b denote the base year prices, quantities and expenditures for the kth item in stratum n where n = 1,2 and k = 1,2,...,K(n). Using the same commodity classification, let $p_{nk}^{y,m}$, $q_{nk}^{y,m}$ and $e_{nk}^{y,m}$ denote the (augmented) year y and month m prices, quantities and expenditures for the kth item in stratum n where n = 1,2; k = 1,2,...,K(n) and m = 1,2,...,13. Thus there are K(1) separate items in the first stratum and K(2) items in the second stratum. Then the *true Lowe index for month m in the augmented year y*, constructed in a single stage, is defined as follows:

(30) $P_{Lo}(p_1^b,p_2^b,p_1^{y,1},p_2^{y,1},p_1^{y,m},p_2^{y,m},s_1^b.s_2^b,e_1^b,e_2^b)$ m= 1,2,...,13

$$\equiv [\textstyle\sum_{k=1}^{K(1)} p_{1k}^{y,m}q_{1k}^b + \sum_{k=1}^{K(2)} p_{2k}^{y,m}q_{2k}^b]/[\sum_{k=1}^{K(1)} p_{1k}^{y,1}q_{1k}^b + \sum_{k=1}^{K(2)} p_{2k}^{y,1}q_{2k}^b]$$

$$= \frac{\sum_{k=1}^{K(1)} (p_{1k}^{y,m}/p_{1k}^{y,1})(p_{1k}^{y,1}/p_{1k}^b)s_{1k}^b e_1^b + \sum_{k=1}^{K(2)} (p_{2k}^{y,m}/p_{2k}^{y,1})(p_{2k}^{y,1}/p_{2k}^b)s_{2k}^{y,1} e_2^b}{\sum_{k=1}^{K(1)} (p_{1k}^{y,1}/p_{1k}^b)s_{1k}^b e_1^b + \sum_{k=1}^{K(2)} (p_{2k}^{y,1}/p_{2k}^b)s_{2k}^b e_2^b}$$

where $p_1^b \equiv [p_{11}^b,p_{12}^b,...,p_{1K(1)}^b]$ and $p_2^b \equiv [p_{21}^b,p_{22}^b,...,p_{2K(2)}^b]$ are the base year price vectors for strata 1 and 2; $p_n^{y,m} \equiv [p_{n1}^{y,m},p_{n2}^{y,m},...,p_{nK(n)}^{y,m}]$ is the augmented year y, month m vector of prices for stratum n for n = 1,2 and m = 1,2,...,13; $s_n^b \equiv [s_{n1}^b,s_{n2}^b,...,s_{nK(n)}^b]$ is the vector of base year expenditure shares for stratum n for n = 1,2 and finally, e_1^b and e_2^b are total expenditures on strata 1 and 2 respectively in the base year. The second expression for the true Lowe index in equations (30) looks rather formidable but it will prove to be useful below.

Now suppose that the statistical agency has constructed *elementary indexes* for the two strata. For stratum n, suppose that the elementary index level for the base year is P_n^b for n = 1,2. The corresponding (approximate) *base year quantities* for the two strata, Q_n^b, are defined as follows:

$$(31)\ Q_n^b \equiv e_n^b/P_n^b\ ; \qquad n = 1,2.$$

We further suppose that the elementary index levels for stratum n for month m in the augmented year y are defined by $P_n^{y,m}$ for n = 1,2 and m = 1,2,...,13.

The following *two stage (approximate) Lowe index*, P_{LoA}, constructed using the elementary indexes and the approximate base year quantities defined by (31), is defined as follows:

$$\begin{aligned}(32)\ P_{LoA}(P^b,P^{y,1},P^{y,m},e^b) &\equiv [P_1^{y,m}Q_1^b + P_2^{y,m}Q_2^b]/[P_1^{y,1}Q_1^b + P_2^{y,1}Q_2^b]\ ; && m = 1,2,...,13 \\ &= [(P_1^{y,m}/P_1^b)e_1^b + (P_2^{y,m}/P_2^b)e_2^b]/[(P_1^{y,1}/P_1^b)e_1^b + [(P_2^{y,1}/P_1^b)e_2^b] && \text{using (31)} \\ &= [(P_1^{y,m}/P_1^{y,1})(P_1^{y,1}/P_1^b)e_1^b + (P_2^{y,m}/P_2^{y,1})(P_2^{y,1}/P_2^b)e_2^b]/[(P_1^{y,1}/P_1^b)e_1^b + [(P_2^{y,1}/P_1^b)e_2^b].\end{aligned}$$

Under what conditions will the approximate Lowe index P_{LoA} defined by (32) be equal to the true Lowe index P_{Lo} defined by (30)? An intuitively appealing set of sufficient conditions for equality are:

- Within each stratum, prices move in a proportional manner and
- The elementary indexes capture these proportional movements in prices.

The price proportionality assumptions for each stratum can formally be represented by the following equations; there exist positive constants α_n^m such that prices within the augmented year y satisfy the following equations:[25]

$$(33)\ p_{nk}^{y,m} = \alpha_n^m p_{nk}^{y,1}; \qquad n = 1,2\ ;\ k = 1,2,...,K(n)\ ;\ m = 1,2,...,13.$$

We will also require that prices within a stratum move in a proportional manner going from the base year b to the first month of year y; i.e., there exist positive constants β_n such that the month 1, year y price vector for stratum n, $p_n^{y,1}$ is proportional to the corresponding base year stratum price vector p_n^b; i.e., we have the existence of positive constants β_1 and β_2 such that the following equations are satisfied:

$$(34)\ p_{nk}^{y,1} = \beta_n p_{nk}^b; \qquad n = 1,2\ ;\ k = 1,2,...,K(n).$$

Finally, the assumption that the elementary indexes capture the same trends in prices that are present in each stratum can be represented algebraically by the following assumptions:

$$(35)\ P_n^{y,m}/P_n^{y,1} = \alpha_n^m; \qquad n = 1,2\ ;\ m = 1,2,...,13;$$

---

[25] When m = 1, equations (33) are automatically satisfied.

(36) $P_n^{y,1}/P_n^{b} = \beta_n$; n = 1,2.

If we substitute assumptions (35) and (36) into the approximate Lowe index defined by (32), we find that:

(37) $P_{LoA} = [\alpha_1^{m}\,\beta_1 e_1^{b} + \alpha_2^{m}\,\beta_2 e_2^{b}]/[\beta_1 e_1^{b} + \beta_2 e_2^{b}]$; m = 1,2,,,,,13.

If we substitute assumptions (33) and (34) into the true Lowe index defined by (30), we find that

(38) $P_{Lo}(p_1^{b},p_2^{b},p_1^{y,1},p_2^{y,1},p_1^{y,m},p_2^{y,m},s_1^{b}.s_2^{b},e_1^{b},e_2^{b})$ m= 1,2,...,13
$= [\sum_{k=1}^{K(1)} \alpha_1^{m}\beta_1 s_{1k}^{b}e_1^{b} + \sum_{k=1}^{K(2)} \alpha_2^{m}\beta_2 s_{2k}^{b}e_2^{b}]/[\sum_{k=1}^{K(1)} \beta_1 s_{1k}^{b}e_1^{b} + \sum_{k=1}^{K(2)} \beta_2 s_{2k}^{b}e_2^{b}]$
$= [\alpha_1^{m}\,\beta_1 e_1^{b} + \alpha_2^{m}\,\beta_2 e_2^{b}]/[\beta_1 e_1^{b} + \beta_2 e_2^{b}]$ since $\sum_{k=1}^{K(n)} s_{nk}^{b} = 1$ for each n
$= P_{LoA}(P^{b},P^{y,1},P^{y,m},e^{b})$ using (37).

Thus if prices within each stratum vary proportionally over time and the elementary indexes capture these proportional movements in prices, then the approximate Lowe index that is constructed in two stages using the elementary indexes in the first stage will be equal to the true Lowe index. While the assumptions underlying this result are not likely to hold in practice, they may be approximately true and so the Lowe type indexes constructed by statistical agencies will approximate true Lowe indexes under these conditions.

3. Problems Associated with the Use of Annual Baskets in a Monthly Index

We conclude our discussion of Lowe indexes with some problematic aspects of Lowe indexes that use annual baskets in the context of producing monthly price indexes.

It should be noted that the problems associated with the Lowe index that uses an annual basket in the context of a monthly price index have been noted in the literature on price indexes when there are seasonal commodities. In the context of seasonal price indexes, the Lowe index is known as the Bean and Stine (1924; 31) Type A index or an Annual Basket (AB) index. The price statistician Andrew Baldwin's comments on this type of index are worth quoting at length:[26]

"For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961; 256-257) calls it an index of 'a hybrid sort'. Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is 'What would have the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices

[26] Balk (1980; 68c) also clearly pointed out the problems associated with using annual weights and monthly prices in the context of seasonal commodities.

nonetheless retained their own seasonal behaviour?' It is hard to believe that this is a question that anyone would be interested in asking." Andrew Baldwin (1990; 258).

Basically, the problem is that households do not purchase (fractions) of their annual basket of purchases for each month of the given year; i.e., there is a *seasonal pattern* to their purchases. Thus for each month of the year, there will be an appropriate *monthly basket* that is relevant for index number construction rather than an annual basket. The problem of seasonal commodities in an annual basket index becomes apparent for *strongly seasonal commodities*; these are commodities that are present in some months of the year but not all months.

Are there solutions to the index number problems generated by seasonal commodities? If not, the Lowe index with some suitable modifications may still be the best index that can be produced under the circumstances. Thus in the following Chapter, we will review where the current state of theory is with respect to producing monthly price indexes when seasonality is present.

References

Baldwin, A. (1990), "Seasonal Baskets in Consumer Price Indexes", *Journal of Official Statistics* 6:3, 251-273.

Bean, L. H. and O. C. Stine (1924), "Four Types of Index Numbers of Farm Prices", *Journal of the American Statistical Association* 19, 30-35.

Diewert, W.E. (2002), "Harmonized Indexes of Consumer Prices: Their Conceptual Foundations", *Swiss Journal of Economics and Statistics* 138:4, 547-637.

Diewert, W.E. (2012), *Consumer Price Statistics in the UK*, Government Buildings, Cardiff Road, Newport, UK, NP10 8XG: Office for National Statistics. http://www.ons.gov.uk/ons/guide-method/userguidance/prices/cpi-and-rpi/index.html

Diewert, W.E., Y. Finkel and Y. Artsev (2009), "Empirical Evidence on the Treatment of Seasonal Products: The Israeli Experience", pp. 53-78 in *Price and Productivity Measurement: Volume 2: Seasonality*, W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (eds.), Victoria, Canada: Trafford Press.

Fisher, I. (1922), *The Making of Index Numbers*, Boston: Houghton-Mifflin.

Hill, T.P. (1996), *Inflation Accounting*, Paris: OECD.

ILO/IMF/OECD/UNECE/Eurostat/World Bank (2004), *Consumer Price Index Manual: Theory and Practice*, Peter Hill (ed.), Geneva: International Labour Office.

Lowe, J. (1823), *The Present State of England in Regard to Agriculture, Trade and Finance*, second edition, London: Longman, Hurst, Rees, Orme and Brown.

Office for National Statistics (ONS) (2012), *Consumer Price Indices Technical Manual*, Newport UK: Office for National Statistics.

Walsh, C.M. (1932), "Index Numbers", pp. 652-658 in *Encyclopedia of the Social Sciences*, Volume 7, E. R. A. Seligman (editor in chief), New York : The Macmillan Co.

Young, A. (1812), *An Inquiry into the Progressive Value of Money in England as Marked by the Price of Agricultural Products*, London.

Zarnowitz, V. (1961), "Index Numbers and the Seasonality of Quantities and Prices", pp. 233-304 in *The Price Statistics of the Federal Government*, G.J. Stigler (Chairman), New York: National Bureau of Economic Research.