INDEX NUMBER THEORY AND MEASUREMENT ECONOMICS

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CHAPTER 12: The Treatment of Owner Occupied Housing and Other Durables in a Consumer Price Index[1]

1. Introduction

When a durable good (other than housing) is purchased by a consumer, national Consumer Price Indexes typically attribute *all* of that expenditure to the period of purchase, even though the use of the good extends beyond the period of purchase.[2] This is known as the *acquisitions approach* to the treatment of consumer durables in the context of determining a pricing concept for the CPI. However, if one takes a *cost of living* approach to the Consumer Price Index, then it may be more appropriate to take the cost of *using* the services of the durable good during the period under consideration as the pricing concept. There are two broad methods for estimating this imputed cost for using the services of a durable good during a period:

- If rental or leasing markets for a comparable consumer durable exist, then this market rental price could be used as an estimate for the cost of using the durable during the period. This method is known as the *rental equivalence approach*.
- If used or second hand markets for the durable exist, then the imputed cost of purchasing a durable good at the beginning of the period and selling it at the end could be computed and this net cost could be used as a estimate for the cost of using the durable during the period. This method is known as the *user cost approach*.

The major *advantages* of the acquisitions approach to the treatment of consumer durables are:

- It is conceptually simple and entirely similar to the treatment of nondurables and services and
- No complex imputations are required.

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[2] This treatment of the purchases of durable goods dates back to Alfred Marshall (1898; 594-595) at least: "We have noticed also that though the benefits which a man derives from living in his own house are commonly reckoned as part of his real income, and estimated at the net rental value of his house; the same plan is not followed with regard to the benefits which he derives from the use of his furniture and clothes. It is best here to follow the common practice, and not count as part of the national income or dividend anything that is not commonly counted as part of the income of the individual."

The major *disadvantage* of the acquisitions approach compared to the other two approaches is that the acquisitions approach is not likely to reflect accurately the consumption services of consumer durables in any period. Thus suppose that real interest rates in a country become very high due to some sort of macroeconomic crisis. Under these conditions, typically purchases of automobiles and houses and other long lived consumer durables drop dramatically, perhaps to zero. However, the actual consumption of automobile and housing services of the country's population will not fall to zero under these circumstances: consumers will still be consuming the services of their existing stocks of autos and houses. Thus for at least some purposes, rather than taking the cost of *purchasing* a consumer durable as the pricing concept, it will be more useful to take the cost of *using* the services of the durable good during the period under consideration as the pricing concept.

The above paragraphs provide a brief overview of the three major approaches to the treatment of consumer durables. In the remainder of this introduction, we explore these approaches in a bit more detail and give the reader an outline of the detailed discussion that will follow in subsequent sections.

We first consider a formal definition of a consumer durable. By definition, a durable good delivers services longer than the period under consideration.[3] The *System of National Accounts 1993* defines a *durable good* as follows:

"In the case of goods, the distinction between acquisition and use is analytically important. It underlies the distinction between durable and non-durable goods extensively used in economic analysis. In fact, the distinction between durable and non-durable goods is not based on physical durability as such. Instead, the distinction is based on whether the goods can be used once only for purposes of production or consumption or whether they can be used repeatedly, or continuously. For example, coal is a highly durable good in a physical sense, but it can be burnt only once. A durable good is therefore defined as one which may be used repeatedly or continuously over a period of more than a year, assuming a normal or average rate of physical usage. A consumer durable is a good that may be used for purposes of consumption repeatedly or continuously over a period of a year or more." *System of National Accounts 1993*, (1993; 208).

According to the above National Accounts definition, durability is more than the fact that a good can physically persist for more than a year (this is true of most goods): a durable good is distinguished from a nondurable good due to its property that it can deliver useful services to a consumer through repeated use over an extended period of time.

Since the benefits of using the consumer durable extend over more than one period, it does not seem to be appropriate to charge the entire purchase cost of the durable to the initial period of purchase. If this point of view is taken, then the initial purchase cost must be distributed somehow over the useful life of the asset. This is *a fundamental problem of accounting*.[4] Hulten (1990) explains the consequences for accountants of the durability of a purchase as follows:

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[3] An alternative definition of a durable good is that the good delivers services to its purchaser for a period exceeding three years: "The Bureau of Economic Analysis defines consumer durables as those durables that have an average life of at least 3 years." Arnold J. Katz (1983; 422).

[4] "The third convention is that of the annual accounting period. It is this convention which is responsible for most of the difficult accounting problems. Without this convention, accounting would be a simple

"Durability means that a capital good is productive for two or more time periods, and this, in turn, implies that a distinction must be made between the value of using or renting capital in any year and the value of owning the capital asset. This distinction would not necessarily lead to a measurement problem if the capital services used in any given year were paid for in that year; that is, if all capital were rented. In this case, transactions in the rental market would fix the price and quantity of capital in each time period, much as data on the price and quantity of labor services are derived from labor market transactions. But, unfortunately, much capital is utilized by its owner and the transfer of capital services between owner and user results in an implicit rent typically not observed by the statistician. Market data are thus inadequate for the task of directly estimating the price and quantity of capital services, and this has led to the development of indirect procedures for inferring the quantity of capital, like the perpetual inventory method, or to the acceptance of flawed measures, like book value." Charles R. Hulten (1990; 120-121).

Thus the treatment of durable goods is more complicated than the treatment of nondurable goods and services due to the simple fact that the period of time that a durable is used by the consumer extends beyond the period of purchase. For nondurables and services, the price statistician's measurement problems are conceptually simple: prices for the same commodity need only be collected in each period and compared. However, for a durable good, the periods of payment and use do not coincide and so complex imputation problems arise if the goal of the price statistician is to measure and compare the price of *using* the services of the durable in two time periods.

As mentioned above, there are 3 main methods for dealing with the durability problem:

- Ignore the problem of distributing the initial cost of the durable over the useful life of the good and allocate the entire charge to the period of purchase. As noted above, this is known as the *acquisitions approach* and it is the present approach used by Consumer Price Index statisticians for all durables with the exception of housing.
- The *rental equivalence approach*. In this approach, a period price is imputed for the durable which is equal to the rental price or leasing price of an equivalent consumer durable for the same period of time.
- The *user cost approach.* In this approach, the initial purchase cost of the durable is decomposed into two parts: one part which reflects an estimated cost of using the services of the durable for the period and another part, which is regarded as an investment, which must earn some exogenous rate of return.

These three major approaches will be discussed more fully in sections 2, 3 and 4 below. However, there is a fourth approach to the treatment of consumer durables that has only been used in the context of pricing owner occupied housing and that is the *payments approach* [5]. This is a kind of a cash flow approach, which is not entirely satisfactory. It

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matter of recording completed and fully realized transactions: an act of primitive simplicity." Stephen Gilman (1939; 26).

"*All* the problems of income measurement are the result of our desire to attribute income to arbitrarily determined short periods of time. Everything comes right in the end; but by then it is too late to matter." David Solomons (1961; 378). Note that these authors do not mention the additional complications that are due to the fact that future revenues and costs must be discounted to yield values that are equivalent to present dollars.

[5] This is the term used by Goodhart (2001; F350-F351).

will be briefly discussed in section 12 after we have discussed the treatment of owner occupied housing in more detail.

The above three approaches to the treatment of durable purchases can be applied to the purchase of any durable commodity. However, historically, it turns out that the rental equivalence and user cost approaches have *only* been applied to owner occupied housing. In other words, the acquisitions approach to the purchase of consumer durables has been universally used by statistical agencies, with the exception of owner occupied housing. A possible reason for this is tradition; i.e., Marshall set the standard and statisticians have followed his example for the past century. However, another possible reason is that unless the durable good has a very long useful life, it usually will not make a great deal of difference in the long run whether the acquisitions approach or one of the two alternative approaches is used. This point is discussed in more detail in section 5 below.

A major component of the user cost approach to valuing the services of owner occupied housing is the depreciation component. In section 6, a general model of depreciation for a consumer durable is presented and then it is specialized to the three most common models of depreciation that are in use. The models presented in section 6 assume that homogeneous units of the durable are produced in each period so that information on the prices of the various vintages of the durable at any point in time can be used to determine the pattern of depreciation. However, many durables (like housing) are custom produced and thus the methods for determining the form of depreciation explained in section 6 are not applicable. The special problems caused by these uniquely produced consumer durables are considered in section 7.

Sections 8, 9, 10 and 11 treat some of the special problems involved in implementing the user cost and rental equivalence methods for valuing the services provided by Owner Occupied Housing (OOH). Section 8 presents a derivation for the user cost of OOH and various approximations to it. Section 9 looks at some of the problems associated with obtaining constant quality prices for housing. Section 10 considers some of the costs that are tied to home ownership while section 11 considers how a landlord's costs might differ from a homeowner's costs. This material is relevant if the rental equivalence approach to valuing the services of OOH is used: care must be taken to remove some costs that are imbedded in market rents that homeowners do not face.

Section 13 tries to bring together all of the material on the problems associated with pricing Owner Occupied Housing and to outline possible CPI measurement strategies.

2. The Acquisitions Approach

The *net acquisitions approach* to the treatment of owner occupied housing is described by Goodhart as follows:

"The first is the net acquisition approach, which is the change in the price of newly purchased owner occupied dwellings, weighted by the net purchases of the reference population. This is an asset based measure, and therefore comes close to my preferred measure of inflation as a change in the value of money, though the change in the price of the stock of existing houses rather than just of net purchases would in some respects be even better. It is, moreover, consistent with the treatment of other durables. A few countries, e.g., Australia and New Zealand, have used it, and it is, I understand, the main contender for use in the Euro-area Harmonized Index of Consumer Prices (HICP), which currently excludes any measure of the purchase price of (new) housing, though it does include minor repairs and maintenance by home owners, as well as all expenditures by tenants." Charles Goodhart (2001; F350).

Thus the weights for the net acquisitions approach are the net purchases of the household sector of houses from other institutional sectors in the base period. Note that in principle, purchases of second-hand dwellings from other sectors are relevant here; e.g., a local government may sell rental dwellings to owner occupiers. However, typically, newly built houses form a major part of these types of transactions. Thus the long term price relative for this category of expenditure will be primarily the price of (new) houses (quality adjusted) in the current period relative to the price of new houses in the base period.[6] If this approach is applied to other consumer durables, it is extremely easy to implement: the purchase of a durable is treated in the same way as a nondurable or service purchase is treated.

One additional implication of the net acquisition approach is that major renovations and additions to owner occupied dwelling units could also be considered as being in scope for this approach. In practice, these costs typically are not covered in a standard consumer price index. The treatment of renovations and additions will be considered in more detail in section 10.4 below.

Traditionally, the net acquisitions approach also includes transfer costs relating to the buying and selling of second hand houses as expenditures that are in scope for an acquisitions type consumer price index. These costs are mainly the costs of using a real estate agent's services and asset transfer taxes. These transfer costs will be further discussed in sections 10.2 and 10.5 below.

The major advantage of the acquisitions approach is that it treats durable and nondurable purchases in a completely symmetric manner and thus no special procedures have to be developed by a statistical agency to deal with durable goods. As will be seen in section 5 below, the major disadvantage of this approach is that the *expenditures* associated with this approach will tend to *understate* the corresponding expenditures on durables that are implied by the rental equivalence and user cost approaches.

Some differences between the acquisitions approach and the other approaches are:

- If rental or leasing markets for the durable exist and the durable has a long useful life, then the expenditure weights implied by the rental equivalence or user cost approaches will typically be much larger than the corresponding

[6] This price index may or may not include the price of the land that the new dwelling unit sits on; e.g., a new house price construction index would typically not include the land cost. The acquisitions approach concentrates on the purchases by households of goods and services that are provided by suppliers from outside the household sector. Thus if the land on which a new house sits was previously owned by the household sector, then presumably, the cost of this land would be excluded from an acquisitions type new house price index.

expenditure weights implied by the acquisitions approach; see Section 5 below.

- If the base year corresponds to a boom year (or a slump year) for the durable, then the base period expenditure weights may be too large or too small. Put another way, the aggregate expenditures that correspond to the acquisitions approach are likely to be more volatile than the expenditures for the aggregate that are implied by the rental equivalence or user cost approaches.
- In making comparisons of consumption across countries where the proportion of owning versus renting or leasing the durable varies greatly,[7] the use of the acquisitions approach may lead to misleading cross country comparisons. The reason for this is that opportunity costs of capital are excluded in the net acquisitions approach whereas they are explicitly or implicitly included in the other two approaches.

More fundamentally, whether the acquisitions approach is the right one or not depends on the overall purpose of the index number. If the purpose is to measure the price of current period *consumption services*, then the acquisitions approach can only be regarded as an approximation to a more appropriate approach (which would be either the rental equivalence or user cost approach). If the purpose of the index is to measure *monetary* (or nonimputed) *expenditures* by households during the period, then the acquisitions approach is preferable, since the rental equivalence and user cost approaches necessarily involve imputations.

3. The Rental Equivalence Approach

The *rental equivalence approach* simply values the services yielded by the use of a consumer durable good for a period by the corresponding market rental value for the same durable for the same period of time (if such a rental value exists). This is the approach taken in the *System of National Accounts: 1993* for owner occupied housing:

"As well-organized markets for rented housing exist in most countries, the output of own-account housing services can be valued using the prices of the same kinds of services sold on the market with the general valuation rules adopted for goods and services produced on own account. In other words, the output of housing services produced by owner-occupiers is valued at the estimated rental that a tenant would pay for the same accommodation, taking into account factors such as location, neighbourhood amenities, etc. as well as the size and quality of the dwelling itself." Eurostat and others (1993; 134).

However, the *System of National Accounts: 1993* follows Marshall (1898; 595) and does *not* extend the rental equivalence approach to consumer durables other than housing. This seemingly inconsistent treatment of durables is explained in the *SNA 1993* as follows:

"The production of housing services for their own final consumption by owner-occupiers has always been included within the production boundary in national accounts, although it constitutes an exception to the general exclusion of own-account service production. The ratio of owner-occupied to rented dwellings can vary significantly between countries and even over short periods of time within a single country, so that

[7] From Hoffmann and Kurz (2002; 3-4), about 60% of German households live in rented dwellings whereas only about 11% of Spaniards rent their dwellings in 1999 (private communication).

both international and intertemporal comparisons of the production and consumption of housing services could be distorted if no imputation were made for the value of own-account services." Eurostat and others (1993; 126).

Eurostat's (2001) *Handbook on Price and Volume Measures in National Accounts* also recommends the rental equivalence approach for the treatment of the dwelling services for owner occupied housing:

"The output of dwelling services of owner occupiers at current prices is in many countries estimated by linking the actual rents paid by those renting similar properties in the rented sector to those of owner occupiers. This allows the imputation of a notional rent for the service owner occupiers receive from their property." Eurostat (2001; 99).

The US statistical agencies, the Bureau of Labor Statistics and the Bureau of Economic Analysis, both use the rental equivalence approach to value the services of owner occupied housing. Katz describes the BEA procedures as follows:

"Basically, BEA measures the gross rent (space rent) of owner occupied housing from data on the rent paid for similar housing with the same market value. To get the service value that is added to GNP (gross housing product), the value of intermediate goods and services included in this figure (e. g., expenditures for repair and maintenance, insurance, condominium fees, and closing costs) are subtracted from the space rent. To obtain a net return (net rental income), depreciation, taxes, and net interest are subtracted from, and subsidies added to, the service value." Arnold J. Katz (1983; 411).

There are some problems with the above treatment of housing and they will be discussed in later sections after the user cost approach to durables has been discussed.[8]

To summarize the above material, it can be seen that the rental equivalence approach to the treatment of durables is conceptually simple: impute a current period rental or leasing price for a comparable product as the price for the purchase of a unit of a consumer durable. For existing stocks of used consumer durables, the rental equivalence approach would entail finding rental prices for comparable used units.[9] To date, as noted above, statistical agencies have not done this, with the single exception of owner occupied housing. However, note that in order to implement the rental equivalence approach, it is necessary that the relevant rental or leasing markets exist and often this will not be the case, particularly when it is recognized that vintage specific rental prices may be required for all vintages of the durable held by households.[10]

[8] To anticipate the later results: the main problem is that the rental equivalence approach to valuing the services of owner occupied housing may give a higher valuation for these services than the user cost approach.

[9] Another method for determining rental price equivalents for stocks of consumer durables is to *ask* households what they think their durables would rent for. This approach is used by the Bureau of Labor Statistics in order to determine expenditure weights for owner occupied housing; i.e., homeowners are asked to estimate what their house would rent for if it were rented to a third party; see the Bureau of Labor Statistics (1983). Lebow and Rudd (2003; 169) note that these consumer expenditure survey based estimates of imputed rents in the US differ considerably from the corresponding Bureau of Economic Analysis estimates for imputed rents, which are based on applying a rent to value ratio for rented properties to the owner occupied stock of housing. Lebow and Rudd feel that the expenditure survey estimates may be less reliable than ratio of rent to value method due to the relatively small size of the consumer expenditure survey plus the difficulties households may have in recalling or estimating expenditures.

[10] However, if the form of depreciation is of the one hoss shay or light bulb type, then the rental price for the durable will be the same for all vintages and hence a detailed knowledge of market rentals by vintage

4. The User Cost Approach

The user cost approach to the treatment of durable goods is in some ways very simple: it calculates the cost of purchasing the durable at the beginning of the period, using the services of the durable during the period and then netting off from these costs the benefit that could be obtained by selling the durable at the end of the period. However, there are several details of this procedure that are somewhat controversial. These details involve the use of opportunity costs, which are usually imputed costs, the treatment of interest and the treatment of capital gains or holding gains.

Another complication with the user cost approach is that it involves making distinctions between current period (flow) purchases within the period under consideration and the holdings of physical stocks of the durable at the beginning and the end of the accounting period. Up to this point, all prices and quantity purchases were thought of as taking place at a single point in time, say the middle of the period under consideration, and consumption was thought of as taking place within the period as well. Thus, there was no need to consider the behavior (and valuation) of stocks of consumer durables that households may have a their disposal. The rather complex problems involved in accounting for stocks and flows are unfamiliar to most price statisticians.

To determine the net cost of using the durable good during say period 0, assume that one unit of the durable good is purchased at the beginning of period 0 at the price P^0. The "used" or "second-hand" durable good can be sold at the end of period 0 at the price P_S^1. It might seem that a reasonable net cost for the use of one unit of the consumer durable during period 0 is its initial purchase price P^0 less its end of period 0 "scrap value" P_S^1. However, *money received at the end of the period is not as valuable as money that is received at the beginning of the period*. Thus in order to convert the end of period value into its beginning of the period equivalent value, it is necessary to *discount* the term P_S^1 by the term $1+r^0$ where r^0 is the beginning of period 0 nominal interest rate that the consumer faces. Hence the *period 0 user cost* u^0 for the consumer durable[11] is defined as

$$(1)\ u^0 \equiv P^0 - P_S^1/(1+r^0)\ .$$

There is another way to view the user cost formula (1): the consumer purchases the durable at the beginning of period 0 at the price P^0 and charges himself or herself the rental price u^0. The remainder of the purchase price, I^0, defined as

$$(2)\ I^0 \equiv P^0 - u^0$$

will not be required. The light bulb model of depreciation dates back to Böhm-Bawerk (1891; 342). For more recent material on this model, see section 6.4 below or Hulten (1990) or Diewert (2003b).

[11] This approach to the derivation of a user cost formula was used by Diewert (1974b) who in turn based it on an approach due to Hicks (1946; 326).

can be regarded as an *investment*, which is to yield the appropriate opportunity cost of capital r^0 that the consumer faces. At the end of period 0, this rate of return could be realized provided that I^0, r^0 and the selling price of the durable at the end of the period P_S^1 satisfy the following equation:

$$(3)\ I^0(1+r^0) = P_S^1 .$$

Given P_S^1 and r^0, (3) determines I^0, which in turn, given P^0, determines the user cost u^0 via (2)[12].

It should be noted that some price statisticians object to the user cost concept as a valid pricing concept for a Consumer Price Index:

"A suitable price concept for a CPI ought to reflect only a ratio of exchange of money for other things, not a ratio at which money in one form or time period can be traded for money in another form or time period. The ratio at which money today can be traded for money tomorrow by paying an interest rate or by enjoying actual or expected holding gains on an appreciating asset has no part in a measure of the current purchasing power of money." Marshall Reinsdorf (2003).

Thus user costs are not like the prices of nondurables or services because the user cost concept involves pricing the durable at *two* points in time rather than at a single point in time.[13] Because the user cost concept involves prices at two points in time, money received or paid out at the first point in time is more valuable than money paid out or received at the second point in time and so *interest rates* creep into the user cost formula. Furthermore, because the user cost concept involves prices at two points in time, *expected prices* can be involved if the user cost is calculated at the beginning of the period under consideration instead of at the end. With all of these complications, it is no wonder that many price statisticians would like to avoid the using user costs as a pricing concept. However, even for price statisticians who would prefer to use the rental equivalence approach to the treatment of durables over the user cost approach, there is some justification for considering the user cost approach in some detail, since this approach gives insights into the economic determinants of the rental or leasing price of a durable. As will be seen in section 11 below, the user cost for a house can differ substantially for a landlord compared to an owner and thus adjustments should be made to market rents for dwelling units if these observed rents are to be used as imputations for owner occupied rents.

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[12] This derivation for the user cost of a consumer durable was also made by Diewert (1974b; 504).

[13] Woolford also suggested that interest should be excluded from an ideal price index that measured inflation. In his view, interest is not a *contemporaneous price*; i.e., an interest rate necessarily refers to *two* points in time; a beginning point when the capital is loaned and an ending point when the capital loaned must be repaid. Thus if one wanted to restrict attention to a domain of definition that consisted of only contemporaneous prices, interest rates would be excluded. Woolford (1999; 535) noted that his ideal inflation measure "would be contemporary in nature, capturing only the current trend in prices associated with transactions in goods and services. It would exclude interest rates on the ground that they are intertemporal prices, representing the relative price of consuming today rather than in the future."

The user cost formula (1) can be put into a more familiar form if the period 0 *economic depreciation rate* δ and the period 0 *ex post asset inflation rate* i^0 are defined. Define δ by:

(4) $(1 - \delta) \equiv P_S^1/P^1$

where P_S^1 is the price of a used asset at the end of period 0 and P^1 is the price of a new asset at the end of period 0. The *period 0 inflation rate* for the new asset, i^0, is defined by:

(5) $1+i^0 \equiv P^1/P^0$.

Eliminating P^1 from equations (4) and (5) leads to the following formula for the end of period 0 used asset price:

(6) $P_S^1 = (1 - \delta)(1 + i^0)P^0$.

Substitution of (6) into (1) yields the following expression for the *period 0 user cost* u^0:

(7) $u^0 = [(1 + r^0) - (1 - \delta)(1 + i^0)]P^0 / (1 + r^0)$.

Note that $r^0 - i^0$ can be interpreted as a period 0 *real interest rate* and $\delta(1+i^0)$ can be interpreted as an *inflation adjusted depreciation rate*.

The user cost u^0 is expressed in terms of prices that are discounted to the *beginning* of period 0. However, it is also possible to express the user cost in terms of prices that are "discounted" to the *end* of period 0.[14] Thus define the *end of period 0 user cost* p^0 as:[15]

(8) $p^0 \equiv (1 + r^0)u^0 = [(1 + r^0) - (1 - \delta)(1 + i^0)]P^0$

[14] Thus the beginning of the period user cost u^0 discounts all monetary costs and benefits into their dollar equivalent at the beginning of period 0 whereas p^0 discounts (or appreciates) all monetary costs and benefits into their dollar equivalent at the end of period 0. This leaves open how flow transactions that take place within the period should be treated. Following the conventions used in financial accounting suggests that flow transactions taking place within the accounting period be regarded as taking place at the end of the accounting period and hence following this convention, end of period user costs should be used by the price statistician.

[15] Christensen and Jorgenson (1969) derived a user cost formula similar to (7) in a different way using a continuous time optimization model. If the inflation rate i equals 0, then the user cost formula (7) reduces to that derived by Walras (1954; 269) (first edition 1874). This zero inflation rate user cost formula was also derived by the industrial engineer A. Hamilton Church (1901; 907-908), who perhaps drew on the work of Matheson: "In the case of a factory where the occupancy is assured for a term of years, and the rent is a first charge on profits, the rate of interest, to be an appropriate rate, should, so far as it applies to the buildings, be equal (including the depreciation rate) to the rental which a landlord who owned but did not occupy a factory would let it for." Ewing Matheson (1910; 169), first published in 1884. Additional derivations of user cost formulae in discrete time have been made by Katz (1983; 408-409) and Diewert (2003b). Hall and Jorgenson (1967) introduced tax considerations into user cost formulae.

where the last equation follows using (7). If the real interest rate r^{0*} is defined as the nominal interest rate r^0 less the asset inflation rate i^0 and the small term δi^0 is neglected, then the end of the period user cost defined by (8) reduces to:

(9) $p^0 = (r^{0*} + \delta)P^0$.

Abstracting from transactions costs and inflation, it can be seen that the end of the period user cost defined by (9) is an *approximate rental cost*; i.e., the rental cost for the use of a consumer (or producer) durable good should equal the (real) opportunity cost of the capital tied up, $r^{0*}P^0$, plus the decline in value of the asset over the period, δP^0. Formulae (8) and (9) thus cast some light on what are the economic determinants of rental or leasing prices for consumer durables.

If the simplified user cost formula defined by (9) above is used, then forming a price index for the user costs of a durable good is not very much more difficult than forming a price index for the purchase price of the durable good, P^0. The price statistician needs only to:

- Make a reasonable assumption as to what an appropriate monthly or quarterly real interest rate r^{0*} should be;
- Make an assumption as to what a reasonable monthly or quarterly depreciation rate δ should be;[16]
- Collect purchase prices P^0 for the durable and
- Make an estimate of the total stock of the durable which was held by the reference population during the base period for quantities. In order to construct a superlative index, estimates of the stock held will have to be made for each period.

If it is thought necessary to implement the more complicated user cost formula (8) in place of the simpler formula (9), then the situation is more complicated. As it stands, the end of the period user cost formula (8) is an *ex post* (or after the fact) *user cost*: the asset inflation rate i^0 cannot be calculated until the end of period 0 has been reached. Formula (8) can be converted into an *ex ante* (or before the fact) *user cost* formula if i^0 is interpreted as an *anticipated asset inflation rate*. The resulting formula should approximate a market rental rate for the asset under inflationary conditions.[17]

[16] The geometric model for depreciation to be explained in more detail in section 6.2 below requires only a single monthly or quarterly depreciation rate. Other models of depreciation may require the estimation of a sequence of vintage depreciation rates. If the estimated annual geometric depreciation rate is δ_a, then the corresponding monthly geometric depreciation rate δ can be obtained by solving the equation $(1 - \delta)^{12} = 1 - \delta_a$. Similarly, if the estimated annual real interest rate is r_a^*, then the corresponding monthly real interest rate r^* can be obtained by solving the equation $(1 + r^*)^{12} = 1 + r_a^*$.

[17] Since landlords must set their rent at the beginning of the period (and in fact, they usually set their rent for an extended period of time), if the user cost approach is used to model the economic determinants of market rental rates, then the asset inflation rate i^0 should be interpreted as an expected inflation rate rather than an after the fact actual inflation rate. This use of ex ante prices in this price measurement context should be contrasted with the preference of national accountants to use actual or ex post prices in the system of national accounts.

Note that in the user cost approach to the treatment of consumer durables, the *entire* user cost formula (8) or (9) is the period 0 price. Thus in the time series context, it is *not* necessary to deflate each component of the formula *separately*; the period 0 price $p^0 \equiv [r^0 - i^0 + \delta(1+i^0)]P^0$ is compared to the corresponding period 1 price, $p^1 \equiv [r^1 - i^1 + \delta(1+i^1)]P^1$ and so on.

In principle, depreciation rates can be estimated using information on the selling prices of used units of the durable good. In section 6 below, this methodology will be explained in more detail. However, before this is done, it will be useful to use the material in this section to explain what the relationship between the user cost and acquisition approaches to the treatment of durables is likely to be. This topic is discussed in the following section.

5. The Relationship Between User Costs and Acquisition Costs

In this section, the user cost approach to the treatment of consumer durables will be compared to the acquisitions approach. Obviously, in the short run, the value flows associated with each approach could be very different. For example, if real interest rates, $r^0 - i^0$, are very high and the economy is in a severe recession or depression, then purchases of new consumer durables, Q^0 say, could be very low and even approach 0 for very long lived assets, like houses. On the other hand, using the user cost approach, existing stocks of consumer durables would be carried over from previous periods and priced out at the appropriate user costs and the resulting consumption value flow could be quite large. Thus in the short run, the monetary values of consumption under the two approaches could be vastly different. Hence, in what follows, a (hypothetical) longer run comparison is considered where real interest rates are held constant.[18]

Suppose that in period 0, the reference population of households purchased q^0 units of a consumer durable at the purchase price P^0. Then *the period 0 value of consumption from the viewpoint of the acquisitions approach* is:

(10) $V_A{}^0 \equiv P^0q^0$.

Recall that the end of period user cost for one new unit of the asset purchased at the beginning of period 0 was p^0 defined by (8) above. In order to simplify the analysis, *declining balance depreciation* is assumed[19]; i.e., at the beginning of period 0, a one period old asset is worth $(1-\delta)P^0$; a two period old asset is worth $(1-\delta)^2P^0$; … ; a t period old asset is worth $(1-\delta)^tP^0$; etc. Under these hypotheses, the corresponding end of period 0 user cost for a new asset purchased at the beginning of period 0 is p^0; the end of period 0 user cost for a one period old asset at the beginning of period 0 is $(1-\delta)p^0$; the corresponding user cost for a two period old asset at the beginning of period 0 is $(1-\delta)^2p^0$; … ; the corresponding user cost for a t period old asset at the beginning of

[18] The following material is based on Diewert (2002a).
[19] This form of depreciation will be discussed in more detail in section 6.2 below.

period 0 is $(1-\delta)^t p^0$; etc.[20] The final simplifying assumption is that household purchases of the consumer durable have been growing at the geometric rate g into the indefinite past. This means that if household purchases of the durable were q^0 in period 0, then in the previous period they purchased $q^0/(1+g)$ new units; two periods ago, they purchased $q^0/(1+g)^2$ new units; … ; t periods ago, they purchased $q^0/(1+g)^t$ new units; etc. Putting all of these assumptions together, it can be seen that *the period 0 value of consumption from the viewpoint of the user cost approach* is:

$$(11)\ V_U^0 \equiv p^0q^0 + [(1-\delta)p^0q^0/(1+g)] + [(1-\delta)^2\, p^0q^0/(1+g)^2] + \ldots$$
$$= (1+g)p^0q^0/(g+\delta) \qquad \text{summing the infinite series}$$
$$(12)\qquad = (1+g)[(1+r^0) - (1-\delta)(1+i^0)]P^0q^0/(g+\delta) \qquad \text{using (8).}$$

Equation (12) can be simplified by letting the asset inflation rate i^0 be 0 (or by replacing $r^0 - i^0$ by the real interest rate r^{0*} and by ignoring the small term δi^0) and under these conditions, the ratio of the user cost flow of consumption (12) to the acquisitions measure of consumption in period 0, (10) is:

$$(13)\ V_U^0/V_A^0 = (1+g)(r^{0*} + \delta)/(g+\delta)\ .$$

Using formula (13), it can be seen that if $1+g > 0$ and $\delta + g > 0$, then V_U^0 / V_A^0 will be greater than unity if

$$(14)\ r^{0*} > g(1-\delta)/(1+g)\ ,$$

a condition that will usually be satisfied.[21] Thus under normal conditions and over a longer time horizon, *household expenditures on consumer durables using the user cost approach will tend to exceed the corresponding money outlays on new purchases of the consumer durable*. The difference between the two approaches will tend to grow as the life of the asset increases (i.e., as the depreciation rate δ decreases).

To get a rough idea of the possible magnitude of the value ratio for the two approaches, V_U^0/V_A^0, equation (13) is evaluated for a "housing" example using annual data where the depreciation rate is 2 % (i.e., $\delta = .02$), the real interest rate is 4 % (i.e., $r^{0*} = .04$) and the growth rate for the production of new houses is 1 % (i.e., $g = .01$). In this base case, the ratio of user cost expenditures on housing to the purchases of new housing in the same period, V_U^0/V_A^0, is 2.02. If the depreciation rate is *increased* to 3 %, then V_U^0/V_A^0 *decreases* to 1.77; if the depreciation rate is *decreased* to 1 %, then V_U^0/V_A^0 *increases* to 2.53. Again looking at the base case, if the real interest rate is *increased* to 5 %, then V_U^0/V_A^0 *increases* to 2.36 while if the real interest rate is *decreased* to 3 %, then V_U^0/V_A^0 *decreases* to 1.68. Finally, if the growth rate for new houses is *increased* to 2 %, then V_U^0/V_A^0 *decreases* to 1.53 while if the growth rate is *decreased* to 0, then V_U^0/V_A^0

[20] For some consumer durables, the one hoss shay assumption for depreciation may be more realistic than the declining balance model; see section 6.4 below or Hulten (1990) or Diewert and Lawrence (2000).
[21] Note that if the real interest rate r^0 equals g, the real rate of growth in the purchases of the durable, then from (13), $V_U^0 / V_A^0 = (1+g)$ and the acquisitions approach will be more or less equivalent to the user cost approach over the long run.

increases to 3.00. *Thus an acquisitions approach to housing in the CPI is likely to give about one half the expenditure weight that a user cost approach would give*.

For shorter lived assets, the difference between the acquisitions approach and the user cost approach will not be so large and hence, this justifies the acquisitions approach as being approximately "correct" as a measure of consumption services.[22]

Here is a list of some of the problems and difficulties that might arise in implementing a user cost approach to purchases of a consumer durable:[23]

- It is difficult to determine what the relevant nominal interest rate r^0 is for each household. If a consumer has to borrow to finance the cost of a durable good purchase, then this interest rate will typically be much higher than the safe rate of return that would be the appropriate opportunity cost rate of return for a consumer who had no need to borrow funds to finance the purchase.[24] It may be necessary to simply use a benchmark interest rate that would be determined by either the government, a national statistical agency or an accounting standards board.
- It will generally be difficult to determine what the relevant depreciation rate is for the consumer durable.[25]
- *Ex post user costs* based on formula (8) will be too volatile to be acceptable to users[26] (due to the volatility of the asset inflation rate i^0) and hence an *ex ante user cost*

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[22] The simplified user cost approach can be used for other consumer durables as well. In formula (13), let $r^{0*} = .04$, $g = .01$ and $\delta = .15$ and under these conditions, $V_U^0/V_A^0 = 1.20$; i.e., for a declining balance depreciation rate of 15%, the user cost approach leads to an estimated value of consumption that is 20% higher than the acquisitions approach under the conditions specified. Thus for consumer durable depreciation rates that are lower than 15%, it would be useful for the statistical agency to produce user costs for these goods and for the national accounts division to produce the corresponding consumption flows as "analytic series". It should be noted that this extends the present national accounts treatment of housing to other long lived consumer durables. Note also that this revised treatment of consumption in the national accounts would tend to make rich countries richer, since poorer countries hold fewer long lived consumer durables on a per capita basis.

[23] For additional material on difficulties with the user cost approach, see Diewert (1980; 475-479) and Katz (1983; 415-422).

[24] Katz (1983; 415-416) comments on the difficulties involved in determining the appropriate rate of interest to use: "There are numerous alternatives: a rate on financial borrowings, on savings, and a weighted average of the two; a rate on nonfinancial investments. e.g., residential housing, perhaps adjusted for capital gains; and the consumer's subjective rate of time preference. Furthermore, there is some controversy about whether it should be the maximum observed rate, the average observed rate, or the rate of return earned on investments that have the same degree of risk and liquidity as the durables whose services are being valued."

[25] It is not necessary to assume declining balance depreciation in the user cost approach: any pattern of depreciation can be accommodated, including one hoss shay depreciation, where the durable yields a constant stream of services over time until it is scrapped. See Diewert and Lawrence (2000) for some empirical examples for Canada using different assumptions about the form of depreciation. For references to the depreciation literature and for empirical methods for estimating depreciation rates, see Hulten and Wykoff (1981a) (1981b) (1996) and Jorgenson (1996).

[26] Goodhart (2001; F351) comments on the practical difficulties of using ex post user costs for housing as follows: "An even more theoretical user cost approach is to measure the cost foregone by living in an owner occupied property as compared with selling it at the beginning of the period and repurchasing it at the end ... But this gives the absurd result that as house prices rise, so the opportunity cost falls; indeed the

concept will have to be used. This creates difficulties in that different national statistical agencies will generally make different assumptions and use different methods in order to construct forecasted structures and land inflation rates and hence the resulting ex ante user costs of the durable may not be comparable across countries.[27]

- The user cost formula (8) should be generalized to accommodate various taxes that may be associated with the purchase of a durable or with the continuing use of the durable.[28]

Some of the problems associated with estimating depreciation rates will be discussed in the next section.

6. Alternative Models of Depreciation

6.1 A General Model of Depreciation for (Unchanging) Consumer Durables

In this subsection, a "general" model of depreciation for durable goods that appear on the market each period without undergoing quality change will be presented. In three subsequent subsections, this general model will be specialized to the three most common models of depreciation that appear in the literature. In section 7 below, the additional problems that occur when the durable is built as a unique good will be discussed.

The main tool that can be used to identify depreciation rates for a durable good is the (cross sectional) sequence of vintage asset prices that units of the good sell for on the second hand market at any point of time.[29]

Some notation is required. Let P^0 be the price of a newly produced unit of the durable good at the beginning of period 0 (this is the same notation as was used earlier). Let $P_v{}^t$ be the second hand market price at the beginning of period t of a unit of the durable good

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more virulent the inflation of housing asset prices, the more negative would this measure become. Although it has some academic aficionados, this flies in the face of common sense; I am glad to say that no country has adopted this method." As will be seen later, Iceland has in fact adopted a simplified user cost framework.

[27] For additional material on the difficulties involved in constructing ex ante user costs, see Diewert (1980; 475-486) and Katz (1983; 419-420). For empirical comparisons of different user cost formulae, see Harper, Berndt and Wood (1989) and Diewert and Lawrence (2000).

[28] For example, property taxes are associated with the use of housing services and hence should be included in the user cost formula; see section 10.2 below. As Katz (1983; 418) noted, taxation issues also impact the choice of the interest rate: "Should the rate of return be a before or after tax rate?" From the viewpoint of a household that is not borrowing to finance the purchase of the durable, an after tax rate of return seems appropriate but from the point of a leasing firm, a before tax rate of return seems appropriate. This difference helps to explain why rental equivalence prices for the durable might be higher than user cost prices; see also section 11.4 below.

[29] Another information source that could be used to identify depreciation rates for the durable good is the sequence of vintage rental or leasing prices that might exist for some consumer durables. In the closely related capital measurement literature, the general framework for an internally consistent treatment of capital services and capital stocks in a set of vintage accounts was set out by Jorgenson (1989) and Hulten (1990; 127-129) (1996; 152-160).

that is v periods old.[30] Let δ_v^0 be the period 0 depreciation rate for a unit of the durable good that is v periods old at the beginning of period 0. These depreciation rates can be defined recursively, starting with the period 0 depreciation rate for a brand new unit, δ_0^0, using the period 0 vintage asset prices P_v^0 as follows:

$$(15)\ 1 - \delta_0^0 \equiv P_1^0/P^0 .$$

Once δ_0^0 has been defined by (15), the period 0 cross sectional depreciation rate for a unit of the durable good that is one period old at the beginning of period 0, δ_1^0, can be defined using the following equation:

$$(16)\ (1 - \delta_1^0)(1 - \delta_0^0) \equiv P_2^0/P^0 .$$

Note that P_2^0 is the beginning of period 0 asset price of a unit of the durable good that is 2 periods old and it is compared to the price of a brand new unit of the durable, P^0 (which is equal to P_0^0 using the vintage good notation).

Given that the period 0 cross sectional depreciation rates for units of the durable that are 0, 1, 2,…, v − 1 periods old at the beginning of period 0 are defined (these are the depreciation rates δ_0^0, δ_1^0, δ_2^0,…, δ_{v-1}^0), then the period 0 cross sectional depreciation rate for units of the durable that are v periods old at the beginning of period 0 can be defined using the following equation:

$$(17)\ (1 - \delta_v^0) \ldots (1 - \delta_1^0)(1 - \delta_0^0) \equiv P_{v+1}^0/P^0 .$$

It should be clear how the sequence of period 0 vintage asset prices P_v^0 can be converted into a sequence of period 0 vintage depreciation rates. It should also be clear that the sequence of equations (15)-(17) can be repeated using the vintage asset price data pertaining to the beginning of period t, P_v^t, in order to obtain a sequence of period t vintage depreciation rates, δ_v^t. In the depreciation literature, it is usually assumed that the sequence of vintage depreciation rates, δ_v^t, is independent of the period t so that:

$$(18)\ \delta_v^t = \delta_v \quad \text{for all periods t and all vintages v.}$$

The above material shows how the sequence of vintage or used durable goods prices at a point in time can be used in order to estimate depreciation rates. This type of methodology, with a few extra modifications to account for differing ages of retirement,

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[30] Using this notation for vintages, it can be seen that the vintage v = 0 price at the beginning of period t = 0, P_0^0, is equal to the price of a new unit of the good, P^0. If these second hand vintage prices depend on how intensively the durable good has been used in previous periods, then it will be necessary to further classify the durable good not only by its vintage v but also according to the intensity of its use. In this case, think of the sequence of vintage asset prices P_v^0 as corresponding to the prevailing market prices of the various vintages of the good at the beginning of period 0 for a assets that have been used at "average" intensities.

was pioneered by Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) (1996).[31]

Recall the user cost formula for a new unit of the durable good under consideration defined by (1) above. The same approach can be used in order to define a sequence of period 0 user costs for all vintages v of the durable. Thus suppose that P_{v+1}^{1a} is the *anticipated end of period 0 price* of a unit of the durable good that is v periods old at the beginning of period 0 and let r^0 be the consumer's opportunity cost of capital. Then the discounted to the beginning of period 0 *user cost* of a unit of the durable good that is v periods old at the beginning of period 0, u_v^0, is defined as follows:

$$(19)\ u_v^0 \equiv P_v^0 - P_{v+1}^{1a}/(1 + r^0)\ ; \qquad v = 0,1,2, \ldots$$

It is now necessary to specify how the *end* of period 0 anticipated vintage asset prices P_v^{1a} are related to their counterpart *beginning* of period 0 vintage asset prices P_v^0. The assumption that is made now is that the entire sequence of vintage asset prices at the end of period 0 is equal to the corresponding sequence of asset prices at the beginning of period 0 times a general anticipated period 0 inflation rate factor, $(1+i^0)$, where i^0 is the anticipated period 0 (general) asset inflation rate. Thus it is assumed that

$$(20)\ P_v^{1a} = (1 + i^0)P_v^0\ ; \qquad v = 1,2, \ldots$$

Substituting (20) and (15)-(18) into (19) leads to the following beginning of *period 0 sequence of vintage user costs*:[32]

$$\begin{aligned}(21)\ u_v^0 &= (1 - \delta_{v-1})(1 - \delta_{v-2}) \ldots (1 - \delta_0)[(1 + r^0) - (1 - \delta_v)(1 + i^0)]P^0/(1 + r^0) \\ &= (1 - \delta_{v-1})(1 - \delta_{v-2}) \ldots (1 - \delta_0)[r^0 - i^0 + \delta_v(1 + i^0)]P^0/(1 + r^0)\ ; \qquad v = 0,1,2,\ldots\end{aligned}$$

Note that if v = 0, then the u_0^0 defined by (21) agrees with the user cost formula for a new purchase of the durable u^0 that was derived earlier in (7).

The sequence of vintage user costs u_v^0 defined by (21) are expressed in terms of prices that are discounted to the *beginning* of period 0. However, as was done in section 4 above, it is also possible to express the user costs in terms of prices that are "discounted" to the *end* of period 0. Thus define the sequence of vintage *end of period 0 user cost* p_v^0 as follows:

$$(22)\ p_v^0 \equiv (1 + r^0)u_v^0 = (1 - \delta_{v-1})(1 - \delta_{v-2}) \ldots (1 - \delta_0)[r^0 - i^0 + \delta_v(1 + i^0)]P^0\ ;\ \ v = 0,1,2,\ldots$$

If the real interest rate r^{0*} is defined as the nominal interest rate r^0 less the asset inflation rate i^0 and the small terms $\delta_v\, i^0$ are neglected in (22), then the sequence of end of the period user costs defined by (22) reduces to:

[31] See also Jorgenson (1996) for a review of the empirical literature on the estimation of depreciation rates.
[32] When v = 0, define $\delta_{-1} \equiv 1$; i.e., the terms in front of the square brackets on the right hand side of (21) are set equal to 1.

(23) $p_v^0 = (1 - \delta_{v-1})(1 - \delta_{v-2}) \ldots (1 - \delta_0)[\, r^{0*} + \delta_v]P^0$; $\qquad v = 0,1,2,\ldots$

Thus if the price statistician has estimates for the vintage depreciation rates δ_v and the real interest rate r^{0*} and is able to collect a sample of prices for new units of the durable good P^0, then the sequence of vintage user costs defined by (23) can be calculated. To complete the model, the price statistician should gather information on the stocks held by the household sector of each vintage of the durable good and then normal index number theory can be applied to these p's and Q's, with the p's being vintage user costs and the Q's being the vintage stocks pertaining to each period. For some worked examples of this methodology under various assumptions about depreciation rates and the calculation of expected asset inflation rates, see Diewert and Lawrence (2000) and Diewert (2003c).[33]

In the following three subsections, the general methodology described above is specialized by making additional assumptions about the form of the vintage depreciation rates δ_v.

6.2 Geometric or Declining Balance Depreciation

The *declining balance method of depreciation* dates back to Matheson (1910; 55) at least.[34] In terms of the algebra presented in section 6.1 above, the method is very simple: all of the cross sectional vintage depreciation rates δ_v^0 defined by (15)-(17) are assumed to be equal to the same rate δ, where δ is a positive number less than one; i.e., for all time periods t and all vintages v, it is assumed that

(24) $\delta_v^t = \delta$; $\qquad v = 0,1,2,\ldots$.

Substitution of (24) into (22) leads to the following formula for the sequence of *period 0 vintage user costs*:

(25) $p_v^0 = (1 - \delta)^v[\, r^0 - i^0 + \delta_v(1 + i^0)]P^0$; $\qquad v = 0,1,2,\ldots$
$\qquad = (1 - \delta)^v\, p_0^0$.

The second set of equations in (25) says *that all of the vintage user costs are proportional to the user cost for a new asset*. This proportionality means that it is not necessary to use an index number formula to aggregate over vintages to form a durable services aggregate. To see this, it is useful to calculate the aggregate value of services yielded by all vintages of the consumer durable at the beginning of period 0. Let q^{-v} be the quantity of the durable purchased by the household sector v periods ago for v = 1,2,…. and let q^0 be the new purchases of the durable during period 0. The beginning of period 0 price for these

[33] Additional examples and discussion can be found in two recent OECD Manuals on productivity measurement and the measurement of capital; see OECD (2001a) (2001b).

[34] A case for attributing the method to Walras (1954; 268-269) could be made but he did not lay out all of the details. Matheson (1910; 91) used the term "diminishing value" to describe the method. Hotelling (1925; 350) used the term "the reducing balance method" while Canning (1929; 276) used the term the "declining balance formula".

vintages of age v will be p_v^0 defined by (25) above. Thus the aggregate services of all vintages of the good, including those purchased in period 0, will have the following value, S^0:

$$(26)\ S^0 = p_0^0q^0 + p_1^0q^{-1} + p_2^0q^{-2} + \ldots$$
$$= p_0^0q^0 + (1-\delta)\, p_0^0q^{-1} + (1-\delta)^2\, p_0^0q^{-2} + \ldots \qquad \text{using (25)}$$
$$= p_0^0\,[q^0 + (1-\delta)q^{-1} + (1-\delta)^2q^{-2} + \ldots\,]$$
$$= p_0^0Q^0$$

where the period 0 aggregate (quality adjusted) quantity of durable services consumed in period 0, Q^0, is defined as

$$(27)\ Q^0 \equiv q^0 + (1-\delta)q^{-1} + (1-\delta)^2q^{-2} + \ldots\,.$$

Thus the *period 0 services quantity aggregate* Q^0 is equal to new purchases of the durable in period 0, q^0, plus one minus the depreciation rate δ times the purchases of the durable in the previous period, q^{-1}, plus the square of one minus the depreciation rate times the purchases of the durable two periods ago, q^{-2}, and so on. The service price that can be applied to this quantity aggregate is p_0^0, the imputed rental price or user cost for a new unit of the durable purchased in period 0.

If the depreciation rate δ and the purchases of the durable in prior periods are known, then the aggregate service quantity Q^0 can readily be calculated using (27). Then using (26), it can be seen that the value of the services of the durable (over all vintages), S^t, decomposes into the price term p_0^0 times the quantity term Q^0. Hence, it is not necessary to use an index number formula to aggregate over vintages using this depreciation model.

6.3 Straight Line Depreciation

Another very common model of depreciation is the *straight line model*.[35] In this model, a most probable length of life for the durable is somehow determined, say L periods, so that after being used for L periods, the durable is scrapped. In the straight line depreciation model, it is assumed that the period 0 cross sectional vintage asset prices P_v^0 follow the following pattern of linear decline relative to the period 0 price of a new asset P^0:

$$(28)\ P_v^0/P^0 = [L - v]/L \qquad \text{for } v = 0, 1, 2, \ldots, L-1.$$

For v = L, L+1, …., it is assumed that $P_v^0 = 0$. Now substitute (20) and (28) into the beginning of the period user cost formula (19) in order to obtain the following sequence of *period 0 vintage user costs* for the durable:

[35] This model of depreciation dates back to the late 1800's; see Matheson (1910; 55), Garcke and Fells (1893; 98) or Canning (1929; 265-266).

$$(29)\ u_v^0 = P_v^0 - (1 + i^0)P_{v+1}^0/(1 + r^0) \qquad \text{for } v = 0, 1, 2, \ldots, L-1$$
$$= [1/L][(L - v) - (L - v - 1)\{(1+i^0)/(1+r^0)\}]P^0$$
$$= [(L - v)r^{0*} + 1]\ P^0/L(1 + r^{0*})$$

where the *asset specific real interest rate for period 0*, r^{0*}, is defined by

$$(30)\ 1 + r^{0*} \equiv (1 + r^0)/(1 + i^0)\ .$$

The user costs for units of the durable good that are older than L periods are zero; i.e., $u_v^0 \equiv 0$ for $v \geq L$. Looking at the terms in square brackets on the right hand side of (29), it can be seen that the first term is a real interest opportunity cost for holding and using the unit of the durable that is v periods old (and this imputed interest cost declines as the durable good ages) and the second term is a depreciation term that is equal to the constant rate 1/L .

In this model of depreciation, it is necessary to keep track of household purchases of the durable for L periods and weight up each vintage quantity q^{-v} of these purchases by the corresponding vintage user cost u_v^0 defined by (29) or the end of period vintage user costs p_v^0 defined as $(1+r^0)u_v^0$ could be used.[36]

6.4 One Hoss Shay or Light Bulb Depreciation

The final model of depreciation that is in common use is the "light bulb" or *one hoss shay model of depreciation*.[37] In this model, the durable delivers the *same* services for each vintage: a chair is a chair, no matter what its age is (until it falls to pieces and is scrapped). Thus this model also requires an estimate of the most probable life L of the consumer durable.[38] In this model, it is assumed that the sequence of vintage beginning of the period user costs u_v^0 defined by the first line of (29) is *constant* for all vintages younger than the asset lifetime L; i.e., it is assumed that

$$(31)\ u^0 = u_v^0 = P_v^0 - (1 + i^0)P_{v+1}^0/(1 + r^0) \qquad \text{for } v = 0, 1, 2, \ldots, L-1$$
$$= P_v^0 - \gamma P_{v+1}^0$$

where the *discount factor* γ is defined as

$$(32)\ \gamma \equiv (1 + i^0)/(1 + r^0) = 1/(1 + r^{0*})$$

[36] A worked example using this model of depreciation can be found in Diewert (2003b)

[37] This model can be traced back to Böhm-Bawerk (1891; 342). For a more comprehensive exposition, see Hulten (1990; 124) or Diewert (2003b).

[38] The assumption of a single life L for a durable can be relaxed using a methodology due to Hulten: "We have thus far taken the date of retirement T to be the same for all assets in a given cohort (all assets put in place in a given year). However, there is no reason for this to be true, and the theory is readily extended to allow for different retirement dates. A given cohort can be broken into components, or subcohorts, according to date of retirement and a separate T assigned to each. Each subcohort can then be characterized by its own efficiency sequence, which depends among other things on the subcohort's useful life T_i." Charles R. Hulten (1990; 125).

and the asset specific real interest rate r^{0*} was defined earlier by (30). Now the second equation in (31) can be used to express the *vintage v asset price* P_v^0 in terms of the common user cost u^0 and the vintage v+1 asset price, P_{v+1}^0, so that

$$(33)\ P_v^0 = u^0 + \gamma P_{v+1}^0 .$$

Now start out using equation (33) with v = 0, then substitute out P_1^0 using (33) with v = 1, then substitute out P_2^0 using (33) with v = 2, etc. until finally the process ends after L such substitutions when P_L^0 is reached and of course, P_L^0 equals zero. The following equation is obtained:

$$\begin{aligned}(34)\ P^0 &= u^0 + \gamma u^0 + \gamma^2 u^0 + \ldots + \gamma^{L-1} u^0 \\ &= u^0 [1 + \gamma + \gamma^2 + \ldots + \gamma^{L-1}] \\ &= \{u^0/(1-\gamma)\} - \{u^0 \gamma^L/(1-\gamma)\} \qquad \text{provided that } \gamma < 1 \\ &= u^0 (1-\gamma^L)/(1-\gamma) .\end{aligned}$$

Now use the last equation in (34) in order to solve for the constant over vintages (beginning of the period) *user cost* for this model, u^0, in terms of the period 0 price for a new unit of the durable, P^0, and the discount factor γ defined by (32):

$$(35)\ u^0 = (1-\gamma)P^0/(1-\gamma^L) .$$

The *end of period 0 user cost*, p^0, is as usual, equal to the beginning of the period 0 user cost, u^0, times the nominal interest rate factor, $1+r^0$:

$$(36)\ p^0 \equiv (1 + r^0)u^0 .$$

The *aggregate services of all vintages* of the good, including those purchased in period 0, will have the following value, S^0:

$$\begin{aligned}(37)\ S^0 &= p_0{}^0 q^0 + p_1{}^0 q^{-1} + p_2{}^0 q^{-2} + \ldots + p_2{}^0 q^{-(L-1)}] \\ &= p^0 [q^0 + q^{-1} + q^{-2} + \ldots + q^{-(L-1)}] \\ &= p^0 Q^0\end{aligned}$$

where the *period 0 aggregate (quality adjusted) quantity of durable services* consumed in period 0, Q^0, is defined as follows for this one hoss shay depreciation model:

$$(38)\ Q^0 \equiv q^0 + q^{-1} + q^{-2} + \ldots + q^{-(L-1)} .$$

Thus in this model of depreciation, the vintage quantity aggregate is the simple sum of household purchases over the last L periods. As was the case with the geometric depreciation model, the one hoss shay model does not require index number aggregation over vintages: there is a constant service price p^0 and the associated period 0 quantity Q^0

is a weighted sum of past purchases for the geometric model and a simple sum over the purchases of the last L periods for the light bulb model.[39]

6.5 The Empirical Estimation of Depreciation Rates

How can the different models of depreciation be distinguished empirically? For durable goods that do not change in quality over time, there are *three possible methods* for determining the sequence of vintage depreciation rates:[40]

- By making a rough estimate of the average length of life L for the durable good and then by *assuming* a depreciation model that seems most appropriate.[41]
- By using cross sectional information on used durable prices at a single point in time and then using equations (15)-(17) above to determine the corresponding sequence of vintage depreciation rates.
- By using cross sectional information on the rental or leasing prices of the durable as a function of the age of the durable and then equations (21) or (22), along with information on the appropriate nominal interest rate and expected durables inflation rate, can be used to determine the corresponding sequence of vintage depreciation rates.

In practice, the third method listed above has not been used (except for rental housing) because the rental markets do not exist or due to difficulties in obtaining the required information on rents by the age of the asset.

Typically, the second method for determining depreciation rates is also not used as described above due to missing information; i.e., not all vintages of the durable are sold on the marketplace at any one point in time. Under these circumstances, an econometric model is constructed that makes use of the limited information on used durable prices but allows the econometrician to estimate the vintage depreciation rates.[42]

7. Unique Durable Goods and the User Cost Approach

In the previous sections, it was assumed that a newly produced unit of the durable good remained the same from period to period. This means that the various vintages of the durable good repeat themselves going from period to period and hence a particular vintage of the good in the current period can be compared with the same vintage in the next period. In particular, consider the period 0 user cost of a new unit of a durable good p_0^0 defined earlier by (8). For convenience, the formula is repeated here:

---

[39] Thus (38) is the quantity aggregate counterpart to (27).

[40] These three classes of methods were noted in Malpezzi, Ozanne and Thibodeau (1987; 373-375) in the housing context.

[41] A length of life L is usually converted into an equivalent geometric depreciation rate δ by setting δ equal to a number between 1/L and 2/L.

[42] See Hall (1971), Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b). See also the discussion of alternative methods for estimating housing depreciation in Malpezzi, Ozanne and Thibodeau (1987; 373).

(39) $p_0^0 = [(1 + r^0) - (1 - \delta_0)(1 + i^0)]P^0 = [r^0 - i^0 + \delta_0(1 + i^0)]P^0$.

Recall that P^0 is the beginning of period 0 purchase price for the durable, r^0 is the nominal opportunity cost of capital that the household faces in period 0, i^0 is the anticipated period 0 inflation rate for the durable good and δ_0 is the one period depreciation rate for a new unit of the durable good. In previous sections, it was assumed that the period 0 user cost p_0^0 for a new unit of the durable could be compared with the corresponding period 1 user cost p_0^1 for a new unit of the durable purchased in period 1. This period 1 user cost can be defined as follows:

(40) $p_0^1 = [(1 + r^1) - (1 - \delta_0)(1 + i^1)]P^0 = [r^1 - i^1 + \delta_0(1 + i^1)]P^0$.

However, many durable goods are produced as *one of a kind* models. For example, a new house may have many features that are specific to that particular house. An exact duplicate of it is unlikely to be built in the following period. Thus if the user cost for the house is constructed for period 0 using formula (39) where the new house price P^0 plays a key role, then since there will not necessarily be a comparable new house price for the same type of unit in period 1, it will not be possible to construct the period 1 user cost for a house of the same type, p_0^1 defined by (40), since the comparable new house price P^1 will not be available.

Recall the notation that was introduced in section 6.1 above where P_v^t was the second hand market price at the beginning of period t of a unit of a durable good that is v periods old. Define δ_v to be the depreciation rate for a unit of the durable good that is v periods old at the beginning of the period under consideration. Using this notation, the user cost of the house (which is now one period old) for period 1, p_1^1 can be defined as follows:

(41) $p_1^1 \equiv (1 + r^1)P_1^1 - (1 - \delta_1)(1 + i^1)P_1^1$

where P_1^1 is the beginning of period 1 price for the house that is now one period old, r^1 is the nominal opportunity cost of capital that the household faces in period 1, i^1 is the anticipated period 1 inflation rate for the durable good and δ_1 is the one period depreciation rate for a house that is one period old. For a unique durable good, there is no beginning of period 1 price for a new unit of the durable, P^1, but it is natural to impute this price as the potentially observable market price for the used durable, P_1^1, divided by one minus the period 0 depreciation rate, δ_0; i.e., define an imputed period 1 price for a new unit of the unique durable as follows:

(42) $P^1 \equiv P_1^1/(1 - \delta_0)$.

If (42) is solved for P_1^1 and the solution is substituted into the user cost defined by (41), then the following expression is obtained for p_1^1, *the period 1 user cost of a one period old unique consumer durable*:

(43) $p_1^1 \equiv (1 - \delta_0)[(1 + r^1) - (1 - \delta_1)(1 + i^1)]P^1$

If it is further assumed that the unique consumer durable follows the geometric model of depreciation, then

(44) $\delta \equiv \delta_0 = \delta_1$.

Substituting (44) into (43) and (40) leads to the following relationship between *the imputed rental cost in period 1 of a new unit of the consumer durable*, p_0^1, and *the period 1 user cost of the one period old consumer durable*, p_1^1:

(45) $p_1^0 = p_1^1/(1 - \delta)$.

Thus in order to obtain an imputed rental price for the unique consumer durable for period 1, p_0^1, that is *comparable* to the period 0 rental price for a new unit of the consumer durable, p_0^0, it is necessary to make a *quality adjustment* to the period 1 rental price for the one period old durable, p_1^1, by dividing this latter price by one minus the one period geometric depreciation rate, δ. This observation has implications for the quality adjustment of observed market rents of houses. Without this type of quality adjustment, observed dwelling unit rents will have a *downward bias*, since the observed rents do not adjust for the gradual lowering of the quality of the unit due to depreciation of the unit.[43]

Note also that in order to obtain an imputed purchase price for the unique consumer durable for period 1, P^1, that is *comparable* to the period 0 purchase price for a new unit of the consumer durable, P^0, it is necessary to make a *quality adjustment* to the period 1 used asset price for the one period old durable, P_1^1, by dividing this latter price by one minus the period 0 depreciation rate, δ_0; recall equation (42) above.[44]

This section is concluded with some observations on the difficulties for economic measurement that occur when it is attempted to determine depreciation rates empirically for unique assets. Consider again equation (42), which allows one to express the potentially observable market price of the unique asset at the beginning of period 1, P_1^1, as being equal to $(1-\delta_0)P^1$, where P^1 is a hypothetical period 1 price for a new unit of the unique asset. If it is assumed that this hypothetical period 1 new asset price is equal to the period 0 to 1 inflation rate factor $(1+i^0)$ times the observable period 0 asset price P^0, then the following relationship between the two observable asset prices is obtained:

(46) $P_1^1 = (1 - \delta_0)(1 + i^0)P^0$.

Thus the potentially observable period 1 used asset price P_1^1 is equal to the period 0 new asset price P^0 times the product of two factors: $(1-\delta_0)$, *a quality adjustment factor* that

[43] There is an exception to this general observation: if housing depreciation is of the one hoss shay type, then there is no need to quality adjust observed rents for the same unit over time. However, one hoss shay depreciation is empirically unlikely in the housing market since renters are generally willing to pay a rent premium for a new unit over an older unit of the same type. For empirical evidence of this age premium, see Malpezzi, Ozanne and Thibodeau (1987; 378) and Hoffman and Kurz (2002; 19).

[44] This type of quality adjustment to the asset prices for unique consumer durables will always be necessary; i.e., there is no exception to this rule as was the case for one hoss shay depreciation in the context of quality adjusting rental prices.

takes into account the effects of aging on the unique asset, and $(1+i^0)$, a period to period *pure price change factor* holding quality constant. The problem with unique assets is that cross sectional information on used asset prices at any point in time is no longer available to enable one to sort out the separate effects of these two factors. Thus there is a fundamental identification problem with unique assets; without extra information or assumptions, *it will be impossible to distinguish the separate effects of asset deterioration and asset inflation*.[45] In practice, this identification problem is solved by making somewhat arbitrary assumptions about the form of depreciation that the asset is expected to experience.[46]

Housing is the primary example of a unique asset. But in addition to the problems outlined in this section, there are other major problems associated with this particular form of unique asset. These problems will be discussed in the following sections.

8. The User Cost of Owner Occupied Housing

Owner occupied housing is typically an example of a *unique* consumer durable so that the material on the quality adjustment of both stock and rental prices developed in the previous section applies to this commodity. However, owner occupied housing is also an example of a *composite* good; i.e., two distinct commodities are bundled together and sold (or rented) at a single price. The two distinct commodities are:

- the structure and
- the land that the structure sits on.

To model this situation, consider a particular newly constructed dwelling unit that is purchased at the beginning of period 0. Suppose that the purchase price is V^0. This value can be regarded as the sum of a cost of producing the structure, $P_S^0Q_S^0$, where Q_S^0 is the number of square meters of floor space in the structure and P_S^0 is the beginning of period 0 price of construction per square meter, and the cost of the land, $P_L^0Q_L^0$, where Q_L^0 is the number of square meters of the land that the structure sits on and the associated yard and P_L^0 is the beginning of period 0 price of the land per square meter.[47] Thus at the beginning of period 0, *the value of the dwelling unit* is V^0 defined as follows:

[45] Special cases of this fundamental identification problem have been noted in the context of various econometric housing models: "For some purposes one might want to adjust the price index for depreciation. Unfortunately, a depreciation adjustment cannot be readily estimated along with the price index using our regression method. … In applying our method, therefore, additional information would be needed in order to adjust the price index for depreciation." Martin J. Bailey, Richard F. Muth and Hugh O. Nourse (1963; 936). "The price index and depreciation are perfectly collinear, so if one cares about the price index, it is necessary to use external information on the geometric depreciation rate of houses." Raymond B. Palmquist (2003; 43).

[46] For example, if the unique asset is a painting by a master, then the depreciation rate can be assumed to be very close to zero. As another example, a reasonable guess at the likely length of life L of the unique asset could be made and then the one hoss shay or straight line depreciation models could be implemented. Alternatively, the length of life L could be converted into an equivalent geometric depreciation rate δ using the conversion rule $\delta = n/L$ where n is a number between 1 and 2.

[47] If the dwelling unit is part of a multiple unit structure, then the land associated with it will be the appropriate share of the total land space.

(47) $V^0 = P_S{}^0 Q_S{}^0 + P_L{}^0 Q_L{}^0$.

Suppose that the anticipated price of a unit of a new structure at the beginning of period 1 is $P_S{}^{1a}$ and that the anticipated price of a unit of land at the beginning of period 1 is $P_L{}^{1a}$. Define the *period 0 anticipated inflation rates for new structures and land*, $i_S{}^0$ and $i_L{}^0$ respectively, as follows:

(48) $1 + i_S{}^0 \equiv P_S{}^{1a}/P_S{}^0$;
(49) $1 + i_L{}^0 \equiv P_L{}^{1a}/P_L{}^0$.

Let δ_0 be the period 0 depreciation rate for the structure. Then the anticipated beginning of period 1 value for the structure and the associated land is equal to

(50) $V^{1a} = P_S{}^{1a}(1 - \delta_0)Q_S{}^0 + P_L{}^{1a}Q_L{}^0$.

Note the presence of the depreciation term $(1-\delta_0)$ on the right hand side of (50). Should this term be associated with the expected beginning of period 1 price for a new unit of structures $P_S{}^{1a}$ or with the structures quantity term $Q_S{}^0$? On the principle that like should be compared to like for prices, it seems preferable to associate $(1-\delta_0)$ with the quantity term $Q_S{}^0$. This is consistent with the treatment of unique assets that was suggested in the previous section; i.e., the initial quantity of structures $Q_S{}^0$ should be quality adjusted downwards to the amount $(1-\delta_0)\, Q_S{}^0$ at the beginning of period 1.

Now calculate the cost (including the imputed opportunity cost of capital r^0) of buying the dwelling unit at the beginning of period 0 and (hypothetically) selling it at the end of period 0. The following *end of period 0 user cost or imputed rental cost* R^0 for the dwelling unit is obtained using (47)-(50):

$$
\begin{aligned}
(51)\ R^0 &\equiv V^0(1 + r^0) - V^{1a} \\
&= [P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0](1 + r^0) - [P_S{}^{1a}(1 - \delta_0)Q_S{}^0 + P_L{}^{1a}Q_L{}^0] \\
&= [P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0](1 + r^0) - [P_S{}^0(1 + i_S{}^0)(1 - \delta_0)Q_S{}^0 + P_L{}^0(1 + i_L{}^0)Q_L{}^0] \\
&= p_S{}^0Q_S{}^0 + p_L{}^0Q_L{}^0
\end{aligned}
$$

where separate period 0 *user costs of structures and land*, $p_S{}^0$ and $p_L{}^0$, are defined as follows:

(52) $p_S{}^0 = [(1 + r^0) - (1 + i_S{}^0)(1 - \delta_0)]\, P_S{}^0 = [r^0 - i_S{}^0 + \delta_0(1 + i_S{}^0)]\, P_S{}^0$;
(53) $p_L{}^0 = [(1 + r^0) - (1 + i_L{}^0)]\, P_L{}^0 \qquad = [r^0 - i_L{}^0]P_L{}^0$.

Note that the above algebra indicates some of the major determinants of market rents for rental properties. The user cost formulae defined by (52) and (53) can be further simplified if the same approximations that were made in section 4 above are made here (recall equation (9) above); i.e., assume that the terms $r^0 - i_S{}^0$ and $r^0 - i_L{}^0$ can be approximated by a real interest rate r^{0*} and neglect the small term δ_0 times $i_S{}^0$ in (52). Then the user costs defined by (52) and (53) simplify to:

(54) $p_S^0 = [(r^{0*} + \delta_0)] P_S^0$;
(55) $p_L^0 = r^{0*} P_L^0$.

Thus *the imputed rent for an owner occupied dwelling unit* is made up of three main costs:

- The real opportunity cost of the financial capital tied up in the structure;
- The real opportunity cost of the financial capital tied up in the land;
- The depreciation cost of the structure.

The above simplified approach to the user cost of housing can be even further simplified by assuming that the ratio of the quantity of land to structures is fixed and so the aggregate user cost of housing is equal to $[r^{0*} + \delta]P_H^0$, where P_H is a quality adjusted housing price index that is based on all properties sold in the country to households during the period under consideration and δ is a geometric depreciation rate that applies to the composite of household structures and land. This super simplified approach is used by Iceland; see Gudnason (2003; 28-29).[48] A variant of this approach is used by the Bureau of Economic Analysis: Lebow and Rudd (2003; 168) note that the US national accounts imputation for the services of owner occupied housing is obtained by applying rent to value ratios for tenant occupied housing to the stock of owner occupied housing. The rent to value ratio can be regarded as an estimate of the applicable real interest rate plus the depreciation rate.[49]

Returning to the period 0 imputed rental cost model for a new structure defined by (47)-(53), now calculate the cost (including the imputed opportunity cost of capital r^1) of buying the used dwelling unit at the beginning of period 1 and (hypothetically) selling it at the end of period 1. Thus at the beginning of period 1, the value of the depreciated dwelling unit is V^1 defined as follows:

(56) $V^1 = P_S^1(1 - \delta_0)Q_S^0 + P_L^1Q_L^0$

where P_S^1 is the beginning of period 1 construction price for building a new dwelling unit of the same type and P_L^1 is the beginning of period 1 price of land for the dwelling unit. Note that (56) is an *end of period 0 ex post or actual value* of the dwelling unit whereas the similar expression (50) defined a *beginning of period 0 ex ante or anticipated value* of the dwelling unit.

[48] The real interest rate that is used is approximately 4% per year and the combined depreciation rate for land and structures is assumed to equal 1.25% per year. The depreciation rate for structures alone is estimated to be 1.5% per year. Property taxes are accounted for separately in the Icelandic CPI. Housing price information is provided by the State Evaluation Board based on property sales data of both new and old housing. The SEB also estimates the value of the housing stock and land in Iceland, using a hedonic regression model based on property sales data. The value of each household's dwelling is collected in the Household Budget Survey.
[49] However, as will be seen in sections 10 and 11 below, this method of imputing the value of Owner Occupied Housing services is likely to give a weight to OOH that is too large.

Suppose that the anticipated price of a unit of a new structure at the beginning of period 2 is P_S^{2a} and that the anticipated price of a unit of land at the beginning of period 2 is P_L^{2a}. Define *the period 1 anticipated inflation rates for new structures and land*, i_S^1 and i_L^1 respectively, as follows:

$$(57)\ 1 + i_S^1 \equiv P_S^{2a}/P_S^1\ ;$$
$$(58)\ 1 + i_L^1 \equiv P_L^{2a}/P_L^1\ .$$

Let δ_1 be the period 1 depreciation rate for the structure. Then *the anticipated beginning of period 2 value for the structure and the associated land* is equal to

$$(59)\ V^{2a} = P_S^{2a}(1-\delta_0)(1-\delta_1)Q_S^0 + P_L^{2a}Q_L^0\ .$$

The following end of period 1 *user cost or imputed rental cost* R_1^1 *for a one period old dwelling unit* is obtained using (56)-(59):

$$\begin{aligned}(60)\ R_1^1 &\equiv V^1(1+r^1) - V^{2a}\\ &= [P_S^1(1-\delta_0)Q_S^0 + P_L^1Q_L^0](1+r^1) - [P_S^{2a}(1-\delta_0)(1-\delta_1)Q_S^0 + P_L^{2a}Q_L^0]\\ &= [P_S^1(1-\delta_0)Q_S^0 + P_L^1Q_L^0](1+r^1) - [P_S^1(1+i_S^1)(1-\delta_0)(1-\delta_1)Q_S^0 + P_L^1(1+i_L^1)Q_L^0]\\ &= p_{S1}^1(1-\delta_0)Q_S^0 + p_L^1Q_L^0\end{aligned}$$

where the period 1 *user costs of one period old structures and land*, p_{S1}^1 and p_L^1, are defined as follows:

$$(61)\ p_{S1}^1 = [(1+r^1) - (1+i_S^1)(1-\delta_1)]\,P_S^1 = [r^1 - i_S^1 + \delta_1(1+i_S^1)]\,P_S^1\ ;$$
$$(62)\ p_L^1 = [(1+r^1) - (1+i_L^1)]\,P_L^1 = [r^1 - i_L^1]P_L^1\ .$$

Comparing the period 0 user cost of land p_L^0 defined by (53) with the period 1 user cost of land p_L^1 defined by (62), it can be seen that these user costs have exactly the same form and hence are comparable. However, comparing the period 0 user cost for a new structure p_S^0 defined by (52) with the period 1 user cost for a one period old structure p_{S1}^1 defined by (61), it can be seen that these user costs are not quite comparable unless the period 0 depreciation rate δ_0 is equal to the period 1 depreciation rate δ_1. If declining balance depreciation for structures is assumed, then $\delta_0 = \delta_1 = \delta$, where δ is the common depreciation rate across all periods. Under this assumption, p_{S1}^1 is comparable to the period 0 user cost for a new unit of structures p_S^0 since p_{S1}^1 turns out to equal the user cost for the services of a new structure during period 1.[50] However, even under the assumption of geometric depreciation, it can be seen that the period 1 imputed rent for a one period old dwelling unit R_1^1 defined by (60) is *not* comparable to the corresponding period 0 imputed rent for a new dwelling unit R^0 defined by (51). The imputed rent R^1 that would be comparable to R^0 can be defined as follows:

[50] Thus when $\delta_0 = \delta_1 = \delta$, a constant quality price index for the services of the structure over the two periods is $p_{S1}^1/p_S^0 = p_S^1/p_S^0$. Note that the corresponding constant quality quantity index for the services of the structure is equal to $(1-\delta)Q_S^0/Q_S^0 = 1-\delta$. Thus depreciation of the structure is regarded as an effect on the quantity of structure services and not on the price of structure services.

(63) $R^1 \equiv p_L^{\,1}Q_L^{\,0} + p_S^{\,1}Q_S^{\,0} = R_1^{\,1} + p_S^{\,1}\,\delta\,Q_S^{\,0}$

where the period 1 user cost of structures $p_S^{\,1}$ is defined by the right hand side of (61) but with δ_1 equal to the common depreciation rate δ and the period 1 user cost of land $p_L^{\,1}$ is defined by (62). Equation (63) has the following implication for the quality adjustment of the price of a rental property: if R^0 is the observed rent of the unit in period 0 and $R_1^{\,1}$ is the observed rent for the same dwelling unit in period 1, then the observed rent $R_1^{\,1}$ is *too low* compared to R^0 and so the period 1 observed rent should be quality adjusted upwards by the period 1 rental price for structures $p_S^{\,1}$ times the amount of physical depreciation $\delta Q_S^{\,0}$ in the structure that occurred in the previous period. This is the same point that was made in section 7 but in this section, the complications due to fact that housing services are a *mixture* of structure and land services are taken into account.

It is evident that the main drivers for the user costs of structures and land are a price index for new dwelling construction, $P_S^{\,t}$, and a price index for residential land, $P_L^{\,t}$. Most statistical agencies have a constant quality price index for new residential structures, since this index is required in the national accounts in order to deflate investment expenditures on residential structures. This index could be used as an approximation to $P_S^{\,t}$.[51] The national accounts also require an imputation for the services of owner occupied housing and thus the constant quality price component of this imputation may be suitable for Consumer Price Index purposes.[52] If the national accounts division also computes quarterly real balance sheets for the economy, then a price index for residential land may be available to the prices division. However, even if this is the case, there will be problems in producing this price index for land on a timely basis and at a monthly frequency.[53] Another possible source of information on land prices may be found in land title registry offices and in the records of real estate firms.

In the following section, the problems involved in obtaining a constant quality price index for either rents or the purchase price of a housing unit are examined in a bit more detail.

9. The Empirical Estimation of Housing Price Indexes

[51] This index may only be an approximation since it covers the construction of rental properties as well as owner occupied dwellings.

[52] However, the national accounts imputation for the services of Owner Occupied Housing will only be produced on a quarterly basis and so some additional work will be required to produce a price deflator on a monthly basis. Also even though the SNA93 recommends that the imputation for the services of OOH be based on the rental equivalent method, it may be the case that the imputation covers only the imputed depreciation on the structures part of OOH. As was pointed out above, there are two other important additional components that should also be included in OOH services; namely, the imputed real interest on the structures and the land on which the structures sit. These latter two components of imputed expenditures are likely to be considerably larger than the depreciation component.

[53] Another source of information on the value of residential land may be available from the local property tax authorities, particularly if properties are assessed at market values.

There are two broad approaches to constructing constant quality price indexes for the purchase price of a housing unit:

- The repeat sales approach;
- The hedonic regression approach.

Both of these approaches will be discussed below. The hedonic regression approach can also be applied to the problem of constructing constant quality indexes of rent.

We discuss first the *repeat sales approach*, due to Bailey, Muth and Nourse (1963), who saw their procedure as a generalization of the *chained matched model methodology* that was used by the early pioneers in the construction of real estate price indexes like Wyngarden (1927) and Wenzlick (1952). We first describe this matched model methodology for the case of three periods, which will suffice to illustrate the general case.

Suppose that there is a certain set of housing units S(0,1) that are in scope for the index and are sold in both periods 0 and 1. Denote the sales price for property n sold in period t by V_n^t for $n \in S(0,1)$ and $t = 0,1$. Let $P^{0,1}$ be the property price index going from period 0 to 1. Then a reasonable stochastic model that relates the ratio of the sales prices of the properties, V_n^1/V_n^0, to the price index $P^{0,1}$ is:

(64) $V_n^1/V_n^0 = P^{0,1} \exp u_n^{0,1}$; $\qquad n \in S(0,1)$

where $u_n^{0,1}$ is an independently distributed error term with mean 0 and constant variance. Taking logarithms of both sides of (64) leads to the following linear regression model:

(65) $\ln [V_n^1/V_n^0] = \pi^{0,1} + u_n^{0,1}$; $\qquad n \in S(0,1)$

where the single parameter $\pi^{0,1}$ is defined as the logarithm of the price index $P^{0,1}$; i.e.,

(66) $\pi^{0,1} \equiv \ln P^{0,1}$.

The least squares estimator for $\pi^{0,1}$ is the arithmetic average of the logarithms of the sales price ratios. Exponentiating this estimator leads to the following estimator for the property price index going from period 0 to 1:[54]

(67) $P^{0,1*} \equiv \prod_{n \in S(0,1)} [V_n^1/V_n^0]^{1/N(0,1)}$

where N(0,1) is the number of houses in the sample that sold in both periods 0 and 1; i.e., it is the number of houses in the set S(0,1). Thus the estimated property price index is simply the equally weighted geometric mean of sales price ratios V_n^1/V_n^0 for all the

[54] There will be a (typically small) bias associated with exponentiating an unbiased estimator; i.e., $P^{0,1*}$ will be biased. See Goldberger (1968).

properties that sold in both periods 0 and 1. This is a typical matched model estimator for an elementary price index.

The above model can be repeated for sales of houses in the target population that sold in both periods 1 and 2. The equations that correspond to (65)-(67) above are (68)-(70) below:

(68) $\ln [V_n^2/V_n^1] = \pi^{1,2} + u_n^{1,2}$; $\qquad n\in S(1,2)$

where S(1,2) is the set of houses that sold in both periods 1 and 2 and the parameter $\pi^{1,2}$ is defined as the logarithm of the property price index going from period 1 to 2, $P^{1,2}$; i.e.,

(69) $\pi^{1,2} \equiv \ln P^{1,2}$.

The least squares estimator for $\pi^{1,2}$ is the arithmetic average of the logarithms of the sales price ratios. Exponentiating this estimator leads to the following estimator for the property price index going from period 1 to 2:

(70) $P^{1,2*} \equiv \prod_{n\in S(1,2)} [V_n^2/V_n^1]^{1/N(1,2)}$

where N(1,2) is the number of sales of houses in the sample that sold in both periods 1 and 2.

Using the above regression estimates, the *levels* of the property price index, P^t, for periods t = 0,1,2 can be defined as follows:

(71) $P^0 \equiv 1$; $P^1 = P^{0,1*}$; $P^2 = P^{0,1*} P^{1,2*}$.

Thus the price index level P^t is set equal to 1 in the base period 0; in period 1, it is equal to the estimated matched model price index going from period 0 to 1, and in period 2, it is equal to the product of the period 1 level times the estimated matched model price index going from period 1 to 2.

The above material explains the chained matched model method that was used prior to the work of Bailey, Muth and Nourse. The innovation made by Bailey, Muth and Nourse (1963) was to reparameterize the regression model defined by (65) and (68) and to add an additional set of estimating equations for repeat sales that took place in periods 0 and 2. Thus the Bailey, Muth and Nourse estimating equations for the case where there are three periods of data on repeat sales are the following ones:

(72) $\ln [V_n^1/V_n^0] = \pi^1 - \pi^0 + u_n^{0,1}$; $\qquad n\in S(0,1)$;
(73) $\ln [V_n^2/V_n^1] = \pi^2 - \pi^1 + u_n^{1,2}$; $\qquad n\in S(1,2)$;
(74) $\ln [V_n^2/V_n^0] = \pi^2 - \pi^0 + u_n^{0,2}$; $\qquad n\in S(0,2)$

where S(0,2) is the set of housing units in the target population that sold in periods 0 and 2 and the π^t are the logarithms of the housing price levels P^t in each period; i.e.,

(75) $\pi^0 \equiv \ln P^0$; $\pi^1 \equiv \ln P^1$; $\pi^2 \equiv \ln P^2$.

It turns out that not all of the parameters π^0, π^1 and π^2 in (72)-(74) can be identified[55] and hence, it is necessary to impose a normalization on the π^t. The natural normalization is

(76) $\pi^0 = 0$ or $P^0 = 1$.

Substituting the normalization (76) into (72)-(74) leads to a simple linear regression model that can be used to obtain least squares estimates for the parameters π^1 and π^2, which we denote by π^{1*} and π^{2*}. Exponentiating these estimates leads to estimates for the period 1 and 2 price levels, P^{1*} and P^{2*} respectively. Hence the Bailey Muth and Nourse estimates for the *housing price levels* in the three periods are defined as follows:

(77) $P^0 \equiv 1$; $P^{1*} \equiv \exp \pi^{1*}$; $P^{2*} \equiv \exp \pi^{2*}$.

It is clear that the Bailey, Muth and Nourse repeat sales model is a big improvement over the original chained repeat sales model since it utilizes the available information on house sales in a statistically more efficient manner.[56]

The above three period model generalizes readily to the general case of T periods considered by Bailey, Muth and Nourse (1963) but the details will be left to the reader to work out.

We now begin our discussion of the hedonic regression approach to constructing constant quality price indexes for housing. *Hedonic regression models* work with *price levels* rather than *price ratios* as dependent variables. Before we discuss a general hedonic regression model for housing, it will be useful to put the above repeat sales model into a levels framework so that the differences between the repeat sales model and a general hedonic model can be assessed.

Consider the three period case again but now suppose that there is a sample of N houses that sold in each of the three periods. Then a reasonable stochastic model for the prices of the houses V_n^t in each period t might be the following one:

(78) $V_n^t = \alpha_n P^t \exp u_n^t$; $\qquad\qquad$ $n = 1,2,\ldots,N$; $t = 0,1,2$

where P^t is the housing price index level for period t, α_n is a parameter that reflects the quality of housing unit n relative to "average" quality and u_n^t is an independently distributed error term with mean zero and constant variance. Taking logarithms of both sides of (78) leads to the following system of estimating equations:

[55] Adding a constant to each π^t leaves the regression unchanged.

[56] The Bailey, Muth and Nourse regression model has not increased the number of parameters to be estimated (two) but it has added an extra N(0,2) degrees of freedom to the regression model.

(79) $\ln V_n^t = \beta_n + \pi^t + u_n^t$; $\qquad$ $n = 1,2,\ldots,N$; $t = 0,1,2$

where the β_n and π^t are defined as follows:

(80) $\beta_n \equiv \ln \alpha_n$; $\qquad$ $n = 1,2,\ldots,N$;
(81) $\pi^t \equiv \ln P^t$; $\qquad$ $t = 0,1,2$.

Equations (79) define a linear regression model in the unknown parameters β_n and π^t. However, as in the previous model, these parameters are not all identified[57] and so a normalization must be imposed. A natural normalization is (76); i.e., we set π^0 equal to 0.

It turns out that the linear regression model defined by (79) and (76) is precisely the same as the country product dummy model (with complete data) for three countries that was invented by Robert Summers (1973) in the context of making price comparisons between countries. It is also a special case of the product dummy hedonic regression model proposed by Aizcorbe, Corrado and Doms (2001).

It is possible to obtain an explicit formula for the least squares estimators for π^1 and π^2 in the linear regression model defined by (76) and (79). The vector of dependent variables in the regression model can be written as the sum of the vectors of exogenous variables times their corresponding least squares estimates plus the vector of least squares residuals. It is well known that the inner product of each exogenous vector with the vector of least squares residuals is zero.[58] This means that the least squares estimators for the unknown parameters in the regression model satisfy the following N+2 equations:

(82) $\sum_{n=1}^N \ln V_n^1 = \sum_{n=1}^N \beta_n^* + N\,\pi^{1*}$;
(83) $\sum_{n=1}^N \ln V_n^2 = \sum_{n=1}^N \beta_n^* + N\,\pi^{2*}$;
(84) $\ln V_n^0 + \ln V_n^1 + \ln V_n^2 = 3\beta_n^* + \pi^{1*} + \pi^{2*}$; $\qquad$ $n = 1,2,\ldots,N$.

Use equations (84) to eliminate the β_n^* from equations (82) and (83) and the resulting two linear equations involving the unknowns π^{1*} and π^{2*} can readily be solved. The solutions are:

(85) $\pi^{1*} = (1/N) \sum_{n=1}^N \ln [V_n^1/V_n^0]$; $\pi^{2*} = (1/N) \sum_{n=1}^N \ln [V_n^2/V_n^0]$.

Using the inverses of equations (81), the π^{t*} defined by equations (85) translate into the following estimates for the period 1 and 2 price levels, P^{1*} and P^{2*} respectively:

(86) $P^{1*} = \prod_{n=1}^N [V_n^1/V_n^0]^{1/N}$; $P^{2*} = \prod_{n=1}^N [V_n^2/V_n^0]^{1/N}$.

57 A constant can be added to each β_n and subtracted from each π^t without changing the regression model.
58 Write the regression model as $y = X\beta + u$. The vector of least squares estimates β^* for β is defined as $\beta^* \equiv (X^TX)^{-1}X^Ty$. Hence β^* satisfies the system of equations $X^T(X\beta^*) = X^Ty$ or $X^T[y - X\beta^*] = 0_K$ where X has K linearly independent columns. Thus taking the inner product of the kth column of X with $y - X\beta^*$ gives us 0 for each column k.

Thus this complete information country product dummy model leads to the geometric mean of the period 1 values relative to the corresponding period 0 values, $\prod_{n=1}^{N} [V_n^1/V_n^0]^{1/N}$, as the estimate for the period 1 housing price level P^1*, and to the geometric mean of the period 2 values relative to the corresponding period 0 values, $\prod_{n=1}^{N} [V_n^2/V_n^0]^{1/N}$, as the estimate for the period 2 housing price level P^2*. Note that this result is very similar to that of the chained matched model originally proposed by Wyngarden (1927) and Wenzlick (1952), except that instead of using the chain principle, the country product dummy method ends up using the fixed base principle.

We now consider a more realistic model where *not* every house in the sample trades in *each* period. In order to minimize notational complexities, we will consider only the case of two periods. Using our earlier notation, let S(0,1) be the set of housing units that sold in both periods 0 and 1. Taking into account the normalization (76), the estimating equations corresponding to these houses are:

(87) $\ln V_n^0 = \beta_n + u_n^0$; $\quad n \in S(0,1)$;
(88) $\ln V_n^1 = \beta_n + \pi^1 + u_n^1$; $\quad n \in S(0,1)$.

Let S(0~1) denote the set of housing units in the target population that sold in period 0 but not in period 1. The estimating equations for these observations are:

(89) $\ln V_m^0 = \gamma_m + u_m^0$; $\quad m \in S(0\sim 1)$

where γ_m is the logarithm of the quality adjustment factor for the mth housing unit that sold in period 0 but not in period 1. Similarly, let S(1~0) denote the set of housing units in the target population that sold in period 1 but not in period 0. The estimating equations for these observations are:

(90) $\ln V_k^1 = \delta_k + u_k^0$; $\quad k \in S(1\sim 0)$

where δ_k is the logarithm of the quality adjustment factor for the kth housing unit that sold in period 1 but not in period 0. The linear regression model defined by equations (87)-(90) is the same as the two country version of Summer's (1973) *country product dummy model* (with incomplete information); it is also identical to the two period case of the Aizcorbe, Corrado and Doms (2001) *dummy product hedonic regression model*.

Let π^1*, β_n*, γ_m* and δ_k* denote the least squares estimates of the parameters π^1, β_n, γ_m and δ_k that appear in (87)-(90). The stacked vector of dependent variables in equations (87)-(90) can be written as the sum of the vectors of exogenous variables times their corresponding least squares estimates plus the vector of least squares residuals. As noted above, the inner product of each exogenous vector with the vector of least squares residuals is zero. This means that the least squares estimators for the unknown parameters in the regression model satisfy the following equations:[59]

[59] This technique of proof was used by Diewert (2003a) in the context of a hedonic regression model.

$$(91)\ \sum_{n\in S(0,1)} \ln V_n^{\,1} + \sum_{k\in S(1\sim 0)} \ln V_k^{\,1} = \sum_{n\in S(0,1)} \beta_n^* + N(0,1)\,\pi^{1*} + \sum_{k\in S(1\sim 0)} \delta_k^* + N(1\sim 0)\,\pi^{1*} ;$$

$$(92)\ \ln V_n^{\,0} + \ln V_n^{\,1} = 2\beta_n^* + \pi^{1*} ; \qquad n\in S(0,1) ;$$

$$(93)\ \ln V_m^{\,0} = \gamma_m^* ; \qquad m\in S(0\sim 1) ;$$

$$(94)\ \ln V_k^{\,1} = \delta_k^* ; \qquad k\in S(1\sim 0)$$

where N(0,1) is the number of housing units that traded in both periods and N(1~0) is the number of housing units that sold in period 1but not period 0.

Use equations (94) to eliminate the δ_k^* in equation (91) and use equations (92) to eliminate the β_n^* from equation (91). The resulting equation shows that π^{1*} is equal to:

$$(95)\ \pi^{1*} = [1/N(0,1)] \sum_{n\in S(0,1)} \ln [V_n^{\,1}/V_n^{\,0}]$$

which is the arithmetic average of the logarithms of the sales price ratios for the matched models in the two periods. Exponentiating (95) shows that this simple hedonic regression model, where each housing unit has only a single dummy variable characteristic, leads to a period 0 to 1 price index that is equal to the equally weighted geometric mean of the selling prices in period 1 divided by the geometric mean of the corresponding selling prices of the matched models in period 0. Thus *in the two period case, the dummy variable hedonic regression model and the country product dummy model give exactly the same result as the matched model method that is used by statistical agencies*.

We now consider more general hedonic regression models[60]. A more *general hedonic regression model* is the following one:

$$(96)\ \ln V_n^{\,t} = \pi^t + \sum_{k=1}^{K} z_{nk}^{\,t}\, \beta_k + u_n^{\,t} ; \qquad t = 0,1,2,\ldots,T ;\ n\in S(t)$$

where S(t) is the set of housing units in the target population that sold (or were rented) in period t. As usual, $u_n^{\,t}$ is an independently distributed error term with mean 0 and constant variance, $V_n^{\,t}$ is the observed selling price (or rent) of housing unit n in period t and $z_{nk}^{\,t}$ is the amount of characteristic k that this housing unit possesses. The parameter π^t is equal to the logarithm of the constant quality price index for period t, P^t, so that $\pi^t = \ln P^t$ for t = 0,1,…,T. The unknown parameter β_k transforms amounts of the kth characteristic z_k into constant quality utility units for k = 1,…,K.[61]

What are the advantages and disadvantages of the general hedonic regression model defined by (96) compared to the Bailey, Muth and Nourse repeat sales model or to the

[60] The main features of a general hedonic regression model were laid out in Court (1939). This publication was not readily available to researchers and so the technique was not used widely until the work of Griliches (1971a) (1971b) popularized the technique. For a recent survey of the hedonic regression technique for making quality adjustments, see Triplett (2002).
[61] If the vector of characteristics contains a constant term, then there will be exact multicollinearity between this constant term and the time dummy variables, the π^t. Under these conditions, it will be necessary to make a normalization on the parameters such as $\pi^0 = 0$.

closely related product dummy hedonic regression models of Summers and Aizcorbe, Corrado and Doms?

The *main advantage* of the general hedonic regression model is that it uses *all* of the information on housing sales in each sample period in a nontrivial way whereas the repeat sales model does not use any information at all on isolated sales that take place in only one of the sample periods. We showed that in the two period case, the product dummy regression model led to an estimator of quality adjusted price change that did not use any information on unmatched sales. This feature of the product dummy hedonic regression model carries over to the case of many time periods; i.e., it is intuitively obvious that when an observation has its very own dummy variable in a linear regression model, then this observation will not be used to determine any of the other parameters in the model. Thus if the unmatched prices in the sample of housing prices behave differently than the matched prices, it can be seen that a general hedonic regression model can generate quite different price indexes than models that rely only on matched prices.[62] Put another way, a general hedonic regression model uses *all* of the sample data in a nontrivial way and not just the data that can be matched.

The *main disadvantage* of the general hedonic regression model compared to the repeat sales model and the product dummy hedonic regression model is the difficulty in determining just which characteristics should be included in the model. A closely related issue is that it is difficult to determine what the appropriate functional form for the hedonic regression is.[63] In section 10.4 below, some of the functional form problems associated with hedonic regression models for housing will be discussed in more detail.[64] But in general, hedonic regression models suffer from a lack of *reproducibility*; i.e., different statisticians and econometricians will collect data on different characteristics of housing and assume different functional forms for the hedonic regression model (96) and thus come up with different measures of quality adjusted price change.

We summarize the advantages and disadvantages of the repeat sales model as compared to a general hedonic regression model as follows.

The *main advantage* of the repeat sales model is:

- Reproducibility; i.e., different statisticians given the same data on the sales of housing units will come up with the same estimate of quality adjusted price change.

The *main disadvantages* of the repeat sales model are:

[62] For empirical evidence on this point, see the hedonic regression studies of Silver and Heravi (2001) (2002) (2003).
[63] Functional form problems for hedonic regressions are discussed in Diewert (2003a) (2003c).
[64] In particular, many hedonic regression studies use the logarithm of a transaction price as the dependent variable. This specification of the hedonic model is usually not consistent with the additive nature of the structure and land components of a property and the multiplicative nature of the depreciation adjustment as appears in equations (47) and (56) which defined the value of a specific property in successive periods.

- It does not use all of the available information on housing unit sales; it uses only information on housing units that have sold more than once during the sample period.
- It cannot deal adequately with depreciation of the housing structure; i.e., depreciation is exactly collinear with the time dummy variables π^t and thus cannot be distinguished from the effects of price change. Conversely, a general hedonic regression model for housing can adjust for the effects of depreciation if the age of the structure is known at the time of sale (or rental).
- It cannot deal adequately with housing units that have undergone major repairs or renovations.[65] Conversely, a general hedonic regression model for housing can adjust for the effects of renovations and extensions if (real) expenditures on renovations and extensions are known at the time of sale (or rental).[66]

Both the repeat sales and general hedonic regression approaches to the construction of constant quality price indexes for housing suffer from another problem that has not been mentioned up to now and that is that both of these methods do not allow the prices to be weighted according to their economic importance. Thus if the statistical agency adopts a superlative index[67] as its target index, then prices should be weighted by either quantities or by expenditure shares and from this perspective, equally weighted geometric means of price relatives are not necessarily close to their superlative counterparts. The regression models discussed in this section have not made any mention of weights so the resulting measures of quality adjusted price change could be different from their weighted counterparts.[68] Another problem that has not been discussed is the possibility that house sales prices might exhibit seasonal fluctuations.[69] The general hedonic regression model cold accommodate seasonal prices by having seasonal dummy variables as explanatory variables.

[65] Case and Shiller (1989) use a variant of the repeat sales method using US data on house sales in four major cities over the years 1970-1986. They attempt to deal with the depreciation and renovation problems as follows: "The tapes contain actual sales prices and other information about the homes. We extracted from the tapes for each city a file of data on houses sold twice for which there was no apparent quality change and for which conventional mortgages applied." Karl E. Case and Robert J. Shiller (1989; 125-126).

[66] However, usually information on maintenance and renovation expenditures is not available in the context of estimating a hedonic regression model for housing. Malpezzi, Ozanne and Thibodeau (1987;375-6) comment on this problem as follows: "If all units are identically constructed, inflation is absent, and the rate of maintenance and repair expenditures is the same for all units, then precise measurement of the rate of depreciation is possible by observing the value or rent of two or more units of different ages. … To accurately estimate the effects of aging on values and rents, it is necessary to control for inflation, quality differences in housing units, and location. The hedonic technique controls for differences in dwelling quality and inflation rates but cannot control for most differences in maintenance (except to the extent that they are correlated with location)."

[67] Superlative indexes were initially introduced as approximations to economic cost of living indexes; see Diewert (1976) (1978). But it turns out that various superlative indexes emerge as useful target indexes from the perspectives of the fixed basket, axiomatic and stochastic approaches as well; see Diewert (2002a; 565-581).

[68] Diewert (2002b) (2003c) discusses how weights can be introduced into both the product dummy and general hedonic regression models.

[69] Case and Shiller (1989; 127) note that US house prices tend to have a seasonal peak in July.

Our conclusion at this point is that there is no completely satisfactory solution to the problems involved in constructing constant quality price indexes for the stock of owner occupied housing. The hedonic regression approach seems to be superior in principle to the repeat sales approach since the latter approach cannot deal adequately with depreciation and renovations to the structure part of a housing unit. However, in practice, the hedonic regression approach has limitations due to its lack of reproducibility and the lack of information on repairs and renovations.

There are many other difficulties associated with measuring the price and quantity of Owner Occupied Housing services. The following section discusses some of the problems involved in modeling the costs of certain expenditures that are tied to the ownership of a home.

10. The Treatment of Costs Tied to Owner Occupied Housing

There are many costs that are quite directly tied to home ownership. However, it is not always clear how these costs can be decomposed into price and quantity components. Several of these cost components are listed below and some suggestions for forming their associated prices are suggested.

10.1 The Treatment of Mortgage Interest Costs

The derivation of the user cost or expected rental price that an owner of a home should charge for the use of the dwelling unit for one period implicitly assumed that the owner had no mortgage interest costs so that the interest rate r^0 referred to the owner's opportunity cost of equity capital. In this section, the case where the owner has a mortgage on the property is considered.

Recall the notation in the previous section where the user cost or imputed rental cost, R^0, for an equity financed dwelling unit was obtained; see (51). Suppose now that the property purchase is partly financed by a mortgage of M^0 dollars at the beginning of period 0. Let f^0 be the fraction of the beginning of period 0 market value of the property that is financed by the mortgage so that

$$(97)\ M^0 = f^0V^0 = f^0[P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0].$$

Let the one period nominal mortgage interest rate be $r_M{}^0$. The owner's period 0 benefits of owning the dwelling unit remain the same as in section 8 and are equal to V^{1a} defined by (50). However, the period 0 costs are now made up of an explicit mortgage interest cost equal to $M^0(1+r_M{}^0)$ plus an imputed equity cost equal to $(1-f^0)V^0(1+r^0)$. Thus the new imputed rent for using the property during period 0 is now

$$\begin{aligned}(98)\ R^0 &\equiv (1-f^0)V^0(1+r^0) + M^0(1+r_M{}^0) - V^{1a} \\ &= (1-f^0)[P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0](1+r^0) + f^0[P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0](1+r_M{}^0) \\ &\quad - [P_S{}^{1a}(1-\delta_0)Q_S{}^0 + P_L{}^{1a}Q_L{}^0]\end{aligned}$$

$= p_S{}^{0*}Q_S{}^0 + p_L{}^{0*}Q_L{}^0$

where the new mortgage interest adjusted period 0 *user costs of structures and land*, $p_S{}^{0}*$ and $p_L{}^{0}*$, are defined as follows:

$$(99)\quad p_S{}^{0}* \equiv [(1 + r^0)(1 - f^0) + (1 + r_M{}^0)f^0 - (1 + i_S{}^0)(1 - \delta_0)]P_S{}^0$$
$$= [(r^0 - i_S{}^0)(1 - f^0) + (r_M{}^0 - i_S{}^0)f^0 + \delta_0(1 + i_S{}^0)]P_S{}^0 ;$$

$$(100)\quad p_L{}^{0}* \equiv [(1 + r^0)(1 - f^0) + (1 + r_M{}^0)f^0 - (1 + i_L{}^0)]P_S{}^0$$
$$= [(r^0 - i_L{}^0)(1 - f^0) + (r_M{}^0 - i_L{}^0)f^0]P_S{}^0 .$$

Comparing the new user costs for structures and land defined by (99) and (100) with the corresponding equity financed user costs defined by (52) and (53) in the previous section, it can be seen that the old equity opportunity cost of capital r^0 is now replaced by a weighted average of this equity opportunity cost and the mortgage interest rate, $r^0(1-f^0) + r_M{}^0f^0$, where f^0 is the fraction of the beginning of period 0 value of the dwelling unit that is financed by the mortgage.

Central bankers often object to the inclusion of mortgage interest in a Consumer Price Index. However, examination of the last equation in (99) and in (100) shows that the *nominal* mortgage interest rate $r_M{}^0$ has an offsetting benefit due to *anticipated price inflation* in the price of structures, $i_S{}^0$ in (99), and in the price of land, $i_L{}^0$ in (100), so as usual, what counts in these user cost formulae are *real* interest costs rather than *nominal* ones.

10.2 The Treatment of Property Taxes

Recall the user costs of structures and land defined by (52) and (53) in section 8 above. It is now supposed that the owner of the housing unit must pay the property taxes $T_S{}^0$ and $T_L{}^0$ for the use of the structure and land respectively during period 0.[70] Define *the period 0 structures tax rate* $\tau_S{}^0$ and *land tax rate* $\tau_L{}^0$ as follows:

$$(101)\quad \tau_S{}^0 \equiv T_S{}^0/P_S{}^0Q_S{}^0 ;$$
$$(102)\quad \tau_L{}^0 \equiv T_L{}^0/P_L{}^0Q_L{}^0 .$$

The *new imputed rent for using the property during period 0*, R^0, including the property tax costs, is defined as follows:

$$(103)\quad R^0 \equiv V^0(1 + r^0) + T_S{}^0 + T_L{}^0 - V^{1a}$$
$$= [P_S{}^0Q_S{}^0 + P_L{}^0Q_L{}^0](1 + r^0) + \tau_S{}^0P_S{}^0Q_S{}^0 + \tau_L{}^0P_L{}^0Q_L{}^0$$
$$- [P_S{}^0(1 + i_S{}^0)(1 - \delta_0)Q_S{}^0 + P_L{}^0(1 + i_L{}^0)Q_L{}^0]$$
$$= p_S{}^0Q_S{}^0 + p_L{}^0Q_L{}^0$$

[70] If there is no breakdown of the property taxes into structures and land components, then just impute the overall tax into structures and land components based on the beginning of the period values of both components.

where separate period 0 *tax adjusted user costs of structures and land*, p_S^0 and p_L^0, are defined as follows:

$$(104)\ p_S^0 \equiv [(1 + r^0) - (1 + i_S^0)(1 - \delta_0) + \tau_S^0]P_S^0$$
$$= [r^0 - i_S^0 + \delta_0(1 + i_S^0) + \tau_S^0]P_S^0\ ;$$

$$(105)\ p_L^0 \equiv [(1 + r^0) - (1 + i_L^0) + \tau_L^0]P_L^0$$
$$= [r^0 - i_L^0 + \tau_S^0]P_L^0\ .$$

Thus the property tax rates, τ_S^0 and τ_L^0 defined by (101) and (102), enter the user costs of structures and land, p_S^0 and p_L^0 defined by (104) and (105), in a simple additive manner; i.e., these terms are additive to the previous depreciation and real interest rate terms.[71]

10.3 The Treatment of Property Insurance

At first glance, it would seem that *property insurance* could be treated in the same manner as the treatment of property taxes in the previous subsection. Thus let C_S^0 be the cost of insuring the structure at the beginning of period 0 and define *the period 0 structures premium rate* γ_S^0 as follows:

$$(106)\ \gamma_S^0 \equiv C_S^0/P_S^0Q_S^0\ .$$

The new *imputed rent* for using the property during period 0, R^0, including property tax and insurance costs, is defined as follows:

$$(107)\ R^0 \equiv V^0(1 + r^0) + T_S^0 + T_L^0 + C_S^0 - V^{1a}$$
$$= [P_S^0Q_S^0 + P_L^0Q_L^0](1 + r^0) + \tau_S^0P_S^0Q_S^0 + \tau_L^0P_L^0Q_L^0 + \gamma_S^0\, P_S^0Q_S^0$$
$$- [P_S^0(1 + i_S^0)(1 - \delta_0)Q_S^0 + P_L^0(1 + i_L^0)Q_L^0]$$
$$= p_S^0Q_S^0 + p_L^0Q_L^0$$

where separate period 0 *tax and insurance adjusted user costs of structures and land*, p_S^0 and p_L^0, are defined as follows:

$$(108)\ p_S^0 \equiv [(1 + r^0) - (1 + i_S^0)(1 - \delta_0) + \tau_S^0 + \gamma_S^0]P_S^0$$
$$= [r^0 - i_S^0 + \delta_0(1 + i_S^0) + \tau_S^0 + \gamma_S^0]P_S^0\ ;$$

$$(109)\ p_L^0 \equiv [(1 + r^0) - (1 + i_L^0) + \tau_L^0]P_L^0$$
$$= [r^0 - i_L^0 + \tau_S^0]P_L^0\ .$$

Thus the insurance premium rate γ_S^0 appears in the user cost of structures, p_S^0 defined by (108), in an additive manner, analogous to the additive property tax rate term.[72] If it is

[71] If the price statistician uses the national accounts imputation for the value of owner occupied housing services, care should be taken to ensure that the value of property taxes is included in this imputation.

[72] This treatment of property insurance dates back to Walras (1954; 268-269).

desired to have a separate CPI price component for insurance, then the corresponding period 0 and 1 prices can be defined as $\gamma_S^0 P_S^0$ and $\gamma_S^1 P_S^1$ respectively while the corresponding period 0 and 1 expenditures can be defined as $\gamma_S^0 P_S^0 Q_S^0$ and $\gamma_S^1 P_S^1 (1-\delta) Q_S^0$ respectively.[73] Of course, if this separate treatment is implemented, then these terms have to be dropped from the corresponding user costs of structures.

The above treatment of property taxation and insurance assumes that the property taxes and the premium payments are made at the *end* of the period under consideration; see (107) above. While this may be an acceptable approximation for the payment of property taxes, it is not acceptable for the payment of insurance premiums: the premium *must* be paid at the *beginning* of the period of protection rather than at the end. When this complication is taken into account, the user cost of structures becomes

$$(110)\ p_S^0 \equiv [(1 + r^0) - (1 + i_S^0)(1 - \delta_0) + \tau_S^0 + \gamma_S^0(1 + r^0)]P_S^0$$
$$= [r^0 - i_S^0 + \delta_0(1 + i_S^0) + \tau_S^0 + \gamma_S^0(1 + r^0)]P_S^0 .$$

There are some additional problems associated with the modeling of property insurance:

- The above user cost derivations assume that the *risk* of property damage remains constant from period to period. If the risk of damage changes, then an argument can be made for quality adjustment of the premium to hold constant the risk so that like can be compared with like.
- The *gross premium approach* to insurance is taken in the above treatment; i.e., it is assumed that dwelling owners pay premiums for property protection services, no matter whether they have a claim or not. In the *net premium approach*, payments to settle claims are subtracted from the gross premium payments.
- The property protection may not be complete; i.e., the insurance policy may have various limitations on the type of claim that is allowed and there may be a deductible or damage threshold, below which no claim is allowed. If the deductible changes from period to period, then the price statistician is faced with a rather complex quality adjustment problem.

Thus it can be seen that there are many difficult problems that remain to be resolved in this area.

10.4 The Treatment of Maintenance and Renovation Expenditures

Another problem associated with home ownership is the treatment of *maintenance expenditures, major repair expenditures and expenditures associated with renovations or additions*.

Empirical evidence suggests that the normal decline in a structure due to the effects of aging and use can be offset by maintenance and renovation expenditures. How exactly

[73] Similarly, if it is desired to have a separate CPI price component for property taxes on structures, then the corresponding period 0 and 1 prices can be defined as $\tau_S^0 P_S^0$ and $\tau_S^1 P_S^1$ respectively while the corresponding period 0 and 1 expenditures can be defined as $\tau_S^0 P_S^0 Q_S^0$ and $\tau_S^1 P_S^1 (1-\delta) Q_S^0$ respectively.

should these expenditures be treated in the context of modeling the costs and benefits of home ownership?

A common approach in the national accounts literature is to treat major renovation and repair expenditures as capital formation and smaller routine maintenance and repair expenditures as current expenditures. If this approach is followed in the CPI context, then these smaller routine maintenance expenditures can be treated in the same manner as other nondurable goods and services. The major renovation and repair expenditures do not enter the CPI in the period that they are made but these expenditures are capitalized and added to expenditures on new structures for the period under consideration, so that period 0 investment in structures in constant dollars, I_S^0 say[74], would include both types of expenditures. Let Q_S^0 and Q_S^1 be the stocks (in constant quality units) of owner occupied structures in the reference population at the beginning of period 0 and 1 respectively. Then if the geometric model of depreciation is used, so that the constant period to period depreciation rate δ is applicable, then the beginning of period 1 stock of owner occupied structures Q_S^1 is related to the beginning of period 0 stock of structures Q_S^0 and the period 0 investment in structures I_S^0 according to the following equation:

$$(111)\ Q_S^1 = (1 - \delta)Q_S^0 + I_S^0 .$$

Thus if declining balance depreciation is assumed for structures, then the treatment of major repair and renovation expenditures does not pose major *conceptual* problems using a conventional capital accumulation model: it is only necessary to have an estimate for the monthly or quarterly depreciation rate δ, a starting value for the stock of owner occupied structures for some period, information on new purchases of residential housing structures by the household sector, information on expenditures by owners on major repairs and renovations and a construction price index for new residential structures. With this information on a timely basis, up to date CPI weights for the stock of owner occupied structures could be constructed.[75]

We now look at how major repair and renovation expenditures could be treated in a repeat sales regression model that used transactions data on the sale of the *same* housing unit in two or more periods. In order to minimize notational complexities, consider a highly simplified situation where data on the sale of N houses of a relatively homogeneous type for two consecutive periods are available. Suppose these *sale prices* are V_n^0 for period 0 and V_n^1 for period 1, for n = 1,2,…,N. Suppose that a *price index for structures* of this type of property in period 0, P_S^0, and a corresponding *price index for land* in period 0, P_L^0, have been constructed.[76] The price statistician's problem is to use the data on the matched sales for the two periods in order to construct estimates of these two indices for period 1; i.e., the problem is to construct P_S^1 and P_L^1.

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[74] Let VI_S^0 be the nominal value of investment in new owner occupied structures in period 0 plus the value of major renovation expenditures made during period 0. Then the constant dollar quantity of investment could be defined as $I_S^0 \equiv VI_S^0/P_S^0$ where P_S^0 is the period 0 construction price index for new structures.

[75] However, the *practical* problems involved in obtaining all of this information on a timely basis are not trivial. Variants of this approach were used by Christensen and Jorgenson (1969) and Leigh (1980) in order to construct estimates of the stock of residential structures in the US.

[76] If these period 0 indices are not available, then set P_S^0 and P_L^0 equal to 1.

The period 0 *dwelling unit values* for the N properties can be decomposed into the structure and land components as follows:

(112) $V_n^0 = V_{Sn}^0 + V_{Ln}^0 = \alpha_n P_S^0 Q_{Sn}^0 + \beta_n P_L^0 Q_{Ln}^0$; $\qquad$ n = 1,2,…,N

where V_{Sn}^0 and V_{Ln}^0 are the estimated period 0 values of the structure and land of property n in period 0, P_S^0 and P_L^0 are the (known) price index values for structures and land for all properties of this type in period 0 and Q_{Sn}^0 and Q_{Ln}^0 are (known) estimates of the quantity of structures and land for property n. The numbers α_n and β_n are *property n quality adjustment factors* that convert the property standardized values of structures and land, $P_S^0 Q_{Sn}^0$ and $P_L^0 Q_{Sn}^0$ respectively, into the period 0 actual market values, V_{Sn}^0 and V_{Ln}^0 respectively; i.e., if estimates of the period 0 market values of the structures and land for property n are available, then α_n and β_n can be defined as follows:

(113) $\alpha_n \equiv V_{Sn}^0/P_S^0 Q_{Sn}^0$; $\beta_n \equiv V_{Ln}^0/P_L^0 Q_{Ln}^0$; $\qquad$ n = 1,…,N.

Suppose that information on the dollar amount of major repairs and renovations made to property n during period 0, VR_n^0, is also available for each property n in the sample of properties. Then the period 1 value for property n, V_n^1, should be approximately equal to

(114) $V_n^1 = \alpha_n P_S^1 (1-\delta) Q_{Sn}^0 + VR_n^0 + \beta_n P_L^1 Q_{Ln}^0$; $\qquad$ n = 1,2,…,N

where δ is the geometric depreciation rate for structures. All of the variables on the right hand side of (114) are assumed to be known with the exception of the period 1 price index values for structures and land, P_S^1 and P_L^1 respectively, and the one period geometric depreciation rate, δ. If the number of observations N is greater than three, then it would appear that these three parameters, P_S^1, P_L^1 and δ, could be estimated by a linear regression using the N equations in (114) as estimating equations. However, it turns out that this is not quite correct. The problem is that the parameters P_S^1 and $(1-\delta)$ appear in (114) in a multiplicative fashion so that while the product of these two terms will be nicely identified by the regression, *the individual terms cannot be uniquely identified*. This is just a reappearance of the same problem that was discussed earlier in section 7 on unique consumer durables: the separate effects of aging of the asset (depreciation or capital consumption) and price appreciation over time *cannot* be separately identified using just market data on resales if the housing unit is regarded as a unique asset.[77]

There are three possible solutions to this identification problem:

[77] Recall equation (46) above. This fundamental identification problem was recognized by Bailey, Muth and Nourse (1963; 936) in the original repeat sales housing article but it was ignored by them and subsequent users of the repeat sales methodology. Another problem with the housing hedonic regression literature is that usually, the logarithm of the purchase price is taken as the dependent variable in the regression. While this specification has some advantages, it does not recognize properly the additive nature of the structure and land components of the housing property. A final problem with the traditional hedonic housing literature is that usually, separate price indices for land and structures are not estimated. It is important to allow for separate price indices for these two components since usually, the price of land is more volatile and tends to increase faster than the price of structures over long periods of time.

- Use an external estimate of the depreciation rate δ;
- Use an external construction price index P_S^1 instead of estimating it as a parameter in equations (114);
- Abandon the repeat sales approach and use a hedonic regression approach instead.

What would a hedonic regression model look like, taking into account the approximate additivity of the value of the housing structure and the value of the land that the structure sits on? If the renovations problem is ignored and geometric depreciation of the structure is assumed, then the value of a housing unit n in period t that is v periods old, V_n^t, should be approximately equal to the depreciated value of the structure plus the value of the land plus an error term; i.e., the following relationship should hold approximately:

$$(115)\ V_n^t = P_S^t(1-\delta)^v Q_{Sn} + P_L^t Q_L + u_n^t$$

where δ is the one period geometric depreciation rate, Q_{Sn} is the number of square meters of floor space of the original structure for housing unit n, Q_L is the number of square meters of land that the housing structure sits on and u_n^t is an error term. P_S^t is the beginning of *period t price level for structures* of this type and P_L^t is the corresponding *price of land* for this class of housing units. As long as there is more than one vintage of structure in the sample (i.e., more than one v), then the parameters P_S^t, P_L^t and δ can be identified by running a nonlinear regression model using equations (115). Why can the price levels be identified in the present hedonic regression model whereas they could not be identified in the repeat sales model? The answer is that the hedonic model (115) does *not* assume property specific quality adjustment factors for each housing unit; instead, *all* of the housing units in the sample are assumed to be of comparable quality once prices are adjusted for the age of the unit and the quantity (in square meters) of original structure and the quantity of land.

Unfortunately, many housing structures that may have started their lives as identical structures do not remain the same over time, due to differing standards of maintenance as well as major renovations and additions to some of the structures. To model this phenomenon, let R_n^t be real maintenance, repair and renovation expenditures on housing unit n during period t and suppose that these real expenditures depreciate at the geometric rate δ_R. It is reasonable to assume that these expenditures *add* to the value of the housing unit and so equations (115) should be replaced by the following equations:

$$(116)\ V_n^t = P_S^t(1-\delta)^v Q_{Sn} + P_R^t[R_n^t + (1-\delta_R)R_n^{t-1} + (1-\delta_R)^2 R_n^{t-2} + \ldots + (1-\delta_R)^v R_n^{t-v}] \\ + P_L^t Q_L + u_n^t$$

where P_R^t is the period t price level for real maintenance, repair and renovation expenditures on this class of housing units. If information on these real renovation and repair expenditures, R_n^t, R_n^{t-1}, R_n^{t-2},…,R_n^{t-v}, is available for each housing unit in the sample of housing units that sold in period t, then the parameters P_S^t, P_L^t, P_R^t, δ and δ_R can be identified by running a nonlinear regression model using equations (116).[78]

However, a major practical problem with implementing a hedonic regression model along the above lines is that usually accurate data on renovation and repair expenditures on a particular dwelling unit between the construction of the initial housing unit and the present period are not available. Without accurate data on repairs and renovations, it will be impossible to obtain accurate estimates of the unknown parameters in the hedonic regression model.

A final practical problem with the above hedonic regression model will be mentioned. Theoretically, "normal" maintenance expenditures could be included in the renovation expenditure terms R_n^t in (116). If this is done, then including normal maintenance expenditures in R_n^t will have the effect of increasing the estimated depreciation rates δ and δ_R. Thus different statistical agencies that have different criteria for deciding where to draw the line between "normal" maintenance and "major" repair and renovations will produce different estimated depreciation rates.

It can be seen that there are many unresolved issues in this area: statistical agency best practice has not yet emerged.

10.5 The Treatment of the Transactions Costs of Home Purchase

Another cost of home ownership needs to be discussed. Normally, when a family purchases a dwelling unit, they have to pay certain fees and costs, which can include:

- The commissions of real estate agents who help the family find the "right" property.
- Various transactions taxes that governments can impose on the sale of the property.
- Various legal fees that might be associated with the transfer of title for the property.

Should the above fees be immediately expensed in the period of purchase or should they simply be regarded as part of the purchase price of the property and hence be depreciated over time in a manner analogous to the treatment of structures in the national accounts?

An argument can be made for either treatment. From the viewpoint of the opportunity cost treatment of purchases of durable goods, the relevant price of the dwelling unit in the periods following the purchase of the property is the after tax and transactions fees value of the property. This viewpoint suggests that the transactions costs of the purchaser should be immediately expensed in the period of purchase. However, from the viewpoint of a landlord who has just purchased a dwelling unit for rental purposes, it would not be sensible to charge the tenant the full cost of these transactions fees in the first month of

[78] Alternatively, if price levels are available for P_S^t and P_R^t from construction price indexes, then these parameters do not have to be estimated.

rent. The landlord would tend to capitalize these costs and recover them gradually over the time period that the landlord expects to own the property. Thus either treatment could be justified and the statistical agency will have to decide which treatment is most convenient from their particular perspective.

11. User Costs for Landlords versus Owners

In the previous section, the various costs associated with home ownership were discussed. Both home owners and landlords face these costs. Thus they will be reflected in market rents and this "fact" must be kept in mind if the imputed rent approach is used to value the services of Owner Occupied Housing. If some or all of these associated costs of OOH are covered elsewhere in the CPI (e.g., home insurance could be separately covered), then the value of imputed rents for OOH must be *reduced* by the amount of these expenditures covered elsewhere.

However, in addition to the costs of home ownership that were covered in the previous section, landlords face a number of *additional costs* compared to the home owner. These additional costs will be reflected in market rents and thus if market rents are used to impute the services provided by the ownership of a dwelling unit, then these extra costs should also be *removed* from the market rents that are used for imputation purposes, since they will not be relevant for owner occupiers. These additional landlord specific costs will be discussed in sections 11.1 to 11.5 below.

11.1 Damage Costs

Tenants do not have the same incentive to take care of a rental property compared to an owned property and so depreciation costs for a rental property are likely to exceed depreciation rates for comparable owned properties. Usually, landlords demand damage deposits but often these deposits are not sufficient to cover the costs of the actual damages that some tenants inflict.

11.2 Nonpayment of Rent and Vacancy Costs

At times, tenants run into financial difficulties and are unable to pay landlords the rent that is owned. Usually, eviction is a long drawn out process and so landlords can lose several months of rent before a nonpaying tenant finally leaves. The landlord also incurs extra costs compared to a homeowner when a rental property remains vacant due to lack of demand.[79] These extra costs will be reflected in market rents but should not be reflected in the user costs of OOH.

11.3 Billing and Maintenance Costs

[79] The demand for rental properties can vary substantially over the business cycle and this can lead to depressed rents or very high rents compared to the user costs of home ownership. Thus imputed rents based on market rents of similar properties can differ substantially from the corresponding user costs of OOH over the business cycle.

A (large) landlord may have to rent office space and employ staff to send out monthly bills to tenants and employ staff to respond to requests for maintenance. A homeowner who provides his or her time in order to provide maintenance services[80] provides this time at his or her *after income tax wage rate* which may be lower than the *before income tax wage rate* that a landlord must pay his or her employees. The net effect of these factors leads to higher market rents compared to the corresponding owner occupied user cost.

11.4 The Opportunity Cost of Capital

The homeowner's *after tax* opportunity cost of capital that appeared in the various user cost formulae considered earlier in this Chapter will typically be *lower* than the landlord's *before tax* opportunity cost of capital. Put another way, the landlord has an extra income tax cost compared to the homeowner. In addition, the landlord may face a higher risk premium for the use of capital due to the risks of damage and nonpayment of rent. However, care must be taken so that these additional landlord costs are not counted twice; i.e., in the present subsection as well as in subsections 11.1 and 11.2 above.

11.5 The Supply of Additional Services for Rental Properties

Often, rental properties will contain some major consumer durables that homeowners have to provide themselves, such as refrigerators, stoves, washing machines, driers and air conditioning units. In addition, landlords may pay for electricity or fuel in some rental apartments. Thus to make the market rental comparable to an owner occupied imputed rent, the market rental should be adjusted downwards to account for the above factors (which will appear elsewhere in the expenditures of owner occupiers).

The factors listed above will tend to make observed market rental prices *higher* than the corresponding user cost for an owner occupier of a property of the same quality. Thus if the imputed rental approach is used to value the services of OOH, then these market based rents should be adjusted downward to account for the above factors.

Although all of the above factors will tend to lead to an *upward* bias if unadjusted market rental rates are used to impute the services of OOH, there is another factor not discussed thus far that could lead to a large *downward* bias. That factor is *rent controls*.

Under normal conditions, the acquisitions approach to the treatment of OOH will give rise to the smallest expenditures, the user cost approach will give rise to the next highest level of expenditures and the use of imputed market rentals will give the largest level of expenditures for owner occupied housing. For the first two approaches, a main driver of the price of OOH is the price of new housing construction. For the user cost approach, another main driver is the price of land. For the imputed rent approach, the main driver of the price of OOH is the rental price index.

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[80] Typically, these imputed maintenance costs will not appear in the CPI but if the user cost of an owned dwelling unit is to be comparable with the market rent of a similar property, these imputed labour costs should be included.

The above discussion is far from being complete and definitive but it does illustrate that it is not completely straightforward to impute market rental rates to owner occupied dwelling units. Care must be taken to ensure that the "correct" expenditure weights are constructed.[81]

As can be seen from the material above, the treatment of owner occupied housing presents special difficulties. Astin discussed some of the difficulties that the European Union encountered in trying to find the "best" approach to use in its Harmonized Index of Consumer Prices as follows:

"A special coverage problem concerns owner-occupied housing. This has always been one of the most difficult sectors to deal with in CPIs.

Strictly, the price of housing should not be included in a CPI because it is classified as capital. On the other hand, the national accounts classifies imputed rents of owner-occupiers as part of consumers' expenditure. This is a reasonable thing to do if the aim is to measure the volume of consumption of the capital resource of housing. But that is not what a CPI is measuring.

Some countries, following the compensation index concept, would prefer to have mortgage interest included in the HICP. This approach could indeed be defended for a compensation index, because there is no doubt that the monthly mortgage payment is an important element in the budget of many households: a rise in the interest rate acts in exactly the same way as a price increase from the point of view of the individual household. But this is not acceptable for a wider inflation index.

So, after many hours of debate, the Working Party came to the conclusion that there were just two options. The first was to simply exclude owner-occupied housing from the HICP. One could at least argue that this was a form of harmonization, although it is worrying that there are such large differences between Member States in the percentages of the population which own or rent their dwellings. Exclusion also falls in line with the international guideline issued 10 years ago by the ILO. Furthermore, it would be possible to supplement the HICP with a separate house price index, which could be used by analysts as part of a battery of inflation indicators.

The second option was to include owner-occupied housing on the basis of acquisition costs, essentially treating them like any other durable. Most secondhand housing would be excluded: in practice the index would include new houses plus a small volume of housing new to the household sector (sales from the company or government sectors to the household sector).

The main problem here is practical: several countries do not have new house price indices and their construction could be difficult and costly. A Task Force is at present examining these matters. Final recommendations are due at the end of 1999." John Astin (1999; 5).

Due to the complexities involved in modeling the treatment of OOH, final recommendations have still not emerged for the HICP.

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[81] Crone, Nakamura and Voith (2000) have a very useful paper using hedonic techniques to estimate both a rent index and a selling price index for housing in the U.S. They also suggest that *capitalization rates* (i.e., the ratio of the market rent of a housing property to its selling price) can be applied to an index of housing selling prices in order to obtain an imputed rent index for OOH. This is adequate as a first approximation but as the authors note, capitalization rates can change over time (due to changes in nominal interest rates, depreciation rates and expected housing inflation rates). Also, as we have seen in this section and the previous section, actual market housing rents can be expected to be considerably higher than the corresponding imputed rents for owner occupied units of the same quality and hence the use of unadjusted capitalization rates to convert the value of the owner occupied stock of housing into imputed rents can lead to a considerable *weighting bias*.

A fourth approach to the treatment of housing will be studied in the following section. Since this approach has only been applied to owner occupied dwellings, it is not as "universal" as the other 3 approaches.[82]

12. The Payments Approach

A fourth possible approach to the treatment of owner occupied housing, the *payments approach*, is described by Goodhart as follows:

"The second main approach is the payments approach, measuring actual cash outflows, on down payments, mortgage repayments and mortgage interest, or some subset of the above. This approach always, however, includes mortgage interest payments. This, though common, is analytically unsound. First, the procedure is not carried out consistently across purchases. Other goods bought on the basis of credit, e.g., credit card credit, are usually not treated as more expensive on that account (though they have been in New Zealand). Second, the treatment of interest flows is not consistent across persons. If a borrower is worse off in some sense when interest rates rise, then equivalently a lender owning an interest bearing asset is better off; why measure one and not the other? If I sell an interest earning asset, say a money market mutual fund holding, to buy a house, why am I treated differently to someone who borrows on a (variable rate) mortgage? Third, should not the question of the price of any purchase be assessed separately from the issue of how that might be financed? Imports, inventories and all business purchases tend to be purchased in part on credit. Should we regard imports as more expensive, when the cost of trade credit rises? Money, moreover, is fungible. As we know from calculations of mortgage equity withdrawal, the loan may be secured on the house but used to pay for furniture. When interest rates rise, is the furniture thereby more expensive? Moreover, the actual cash out-payments totally ignore changes in the on going value of the house whether by depreciation, or capital loss/gain, which will often dwarf the cash flow. Despite its problems, such a cash payment approach was used in the United Kingdom until 1994 and still is in Ireland." Charles Goodhart (2001; F350-F351).

Thus the *payments approach* to owner occupied housing is a kind of a *cash flow approach* to the costs of operating an owner occupied dwelling. Possible objections to this approach are that it ignores the opportunity costs of holding the equity in the owner occupied dwelling, it ignores depreciation and it uses nominal interest rates without any offset for inflation. However, if the payments approach is adjusted for these imputed costs, then the result is a rather complicated user cost approach to the treatment of housing. Nevertheless, as was mentioned in Chapter 10, under some conditions, the payments approach to the treatment of owner occupied housing may be a reasonable compromise. In general, the payments approach will tend to lead to much smaller monthly expenditures on owner occupied housing than the other 3 main approaches, except during periods of high inflation, when the nominal mortgage rate term becomes very large without any offsetting item for inflation.[83]

13. Alternative Approaches for Pricing Owner Occupied Housing

[82] The acquisitions, user cost and rental equivalence approaches can be applied to any consumer durable but of course, to apply the rental equivalence approach, appropriate rental or leasing markets for the durable must exist.

[83] If there is high inflation, then the statistical agency using the payments approach may want to consider adjusting nominal mortgage interest rates for the inflation component as was done in section 10.1 above.

For consumer durables that have long useful lives, the usual acquisitions approach will not be adequate for CPI users who desire prices that measure the service flows that consumer durables generate. This is particularly true for owner occupied housing. Hence it will be useful to many users if, in addition to the acquisitions approach, the statistical agency implements a variant of either the rental equivalence approach or the user cost approach for long lived consumer durables and for owner occupied housing in particular. Users can then decide which approach best suits their purposes. Any one of the three main approaches could be chosen as the approach that would be used in the "headline" CPI. The other two approaches could be made available to users as "analytic tables".

We conclude this paper by outlining some of the problems involved in implementing the three main approaches to the measurement of price change for Owner Occupied Housing.

13.1 The Acquisitions Approach

In order to implement the acquisitions approach, a constant quality price index for the sales of new residential housing units will be required.

13.2 The Rental Equivalence Approach

Option 1: Using Home Owner's Estimates of Rents

In this option, homeowners would be surveyed and asked to estimate a rental price for their housing unit. Problems with this approach are:

- Homeowners may not be able to provide very accurate estimates for the rental value of their dwelling unit.
- The statistical agency should make an adjustment to these estimated rents over time in order to take into account the effects of depreciation, which causes the quality of the unit to slowly decline over time (unless this effect is offset by renovation and repair expenditures).[84]
- Care must be taken to determine exactly what extra services are included in the homeowner's estimated rent; i.e., does the rent include insurance, electricity and fuel or the use of various consumer durables in addition to the structure? If so, these extra services should be stripped out of the rent, since they are covered elsewhere in the consumer price index.[85]

Option 2: Using a Hedonic Regression Model of the Rental Market to Impute Rents

[84] Recall section 8 above.

[85] However, it could be argued that these extra services that might be included in the rent are mainly a weighting issue; i.e., it could be argued that the *trend* in the homeowner's estimated rent would be a reasonably accurate estimate of the trend in the rents after adjusting for the extra services included in the rent.

In this option, the statistical agency would collect data on rental properties and their characteristics and then use this information to construct a hedonic regression model for the housing rental market.[86] Then this model would be used to impute prices for owner occupied properties. Problems with this approach are:

- It is information intensive; in addition to requiring information on the rents and characteristics of rental properties, information on the characteristics of owner occupied properties would also be required.
- The characteristics of the owner occupied population could be quite different from the characteristics of the rental population. In particular, if the rental market for housing is subject to rent controls, this approach is not recommended.
- Hedonic regression models suffer from a lack of reproducibility in that different researchers will have different characteristics in the model and use different functional forms.
- From the discussion in section 11, it was seen that market rents can be considerably higher than the opportunity costs of home owners and hence using market rents to impute rents for owner occupiers may lead to rents that are too high.[87] On the other hand, if there are rent controls or a temporary glut of rental properties, then market rents could be too low compared to the opportunity costs of home owners.
- There is some evidence that depreciation is somewhat different for rental units compared to owner occupied housing units.[88] If this is so, then the imputation procedure will be somewhat incorrect. However, all studies that estimate depreciation for owner occupied housing suffer from biases due to the inadequate treatment of land and due to the lack of information on repair, renovation and maintenance expenditures over the life of the dwelling unit. Hence, it is not certain that depreciation for rental units is significantly different than that for owner occupied units.

13.3 The User Cost Approach

It is first necessary to decide whether an ex ante or ex post user cost of housing is to be calculated. It seems that the ex ante approach is the more useful one for CPI purposes; these are the prices that should appear in economic models of consumer choice. Moreover, the ex post approach will lead to user costs that fluctuate too much to suit the needs of most users. Of course, the problem with the ex ante approach is that it will be difficult to estimate anticipated inflation rates for house prices.

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[86] See Crone, Nakamura and Voith (2000) and Hoffmann and Kurz (2002) for example of such hedonic models that try to cope with the heterogeneity in the rental market.

[87] Again, it could argued that this is a mainly a weighting issue; i.e., it could be argued that the *trend* in market rents would be a reasonably accurate estimate for the trend in home owner's opportunity costs.

[88] "The average depreciation rate for rental property is remarkably constant, ranging from 0.58% to 0.60% over the 25 year period. Depreciation rates for owner occupied units show more variation than the estimated rates for renter occupied units. The average depreciation rate for owner occupied housing ranges from 0.9% in year 1 to 0.28% in year 20." Stephen Malpezzi, Larry Ozanne and Thomas G. Thibodeau (1987; 382).

Option 3: The Rent to Value Approach

In this option, the statistical agency collects information on market rents paid for a sample of rental properties but it also collects information on the sales price of these rental properties when they are sold. Using these two pieces of information, the statistical agency can form an estimated *rent to value ratio* for rental properties of various types. It can be seen that this rent to value ratio represents an estimate of all the terms that go into an ex ante user cost formula, except the asset price of the property; i.e., the rent to value ratio for a particular property can be regarded as an estimate of the interest rate less anticipated housing inflation plus the depreciation rate plus the other miscellaneous rates that were discussed in section 10, such as insurance and property tax rates. Under the assumption that these rates remain reasonably constant over the short run, changes in user costs are equal to changes in the price of owner occupied housing. Thus this approach can be implemented if a constant quality price index for the stock value of owner occupied housing can be developed. It may be decided to approximate the comprehensive price index for owner occupied housing by a new housing price index, and if this is done, the approach essentially reduces down to the acquisitions approach, except that the weights will generally be larger using this user cost approach than those obtained using the acquisitions approach.[89] Problems with this approach include:

- It will require considerable amount of resources to construct a constant quality price index for the stock of owner occupied housing units. If a hedonic regression model is used, there are problems associated with the reproducibility of the results.
- Rent to value ratios can change considerably over time. Hence it will be necessary to keep collecting information on rents and selling prices of rental properties on an ongoing basis.
- As was noted in section 11 above, the user cost structure of rental properties can be quite different from the corresponding user cost structure of owner occupied properties. Hence, the use of rent to value ratios can give misleading results.[90]

Option 4: The Simplified User Cost Approach

This approach is similar to that of Option 3 above but instead of using the rent to value ratio to estimate the sum of the various rates in the user cost formula, direct estimates are made of these rates. If the simplified Icelandic user cost approach discussed in section 8 is used, all that is required is a constant quality owner occupied housing price index, an estimated real interest rate and an estimated composite depreciation rate on the structure and land together. Problems with this approach are:

- As was the case with Option 3 above, it will require a considerable amount of resources to construct a constant quality price index for the stock of owner

[89] Recall the discussion in section 5 above.

[90] However, this is primarily a weighting issue so that the trend in the constant quality stock of owner occupied housing price index should be an adequate approximation to the trend in owner occupied user costs.

occupied housing units. If a hedonic regression model is used, there are problems associated with the reproducibility of the results.

- It is not known with any degree of certainty what the appropriate real interest rate should be.
- Similarly, it is difficult to determine what the "correct" depreciation rate should be.[91] Moreover, this problem is complicated by the fact that over time, the price of land tends to increase faster than the price of building a residential structure and so the land price component of an owner occupied housing unit will tend to increase in importance which in turn will tend to decrease the composite depreciation rate.

Option5: A National Accounting Approach

This approach makes use of the fact that the national accounts division of the statistical agency will usually collect data on investment in residential housing as well as on repair and renovation expenditures on housing. In addition, many statistical agencies will also construct estimates for the stock of residential dwelling units so that estimates for the structures depreciation rates are available. Finally, if the statistical agency also constructs a national balance sheet, then estimates for the value of residential land will also be available. Thus all of the basic ingredients that are necessary to construct stocks for residential structures and the associated land stocks are available. If in addition, assumptions about the appropriate nominal interest rate and about expected prices for structures and land are made[92], then aggregate user costs of residential structures and residential land can be constructed. The proportion of these stocks that is rented can be deducted and estimates for the user costs and corresponding values for owner occupied residential land and structures can be made. Of course, it would be almost impossible to do all of this on a current basis, but all of the above computations can be done for a base period in order to obtain appropriate weights for owner occupied structures and land. Then, it can be seen that the main drivers for the monthly user costs are the price of a new structure and the price of residential land. Hence if timely monthly indicators for these two prices can be developed, the entire procedure is feasible. Problems with this approach include:

- As was the case with Option 4 above, it will be difficult to determine what the "correct" depreciation rates and real interest rates are.[93]
- It will be difficult to construct a monthly price of residential land index.
- It may be difficult to convert the residential housing investment price deflator from a quarterly to a monthly basis.

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[91] Due to the lack of information on repairs and renovations, estimated housing depreciation rates vary widely: "One striking feature with the results of all three approaches used in these and related studies is their variability: estimates range from about a half percent per year to two and a half percent." Stephen Malpezzi, Larry Ozanne and Thomas G. Thibodeau (1987; 373-375).

[92] Alternatively, an appropriate real interest rate can be assumed.

[93] However, as usual, it can be argued that errors in estimating these parameters will mainly affect the weights used in the price index.

All of the above 5 options have their advantages and disadvantages; there does not appear to be clear "winning" option.[94] Thus each statistical agency will have to decide whether they have the resources to implement any of these five options in addition to the usual acquisitions approach to the treatment of owner occupied housing. From the viewpoint of the cost of living approach to the Consumer Price Index, any one of the 5 options would be an adequate approximation to the ideal treatment from the perspective of measuring the flow of consumption services in each period.

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[94] For consumer durables that do not change in quality over time, Option 5 will probably suffice.

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